RESEARCH CONCERNING THE DYNAMIC ANALYSIS OF A CRANK AND CONNECTING ROD MECHANISM

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ABSTRACT

The paper analyzes the influence of some parameters regarding the configuration and the operation of the crank and connecting rod mechanism on the variation of the motor torque and on the values of the connecting forces in the joints.

Keywords*:* dynamics, connecting forces, motor torque

INTRODUCTION

Generally, the dynamic study of the mechanisms through the obtained results and the expected possibilities of functional optimization remains a very topical field in mechanical engineering. The present paper analyzes the influence of some dimensional and operational parameters of the crank and connecting rod mechanism (Figure 1) on the variation of the motor torque and on the values of the connecting forces in the joints.

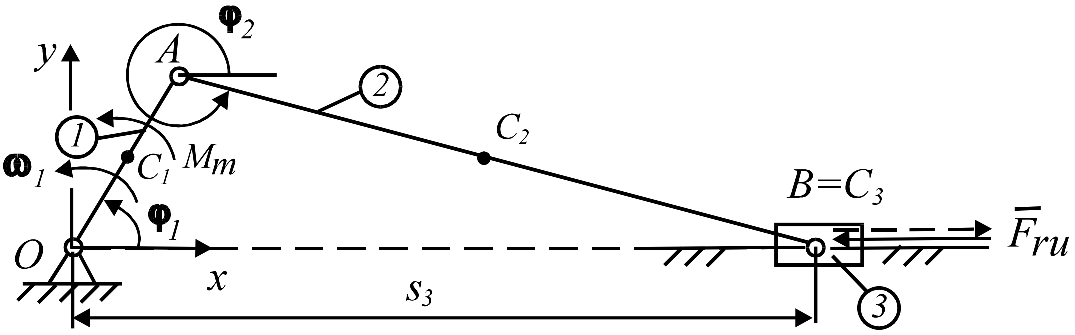


Figure 1 Crank and connecting rod mechanism

THEORETICAL CONSIDERATIONS

Figure 2 shows the loads on each of the component elements of the crank and connecting rod mechanism: M_m is the motor torque acting on the crank; $\overline{F}_{ij} = -m_j \cdot \overline{a}_{Cj};\, j = \overline{1,3}$, are the inertia forces, $m_j,\, j = \overline{1,3}$, are the masses of the component elements and $\overline{a}_{Cj};\, j = \overline{1,3}$, are the accelerations of the mass centres;

$\overline{M}_{i2} = -J_{C2} \cdot \overline{\varepsilon}_2$ is the inertia moment acting on the connecting rod, J_{C2} is the mass moment of inertia of the connecting rod and $\overline{\varepsilon}_2$ is its angular acceleration; $\overline{G}_j$, $j = \overline{1,3}$, are the weights of the elements of the mechanism; $\overline{F}_{01}, \overline{F}_{21}, \overline{F}_{32}$ and $\overline{F}_{03}$ are the connecting forces in the component joints; $F_{ru} = F_r \cdot \sin\varphi_1$ is the technological resistance force acting on the piston *3* and φ_1 is the crank angle.

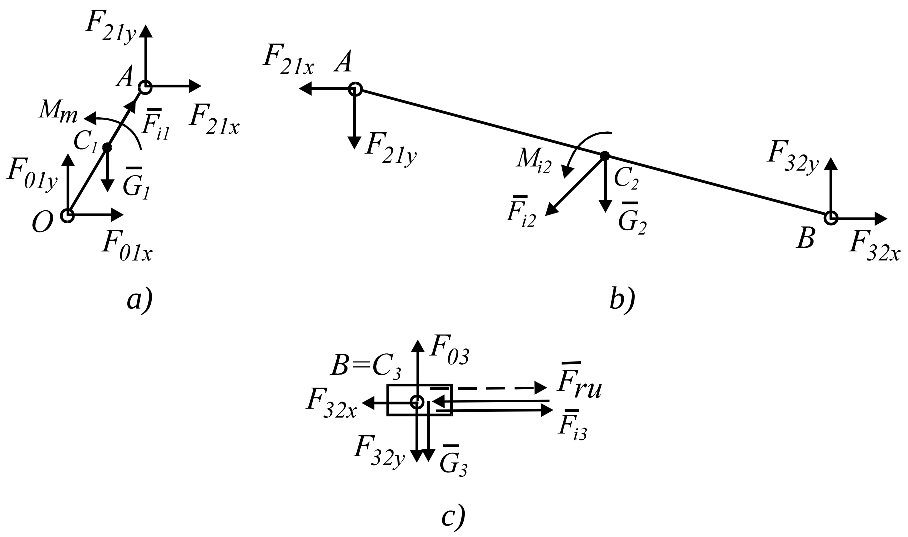


Figure 2 The loads on the component elements of the crank and connecting rod mechanism

The equations of the dynamic equilibrium [1] corresponding to the three elements of the crank and connecting rod mechanism are the following:

- the equations for the crank (Figure 2,*a*):

$$\begin{cases} F_{01x} + F_{21x} + F_{i1x} = 0 \\ F_{01y} + F_{21y} - G_1 + F_{i1y} = 0 \\ M_m + x_A \cdot F_{21y} - y_A \cdot F_{21x} - x_{C1} \cdot G_1 = 0 \end{cases} \tag{1}$$

- the equations for the connecting rod (Figure 2,*b*):

$$\begin{cases} -F_{21x} + F_{32x} + F_{i2x} = 0 \\ -F_{21y} + F_{32y} - G_2 + F_{i2y} = 0 \\ (x_B - x_A) \cdot F_{32y} - (y_B - y_A) \cdot F_{32x} + M_{i2} + (x_{C2} - x_A) \cdot F_{i2y} - \\ -(y_{C2} - y_A) \cdot F_{i2x} - (x_{C2} - x_A) \cdot G_2 = 0 \end{cases} \tag{2}$$

- the equations for the piston (Figure 2,*c*):

$$\begin{cases} -F_{32x}+F_{i3}+F_r\cdot\sin\varphi_1=0 \\ -F_{32y}+F_{03}-G_3=0 \end{cases} \tag{3}$$

The dynamic equilibrium equations 1÷3 may be arranged in the following matrix form:

$$A\cdot X=B \tag{4}$$

where:

$$X=\begin{bmatrix} F_{01x} & F_{01y} & F_{21x} & F_{21y} & F_{32x} & F_{32y} & F_{03} & M_m \end{bmatrix}^T \tag{5}$$

$$A=\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -y_A & x_A & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -(y_B-y_A) & x_B-x_A & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \tag{6}$$

$$B=\begin{bmatrix} -F_{i1x} \\ G_1-F_{i1y} \\ x_{C1}\cdot G_1 \\ -F_{i2x} \\ G_2-F_{i2y} \\ -M_{i2}-(x_{C2}-x_A)\cdot F_{i2y}+(y_{C2}-y_A)\cdot F_{i2x}+(x_{C2}-x_A)\cdot G_2 \\ -F_{i3}-F_r\cdot\sin\varphi_1 \\ G_3 \end{bmatrix} \tag{7}$$

Vector X can be calculated from the equation (4); it contains the motor torque and the projections on the x and y axes of the connecting forces in the joints.

The cinematic analysis of the crank and connecting rod mechanism has been accomplished using the method of projecting the closed and independent vector contours [2, 3]. By projecting the vector equation: $\overline{OA}+\overline{AB}+\overline{BO}=0$ (Figure 1) on the x and y axes, the following system of equations has been obtained:

$$\begin{cases} l_1\cdot\cos\varphi_1+l_2\cdot\cos\varphi_2-s_3=0 \\ l_1\cdot\sin\varphi_1+l_2\cdot\sin\varphi_2=0 \end{cases} \tag{8}$$

where: $l_1=OA$ and $l_2=AB$.

By solving this system of equations, the angle φ_2 and the displacement s_3 may be calculated with the following relations:

$$\sin\varphi_2 = -\frac{l_1}{l_2}\cdot\sin\varphi_1 \tag{9}$$

$$s_3 = l_1\cdot\cos\varphi_1 + l_2\cdot\cos\varphi_2 \tag{10}$$

Then, the coordinates of the points that appear in the dynamic equilibrium equations 1÷3 may be calculated with the relations:

$$\begin{cases} x_A = l_1\cdot\cos\varphi_1;\ y_A = l_1\cdot\sin\varphi_1 \\ x_B = x_{C3} = l_1\cdot\cos\varphi_1 + l_2\cdot\cos\varphi_2;\ y_B = y_{C3} = 0 \\ x_{C1} = OC_1\cdot\cos\varphi_1;\ y_{C1} = OC_1\cdot\sin\varphi_1 \\ x_{C2} = OA\cdot\cos\varphi_1 + AC_2\cdot\cos\varphi_2; y_{C2} = OA\cdot\sin\varphi_1 + AC_2\cdot\sin\varphi_2 \end{cases} \tag{11}$$

The angular speed ω_2 and the angular acceleration ε_2 are calculated with the relations:

$$\omega_2 = \dot{\varphi}_2 = \frac{d\varphi_2}{d\varphi_1}\cdot\frac{d\varphi_1}{dt} = \omega_1\cdot\frac{d\varphi_2}{d\varphi_1} \tag{12}$$

$$\varepsilon_2 = \dot{\omega}_2 = \varepsilon_1\cdot\frac{d\varphi_2}{d\varphi_1} + \frac{d^2\varphi_2}{d\varphi_1^2}\cdot\omega_1^2 \tag{13}$$

where: ω_1 is the angular speed of the crank and ε_1 is its angular acceleration.

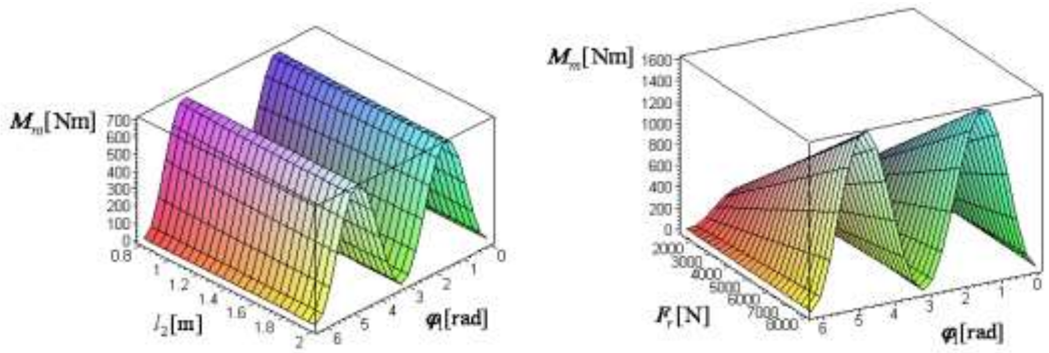
The speeds and the accelerations of the mass centers $C_j, j = \overline{1,3}$, may be calculated by deriving, in relation to time, the coordinates of these points:

$$\begin{cases} v_{Cjx} = \dot{x}_{Cj} = \dfrac{dx_{Cj}}{d\varphi_1}\cdot\dfrac{d\varphi_1}{dt} = \omega_1\cdot\dfrac{dx_{Cj}}{d\varphi_1} \\ v_{Cjy} = \dot{y}_{Cj} = \dfrac{dy_{Cj}}{d\varphi_1}\cdot\dfrac{d\varphi_1}{dt} = \omega_1\cdot\dfrac{dy_{Cj}}{d\varphi_1} \end{cases} \quad j = \overline{1,3} \tag{14}$$

$$\begin{cases} a_{Cjx} = \varepsilon_1\cdot\dfrac{dx_{Cj}}{d\varphi_1} + \dfrac{d^2x_{Cj}}{d\varphi_1^2}\cdot\omega_1^2 \\ a_{Cjy} = \varepsilon_1\cdot\dfrac{dy_{Cj}}{d\varphi_1} + \dfrac{d^2y_{Cj}}{d\varphi_1^2}\cdot\omega_1^2 \end{cases} \quad j = \overline{1,3} \tag{15}$$

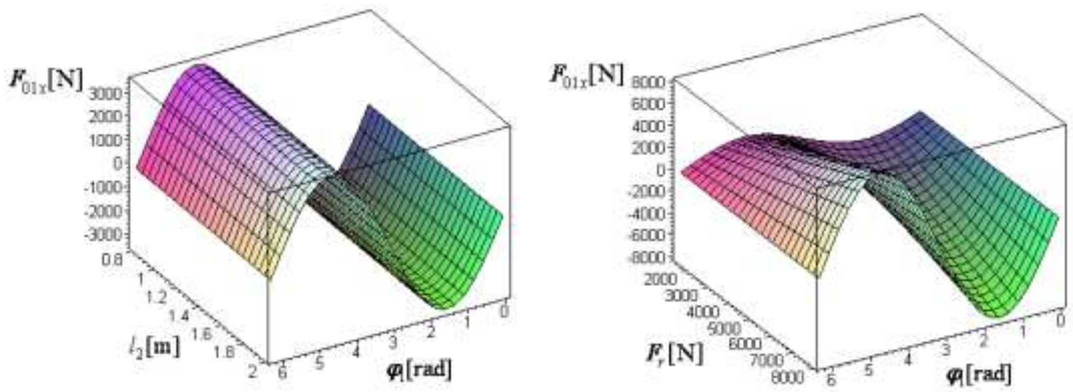
SIMULATION RESULTS

The relations above have been transposed into a computer program using Maple programming environment. It was considered that the elements of the crank and connecting rod mechanism are made of steel, the connecting rod and the crank being made of a bar with a round section, having a radius of 0.015 m. The mass of the piston is of 2 kg. The angular speed of the crank ω_1 is equal to 10 rad/s and the length l_1 of the crank is equal to 0.2 m. Figure 3,*a* represents the variation on a cinematic cycle of the motor torque when $F_r = 3500\mathrm{N}$ and the length l_2 of the connecting rod varies between 0.8 m and 2 m. Figure 3,*b* represents the variation on a cinematic cycle of the motor torque when $l_2 = 1.1\mathrm{m}$ and F_r varies between 1500 N and 8000 N. Figures 4÷10 represent the variations on a cinematic cycle of the projections on the *x* and *y* axes of the connecting forces $\overline{F}_{01}, \overline{F}_{21}, \overline{F}_{32}$ and $\overline{F}_{03}$, when the length l_2 of the connecting rod varies between 0.8 m and 2 m and $F_r = 3500\mathrm{N}$, and when F_r varies between 1500 N and 8000 N and $l_2 = 1.1\mathrm{m}$, respectively.



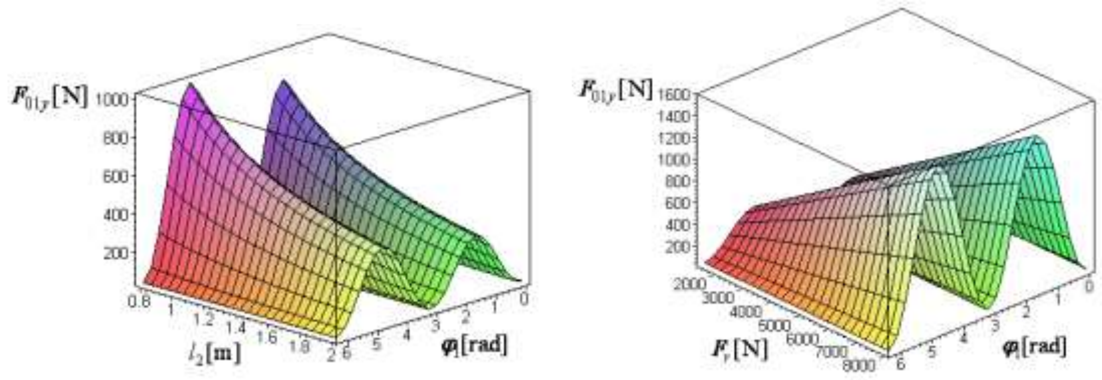
a) *b)*

Figure 3 The variation on a cinematic cycle of the motor torque M_m (a - when $F_r = 3500\mathrm{N}$ and the length l_2 varies between 0.8 m and 2 m; b – when $l_2 = 1.1\mathrm{m}$ and F_r varies between 1500 N and 8000 N)



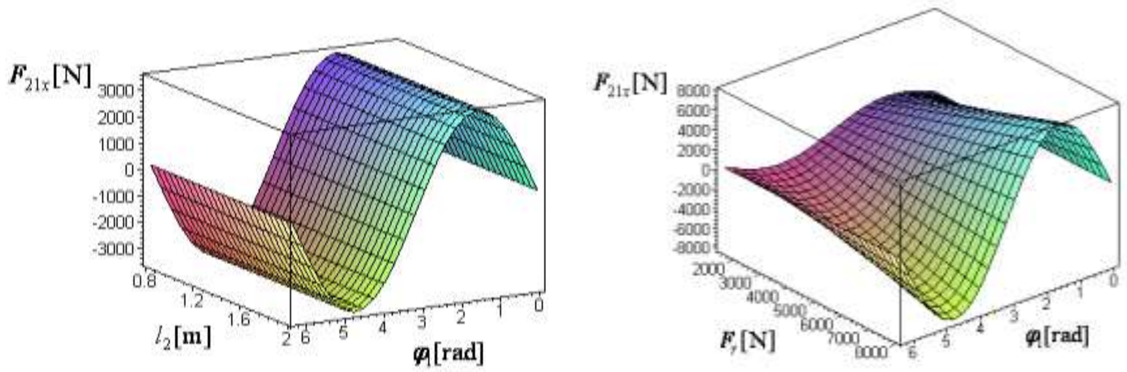
a) *b)*

Figure 4 The variation on a cinematic cycle of F_{01x} (a - when $F_r = 3500\mathrm{N}$ and the length l_2 varies between 0.8 m and 2 m; b – when $l_2 = 1.1\mathrm{m}$ and F_r varies between 1500 N and 8000 N)



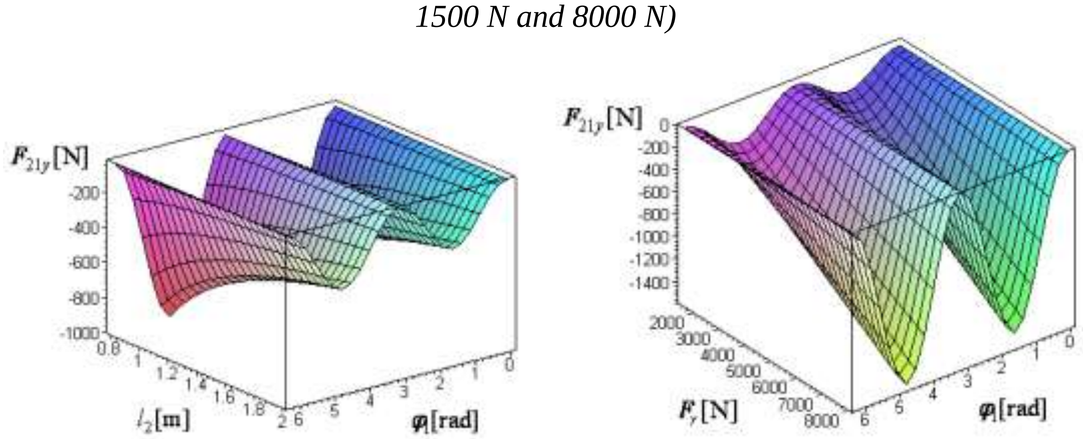
a) *b)*

Figure 5 The variation on a cinematic cycle of F_{01y} *(a - when* $F_r = 3500\text{N}$ *and the length* l_2 *varies between 0.8 m and 2 m; b – when* $l_2 = 1.1\text{m}$ *and* F_r *varies between 1500 N and 8000 N)*



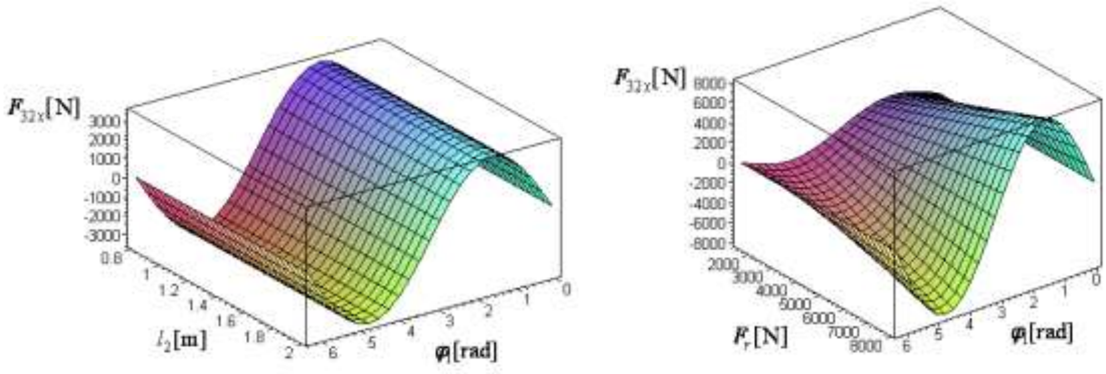
a) *b)*

Figure 6 The variation on a cinematic cycle of F_{21x} *(a - when* $F_r = 3500\text{N}$ *and the length* l_2 *varies between 0.8 m and 2 m; b – when* $l_2 = 1.1\text{m}$ *and* F_r *varies between 1500 N and 8000 N)*



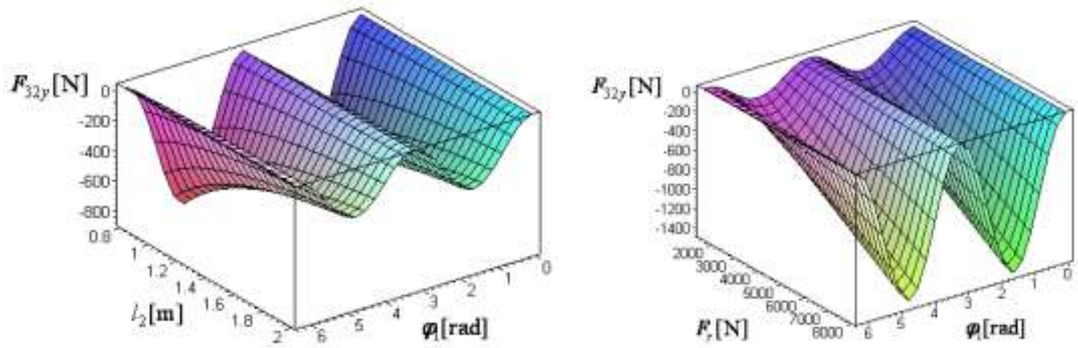
a) *b)*

Figure 7 The variation on a cinematic cycle of F_{21y} *(a - when* $F_r = 3500\text{N}$ *and the length* l_2 *varies between 0.8 m and 2 m; b – when* $l_2 = 1.1\text{m}$ *and* F_r *varies between 1500 N and 8000 N)*



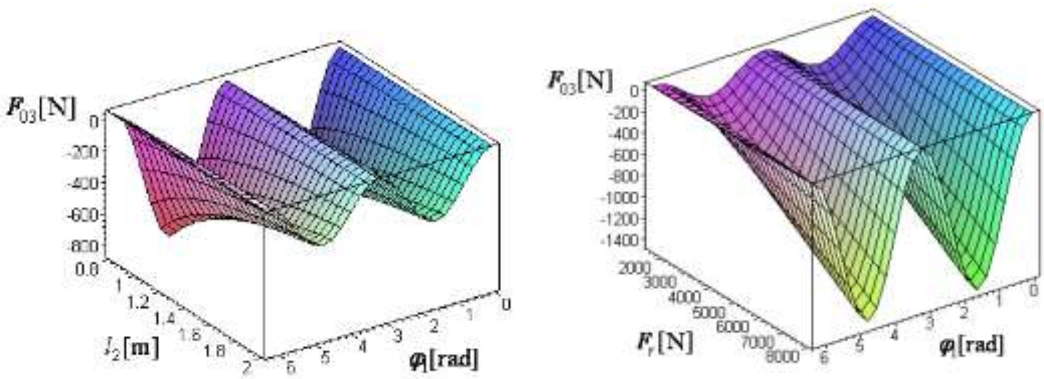
a) *b)*

Figure 8 The variation on a cinematic cycle of F_{32x} *(a - when* $F_r = 3500\text{N}$ *and the length* l_2 *varies between 0.8 m and 2 m; b – when* $l_2 = 1.1\text{m}$ *and* F_r *varies between 1500 N and 8000 N)*



a) *b)*

Figure 9 The variation on a cinematic cycle of F_{32y} *(a - when* $F_r = 3500\text{N}$ *and the length* l_2 *varies between 0.8 m and 2 m; b – when* $l_2 = 1.1\text{m}$ *and* F_r *varies between 1500 N and 8000 N)*



a) *b)*

Figure 10 The variation on a cinematic cycle of F_{03} *(a - when* $F_r = 3500\text{N}$ *and the length* l_2 *varies between 0.8 m and 2 m; b – when* $l_2 = 1.1\text{m}$ *and* F_r *varies between 1500 N and 8000 N)*

CONCLUSIONS

The paper analyzed the dynamics of the crank and connecting rod mechanism. It has been studied the influence of the variation of the length of the connecting rod and of the value of the technological resistance force on the variation of the motor torque and on the values of the connecting forces in the joints. The obtained results are useful for an optimal dimensioning of the mechanism for different loads to which it is subjected.

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