# RESEARCH CONCERNING THE OPTIMIZATION OF A MECHANISM WITH TWO INDEPENDENT CONTOURS 

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#### Abstract

The paper presents a method for achieving dimensional optimization of a planar mechanism with two independent contours when certain values are imposed for some parameters that characterize the operation of the mechanism.


Keywords: optimization, mechanism, kinematics

## INTRODUCTION

Dimensional optimization of the mechanisms in order to meet certain functional conditions, leads to obtaining compact structures, optimal in terms of total weight and having a very precise operation [1]. The present paper presents a method for achieving dimensional optimization of a planar mechanism with two independent contours (Figure 1) when certain values are imposed for the speed and the stroke of the component piston 5.


Figure 1 Planar mechanism with two independent contours

## THEORETICAL CONSIDERATIONS AND SIMULATION RESULTS

Figure 2 represents the graph associated $[2,3]$ with the analyzed mechanism. The graph highlights the two independent contours from the mechanism component: 0-1-2-3-0 and 0-1-2-4-5-0.


Figure 2 The graph associated with the analyzed mechanism
By projecting the vector equation: $\overline{O A}+\overline{A B}+\overline{B O}=0$ (Figure 1) corresponding to the first independent contour $0-1-2-3-0$ on the $x$ and $y$ axes, the following system of equations has been obtained:

$$
\left\{\begin{array}{l}
l_{1} \cdot \cos \varphi_{1}+l_{2} \cdot \cos \varphi_{2}-s_{3}=0  \tag{1}\\
l_{1} \cdot \sin \varphi_{1}+l_{2} \cdot \sin \varphi_{2}=0
\end{array}\right.
$$

where: $l_{1}=O A$ and $l_{2}=A B$.
Then, by projecting the vector equation: $\overline{O A}+\overline{A C}+\overline{C D}+\overline{D O}=0$ corresponding to the second independent contour $0-1-2-4-5-0$ on the $x$ and $y$ axes, the following system of equations has been obtained:
$\left\{\begin{array}{l}l_{1} \cdot \cos \varphi_{1}+l_{2 p} \cdot \cos \varphi_{2}+l_{4} \cdot \cos \varphi_{4}-x_{D}=0 \\ l_{1} \cdot \sin \varphi_{1}+l_{2 p} \cdot \sin \varphi_{2}+l_{4} \cdot \sin \varphi_{4}-s_{5}=0\end{array}\right.$
where: $l_{2 p}=A C$ and $l_{4}=C D$.
By solving these systems of equations, the angles $\varphi_{2}$ and $\varphi_{4}$ and the displacements $s_{3}$ and $s_{5}$ may be calculated from the following relations:

$$
\begin{align*}
& \sin \varphi_{2}=-\frac{l_{1}}{l_{2}} \cdot \sin \varphi_{1}  \tag{3}\\
& s_{3}=l_{1} \cdot \cos \varphi_{1}+l_{2} \cdot \cos \varphi_{2}  \tag{4}\\
& \cos \varphi_{4}=-\frac{1}{l_{4}}\left(l_{1} \cdot \cos \varphi_{1}+l_{2 p} \cdot \cos \varphi_{2}-x_{D}\right) \tag{5}
\end{align*}
$$

$s_{5}=l_{1} \cdot \sin \varphi_{1}+l_{2 p} \cdot \sin \varphi_{2}+l_{4} \cdot \sin \varphi_{4}$
The angular speeds $\omega_{j}, j=2,4$ and the angular acceleration $\varepsilon_{j}, j=2,4$ are calculated with the relations:

$$
\begin{align*}
& \omega_{j}=\phi_{J}=\frac{\mathrm{d} \varphi_{j}}{\mathrm{~d} \varphi_{1}} \cdot \frac{\mathrm{~d} \varphi_{1}}{\mathrm{~d} t}=\omega_{1} \cdot \frac{\mathrm{~d} \varphi_{j}}{\mathrm{~d} \varphi_{1}} ; j=2,4  \tag{7}\\
& \varepsilon_{j}=c \&_{j}=\varepsilon_{1} \cdot \frac{\mathrm{~d} \varphi_{j}}{\mathrm{~d} \varphi_{1}}+\frac{\mathrm{d}^{2} \varphi_{j}}{\mathrm{~d} \varphi_{1}^{2}} \cdot \omega_{1}^{2} ; j=2,4 \tag{8}
\end{align*}
$$

where: $\omega_{1}$ is the angular speed of the crank and $\varepsilon_{1}$ is its angular acceleration.
The speeds and the accelerations of the two component pistons 3 and 5 may be calculated by deriving, in relation to time, their displacements $s_{3}$ and $s_{5}$ :

$$
\begin{align*}
& v_{j}=\&=\frac{\mathrm{d} s_{j}}{\mathrm{~d} \varphi_{1}} \cdot \frac{\mathrm{~d} \varphi_{1}}{\mathrm{~d} t}=\omega_{1} \cdot \frac{\mathrm{~d} s_{j}}{\mathrm{~d} \varphi_{1}} ; j=3,5  \tag{9}\\
& a_{j}=\Leftrightarrow=\varepsilon_{1} \cdot \frac{\mathrm{~d} s_{j}}{\mathrm{~d} \varphi_{1}}+\frac{\mathrm{d}^{2} s_{j}}{\mathrm{~d} \varphi_{1}^{2}} \cdot \omega_{1}^{2} ; j=3,5 \tag{10}
\end{align*}
$$

A computer program that simulates the functioning of the mechanism has been realized using Maple programming environment. It was considered that the elements of the mechanism are made of aluminium, the elements 1,2 and 4 being made of a bar with a round section, having a radius of 0.015 m . The mass of the pistons 3 and 5 is of 1.5 kg . The angular speed of the crank $\omega_{1}$ is equal to $10 \mathrm{rad} / \mathrm{s}$.

The dimensional optimization of the mechanism has been accomplished using the function NLPSolve included in the Optimization package from Maple. It has been considered the following initial dimensions of the component elements of the mechanism: $l_{1}=0.1 \mathrm{~m} ; l_{2}=0.8 \mathrm{~m} ; l_{2 p}=0.5 \mathrm{~m}$ and $l_{4}=0.5 \mathrm{~m}$. For the coordinate $x_{D}$, the value 0.8 m has been considered.

First, we aimed to establish the dimensions of the component elements and the coordinate $x_{D}$ to ensure a minimum value for the total mass of the mechanism, given that the speed of the piston 5 is zero for $\varphi_{1}=60^{\circ}$ and $\varphi_{1}=220^{\circ}$. In the application of the function NLPSolve it was considered that the dimensions of the component elements and the value of the coordinate $x_{D}$ vary between 0.8 and 1.2 of their initial values.

Following the optimization, the following values were obtained: $l_{1}=0.08 \mathrm{~m}$; $l_{2}=0.761 \mathrm{~m} ; l_{2 p}=0.515 \mathrm{~m}, l_{4}=0.495 \mathrm{~m}$ and $x_{D}=0.64 \mathrm{~m}$.

Figures 3 and 4 represent the variation curves on a cinematic cycle, beginning with $\varphi_{1}=60^{\circ}$, of the speed and acceleration of piston 5 .

Then, we established the optimal dimensions of the component elements and the optimal coordinate $x_{D}$ to ensure a minimum value for the total mass of the mechanism, given that the speed of piston 5 is zero for $\varphi_{1}=60^{\circ}$ and $\varphi_{1}=220^{\circ}$ and its stroke is equal
to 0.15 m . Also, in this case it was considered that the dimensions of the component elements and the value of the coordinate $x_{D}$ vary between 0.8 and 1.2 of their initial values.


Figure 3 The variation on a cinematic cycle, beginning with $\varphi_{1}=60^{\circ}$, of the speed of piston 5


Figure 4 The variation on a cinematic cycle, beginning with $\varphi_{1}=60^{\circ}$, of the acceleration of piston 5

Following the optimization, the following values were obtained: $l_{1}=0.12 \mathrm{~m}$; $l_{2}=0.83 \mathrm{~m} ; l_{2 p}=0.435 \mathrm{~m}, l_{4}=0.578 \mathrm{~m}$ and $x_{D}=0.64 \mathrm{~m}$.

Figures 5 and 6 represent the variation of the speed and acceleration of piston 5 on a cinematic cycle in this case, beginning with $\varphi_{1}=60^{\circ}$.


Figure 5 The variation on a cinematic cycle, beginning with $\varphi_{1}=60^{\circ}$, of the speed of piston 5 when its stroke is 0.15 m


Figure 6 The variation on a cinematic cycle, beginning with $\varphi_{1}=60^{\circ}$,
of the acceleration of piston 5 when its stroke is 0.15 m
Also, the present paper analyses the optimization case to ensure a minimum value for the total mass of the mechanism, given that the speed of piston 5 is zero for $\varphi_{1}=60^{\circ}$ and $\varphi_{1}=220^{\circ}$ and its stroke is equal to 0.2 m . In this case, it was considered that the dimensions of the component elements and the value of the coordinate $x_{D}$ vary between 0.7 and 1.3 of their initial values.

The following values were obtained after optimization: $l_{1}=0.13 \mathrm{~m} ; l_{2}=0.852 \mathrm{~m}$; $l_{2 p}=0.345 \mathrm{~m}, l_{4}=0.579 \mathrm{~m}$ and $x_{D}=0.593 \mathrm{~m}$.

Figures 7 and 8 represent the variation of the speed and acceleration of piston 5 on a cinematic cycle in this case, beginning with $\varphi_{1}=60^{\circ}$.

Finally, we analyzed the optimization case to ensure a minimum value for the total mass of the mechanism, given that the speed of piston 5 is zero for $\varphi_{1}=60^{\circ}$ and $\varphi_{1}=220^{\circ}$ and its stroke is equal to the stroke of piston 3 and has the value of 0.3 m . In this case it was considered that the dimensions of the component elements and the value of the coordinate $x_{D}$ vary between 0.5 and 1.5 of their initial values.


Figure 7 The variation on a cinematic cycle, beginning with $\varphi_{1}=60^{\circ}$, of the speed of piston 5 when its stroke is 0.2 m


Figure 8 The variation on a cinematic cycle, beginning with $\varphi_{1}=60^{\circ}$, of the acceleration of piston 5 when its stroke is 0.2 m

The following values were obtained after optimization: $l_{1}=0.15 \mathrm{~m} ; l_{2}=1.098 \mathrm{~m}$; $l_{2 p}=0.25 \mathrm{~m}, l_{4}=0.663 \mathrm{~m}$ and $x_{D}=0.589 \mathrm{~m}$.

Figures 9 and 10 represent the variation of the speed and acceleration of piston 5 on a cinematic cycle in this last case, beginning with $\varphi_{1}=60^{\circ}$.


Figure 9 The variation on a cinematic cycle, beginning with $\varphi_{1}=60^{\circ}$, of the speed of piston 5 when its stroke is 0.3 m (equal to the stroke of the piston 3)


Figure 10 The variation on a cinematic cycle, beginning with $\varphi_{1}=60^{\circ}$, of the acceleration of piston 5 when its stroke is 0.3 m (equal to the stroke of the piston 3)

## CONCLUSIONS

The paper has presented a method for achieving dimensional optimization of a planar mechanism with two independent contours when certain values are imposed for the speed and the stroke of a component piston. The use of the NLP Solve optimization function proved to be easy to implement in the computer program and allowed to obtain the results in a fast and precise way.

## REFERENCES

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