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NUMERICAL WELLBORE STABILITY FOR VERTICAL AND DEVIATED WELLS USING FINITE ELEMENT METHOD

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ABSTRACT

A 3D vertical wellbore model is created in this paper using the ANSYS finite-element software package to examine stress distribution contours and displacement plots, and then developing a comprehensive approach for converting the vertical model into any orientated and 3D deviated wellbore model. Applying isotropic rock elasticity and the Von Mises formulas, both wellbore models can be utilized repeatedly to explore the stability, stress distribution, and deformation surrounding wellbores in any direction. Changing the direction and magnitude of the insitu normal and shear stress acting on the vertical wellbore model is also done to show the effect of inclination on wellbore stability.

Keywords: Numerical simulation, Ansys FEM, stress and deformation, wellbore problems.

INTRODUCTION

A wellbore stability considers one of the main issues regarding drilling operations. It mainly causes non-productive time during operations of drilling wells and globally costs the petroleum industry billions of dollars yearly. Before drilling a well, subsurface geological formations are fully under conditions of well-balanced stresses with three insitu principal stresses known as vertical, maximum horizontal and minimum horizontal stress. These various stresses also produce shear stresses on planes within the formation mass [1]. However, the equilibrium state of the well-balanced rock will varies after cylindrical wellbores of rocks are cut during drilling phases and substituted by drilling fluids at particular mud weights so as to support temporary wellbore wall. If there is no good support for the wellbore, the rock surrounding its wall may become unstable and yield. The drilling fluid can partially support the only normal stresses on the wellbore wall; however it can't the same for shear stresses, as the original rock does. This leads to stress concentration and redistribution around the borehole wall that cause failures [1]. In order to drill a stabilized circular hole in oil and gas fields with applying the appropriate drilling and completion fluids, analytical and numerical wellbore stability models have used to avoid sources of wellbore failures and casing collapse problems.

Recently, the extensive use sophisticated software and computational finite-elements has increased to create wellbore stability models. These techniques combine the mathematical of solid mechanics such as elasticity theory, plasticity theory, poroelasticity theory and viscoelasticity theory, with various failure criteria and borehole elements to develop more realistic and sophisticated different wellbore models [2]. The FEM may display linear or non-linear behavior of rock. Furthermore, it can be tested and adjusted utilizing experimental data to provide realistic visual illustration and investigation about what is really occurring surrounding the wellbore wall. The ability of FEM software to make stress distribution contours and displacement plots with incremental elements throughout the drilling operation allows the visualization of the wellbore stability analysis. Over the past twenty years, the finite-element software package ABAQUS (SIMULIA) [3] has rapidly developed robust non-linear FEM techniques which are appropriate to analyze soil mechanics, complex geomechanics, and rock mechanics. Moreover, it has successfully been utilized to do FE analysis for casing integrity of horizontal wells, sand production, cement sheath integrity, hydraulic fracturing and several applications [3-7]. However, it has been appeared another software called ANSYS finite-element software package which is easier, advanced, and has more applications of the entire drilling process [8-11].

Therefore, the aim of this paper is to simulate the actual stresses and deformation numerically around wellbore using isotropic elasticity of rock behavior and Von Mises equivalent equations in order to describe the actual cases of wellbore instability problems. Description of numerical software is presented. Governing equations are also presented. Further, the experimental data and local insitu stress, which used in the study, are discussed.

1. ANSYS STATIC STRUCTURE

Ansys structure is a computational static materials analysis software which is a part of Ansys, a software company based in the United States. Ansys static Software have a very high capability in terms of modelling which are needed to model any type of flow, heat transfer, material loads, turbulence and many reactions in industrial applications such as combustion, material failure, aircraft wing airflow, oil columns and many others [8-11]. Nowadays, most of the companies uses Ansys as an essential part in designing and optimizing systems. Ansys provides precise failure results in a relatively quick time which can be relied on when a model is developed properly. In static structure, it makes part of solid mechanics which combines data structure and numerical analysis approach to solve problems related to wellbore stability. There are several methods used in simulating yield of rocks which takes different approaches solving the desired problem which sometimes can be very complex [8-11]. The method which lays behind Ansys static if called “Finite Element Method (FEM)”.

1.1. Governing Equations

As stated earlier, the Ansys static structure software uses the finite volume method. In this method there are several equations which are involved the desired problem by the modelled physical phenomenon. For wellbore failure problem, Ansys static structure processes and solves state of stresses around a wellbore equations for isotropic elastic material behavior which can be defined by Kirsch [12-15] as follows:

I. Cylindrical Stresses in three dimensions

$$\sigma_r = \frac{1}{2}(\sigma_x + \sigma_y)\left(1 - \frac{R_w^2}{r^2}\right) + \frac{1}{2}(\sigma_x - \sigma_y)\left(1 + 3\frac{R_w^4}{r^4} - 4\frac{R_w^2}{r^2}\right)\cos 2\theta + \tau_{xy}\left(1 + 33\frac{R_w^4}{r^4} - 4\frac{R_w^2}{r^2}\right)\sin 2\theta + \frac{R_w^2}{r^2} + \Delta P_w \frac{R_w^2}{r^2} \quad (1)$$

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y)\left(1 + \frac{R_w^2}{r^2}\right) - \frac{1}{2}(\sigma_x - \sigma_y)\left(1 + 3\frac{R_w^4}{r^4}\right)\cos 2\theta - \tau_{xy}\left(1 + 3\frac{R_w^4}{r^4}\right)\sin 2\theta + \Delta P_w \frac{R_w^2}{r^2} \quad (2)$$

$$\sigma_z = \sigma_{zz} - 2\nu(\sigma_x - \sigma_y)\frac{R_w^2}{r^2}\cos 2\theta - 4\nu\,\tau_{xy}\frac{R_w^2}{r^2}\sin 2\theta \quad (3)$$

$$\tau_{r\theta} = \left\{\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta\right\}\left(1 - 3\frac{R_w^4}{r^4} + 2\frac{R_w^2}{r^2}\right) \quad (4)$$

$$\tau_{rz} = \{\tau_{xz}\cos\theta + \tau_{yz}\sin\theta\}\left(1 - \frac{R_w^2}{r^2}\right) \quad (5)$$

$$\tau_{rz} = \{-\tau_{xz}\cos\theta + \tau_{yz}\sin\theta\}\left(1 + \frac{R_w^2}{r^2}\right) \quad (6)$$

However, these equations are reduced at the borehole wall ($r=R_w$) as follows:

Radial stress: $\sigma_r = \Delta P_w$ (7)

Tangential stress: $\sigma_\theta = \sigma_x + \sigma_y - \Delta P_w - 2(\sigma_x - \sigma_y)\cos(2\theta) - 4\tau_{xy}\sin(2\theta)$ (8)

Axial stress, plane strain: $\sigma_z = \sigma_{zz} - 2\upsilon(\sigma_x - \sigma_y)\cos(2\theta) - 4\upsilon\tau_{xy}\sin(2\theta)$ (9)

Axial stress, plane stress: $\sigma_z = \sigma_{zz}$ (10)

Shear stress: $\sigma_{\theta z} = 2(\tau_{yz}\cos\theta - \tau_{yz}\sin\theta)$, $\tau_{rz} = \tau_{r\theta}$ (11)

II. Cartesian Stresses in three dimensions

In the petroleum fields, the vertical or overburden stress (σ_v), as well as the maximum and minimum horizontal stresses (σ_H & σ_h) are the main in-situ principle stresses. Because of the Kirsch equations' principles that assume the horizontal, vertical, and the wellbore deviation may be in any direction, these stresses should therefore be converted into Cartesian system x, y and z and symbolized as stresses σ_x, σ_y and σ_z. Equations 12 through 17 represent all converted stress items:

$$\sigma_x = (\sigma_H \cos^2\varphi + \sigma_h \sin^2\varphi)\cos^2\gamma + \sigma_v \sin^2\gamma \quad (12)$$

$$\sigma_y = (\sigma_H \sin^2\varphi + \sigma_h \cos^2\varphi) \quad (13)$$

$$\sigma_{zz} = (\sigma_H \cos^2\varphi + \sigma_h \sin^2\varphi)\sin^2\gamma + \sigma_v \cos^2\gamma \quad (14)$$

$$\tau_{yz} = \frac{1}{2}(\sigma_h - \sigma_H)\sin(2\varphi)\sin\gamma \quad (15)$$

$$\tau_{xz} = \frac{1}{2}(\sigma_H \cos^2\varphi + \sigma_h \sin^2\varphi - \sigma_v)\sin(2\gamma) \quad (16)$$

$$\tau_{xy} = \frac{1}{2}(\sigma_h - \sigma_H)\sin(2\varphi)\cos\gamma \quad (17)$$

Where

σ_v = effective in-situ stress in vertical direction

σ_H = effective max. in-situ stress in horizontal direction

σ_h = effective min. in-situ stress in horizontal direction

ν= rock Poisson's ratio

φ = angle between the drill axis and its projection onto the horizontal plane

γ = angle between the borehole axis and the vertical direction

θ= polar angle in the borehole cylindrical coordinate system

Pw= pressure difference between mud pressure and pore pressure in the borehole

$$\sqrt{J_2} = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}{6}} = \frac{C_0}{3} \quad (18)$$

Where J_2= The invariant of the deviatoric stress

All equations required to analyze failures of boreholes are now defined. Von Mises failure mode (Equation 18) is involved in FEM for calculating stress distribution and deformation in Ansys static material, however, the isotropic elasticity case and many other failure models can be integrated depending on the physics used in approaching the numerical simulation.

2. NUMERICAL SIMULATION METHODOLOGY

In this part, the methodology of approaching the numerical simulation will be explained step by step. A computational material stresses study done with a computer aided software Ansys static structure 2020 educational version is used to study the stress distribution effect of a proposed wellbore. The wellbore geometry is designed in Ansys and connected to Ansys static structural model for numerical simulation. The designed geometry of the wellbore was cut into half for meshing purposes, and the geometry is designed in way that only forces domain is projected, as the main interest of this numerical simulation is to investigate the stress domain. Figure 1 illustrates the flowchart of the steps undertaken in order to approach the desired problem and have an effective numerical simulation.

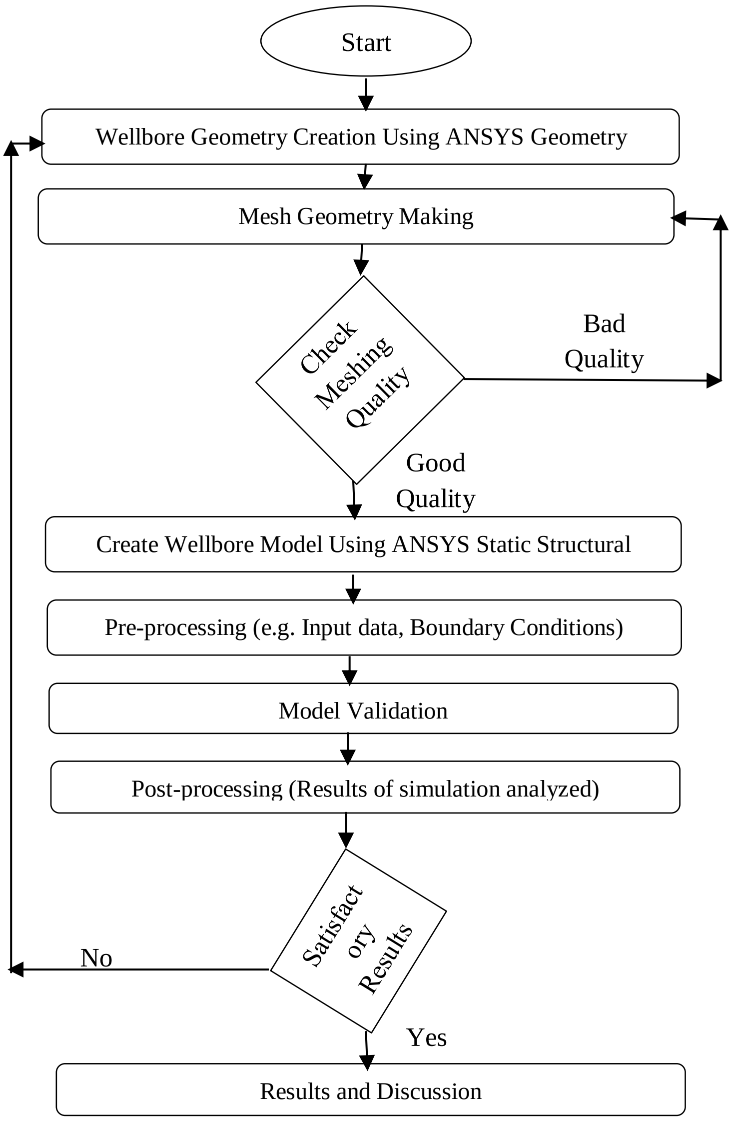


Figure 1. *Methodology of the numerical simulation flowchart.*

3. EXPERIMENTAL DATA

The experimental data published by Gray [2] will be used and modified in this study as shown in Table 1.

Table 1. *Experimental data of rock sample*

Parameter	**Value**	**Unit**
Well data of both cases (vertical and deviated)		
True Vertical Depth of the Well	4572	m
Estimated Hole Diameter	0.4213	m
Overburden Pressure Gradient	22.62	kPa/m
Pore Pressure Gradient	14.0248	kPa/m
Ratio of far-field max. horizontal stress to overburden stress	0.78	--
Ratio of far-field max. horizontal stress to far-field min. horizontal stress	1.18	--
Rock Type and Properties		
Rock Type	Sandstone or shale	--
Young Modulus, E	2.7×10^7	kPa
Poisson Ratio, ν	0.2	--
Friction Angle, ϕ	30	degree
Cohesive Strength, Co	5.93×10^7	kPa
Dilation Angle, Ψ	0.1	degree
Density, ρ	2500	Kg/m^3
Hydraulic Conductivity, k	1.00E-08	m/sec
Porosity, φ_p	33	%

4. RESULTS AND ANALYSIS

4.1. Finite-element wellbore models considered

The half-vertical wellbore model allows shear stress to a make complete loop around the X-Z plane, i.e. the vertical wellbore can be rotated in the X-Z plane. However, the model is limited to rotate perpendicularly to σh or σH as these are the only directions where shear stress around X-Y and Y-Z plane are zero. Otherwise, a full model will be required.

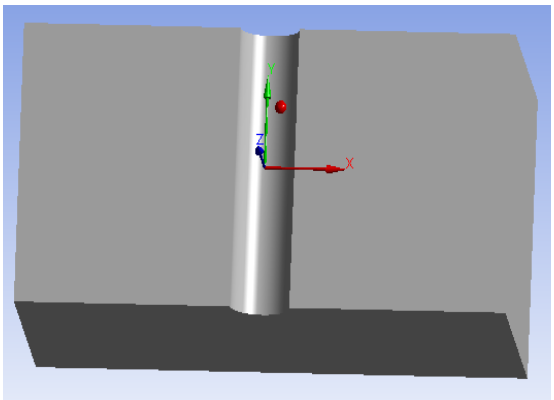


Figure 2. *Symmetrical half-vertical wellbore model.*

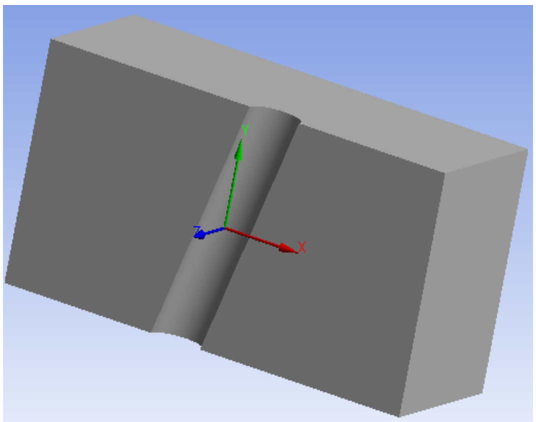


Figure 3. *Inclined wellbore model.*

The full-inclined wellbore model is an alternative to analyze the wellbore stability of an arbitrarily oriented and inclined wellbore. No stress transformation calculation will be required for this model as the far-field stress that acts normally to the block surfaces and the cylindrical wellbore that has been carefully created at a specific angle will do the job.

4.2. Dimensions, meshes and elements of the wellbore model

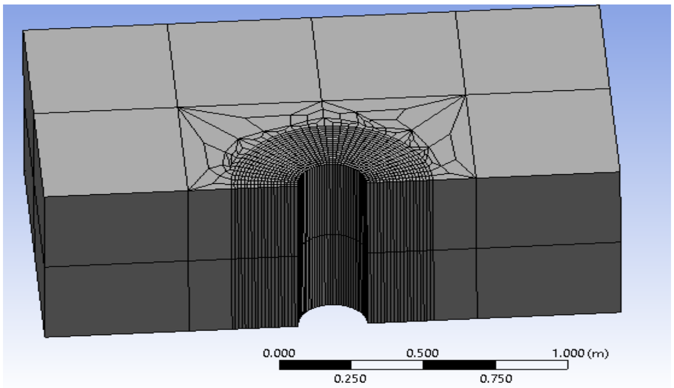


Figure 4. *Mesh and elements of the symmetrical half-vertical wellbore model.*

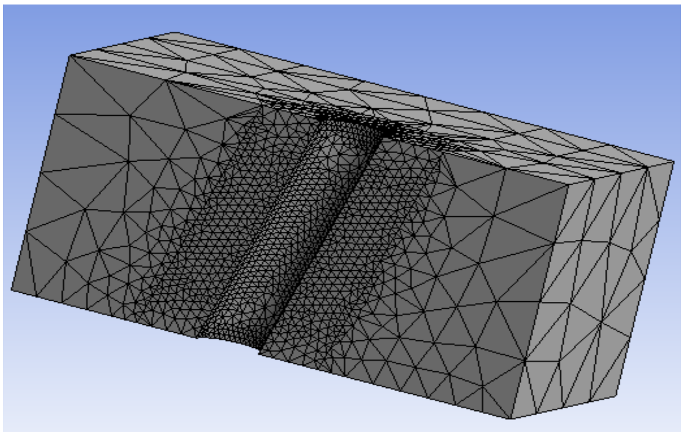
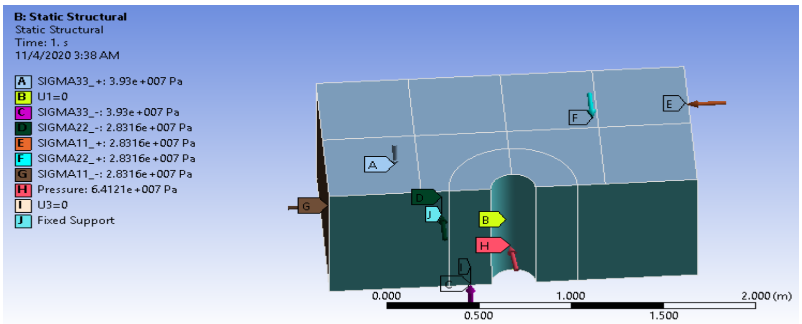
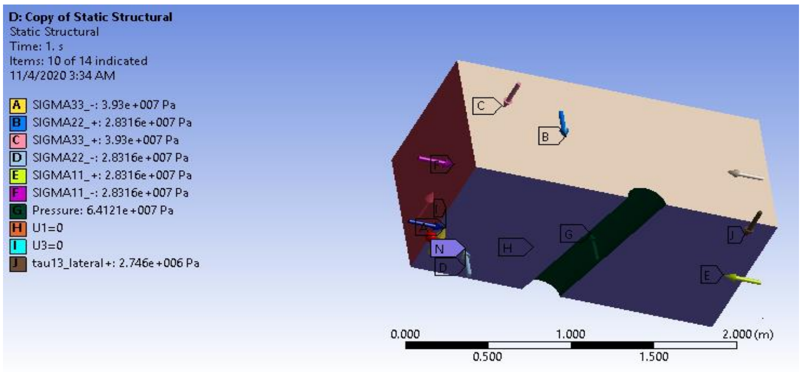


Figure 5. *Mesh and elements of the symmetrical half-inclined wellbore model.*



(a)



(b)

Figure 6. *Boundary conditions at the geostatic stage (a) vertical, (b) inclined.*

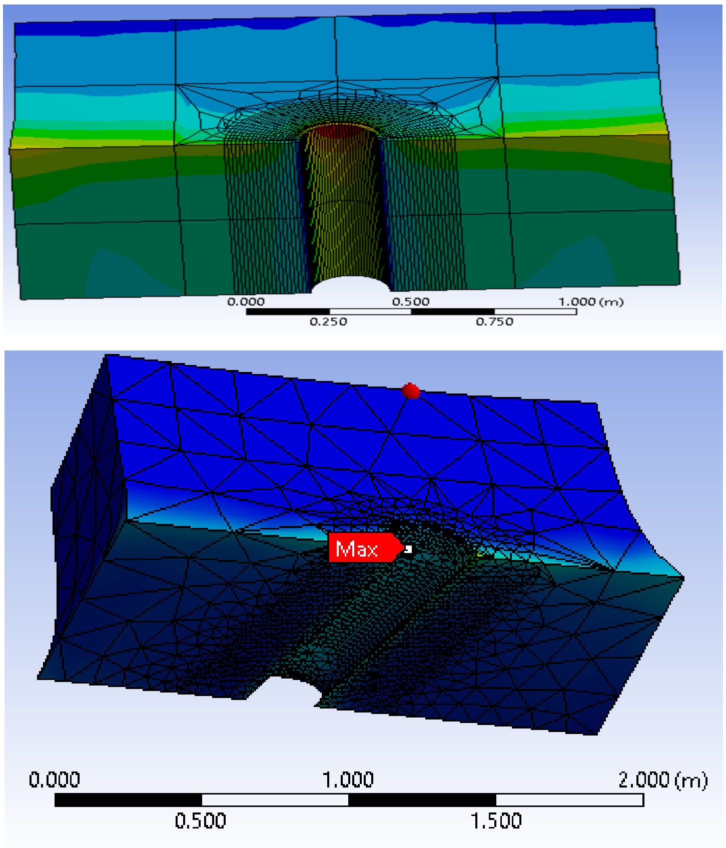


Figure 7. *Effective stress around wellbore (vertical and inclinated).*

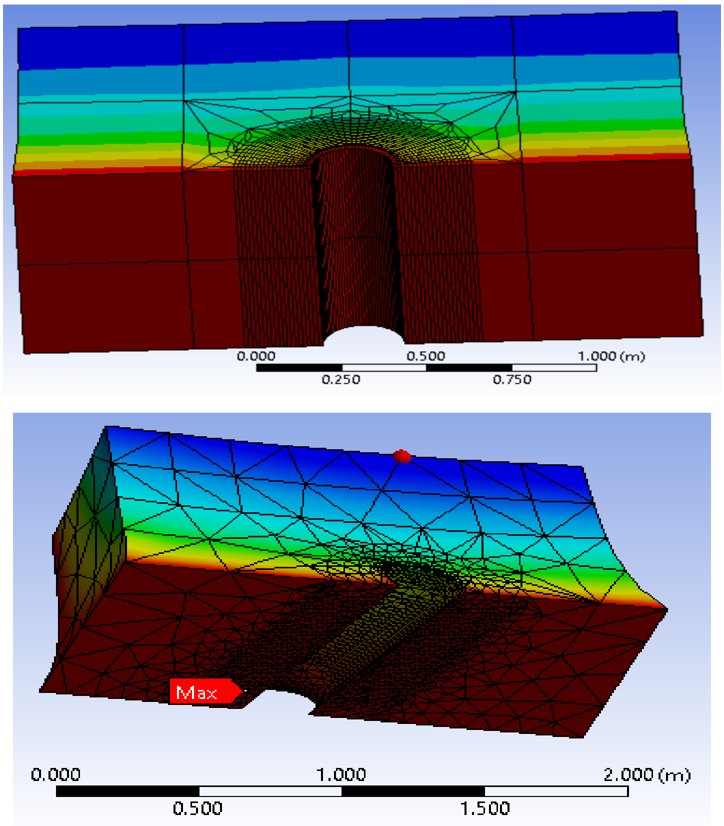


Figure 8. *Rock deformation around wellbore (vertical and inclined)*

Finite-element 3D model was built by ANSYS FEM software package for isotropic elastic, impermeable rock formation of data shown in Table 1 in order to analyse a wellbore stability in case of vertical and deviated well with 15°. Figure 1 show the methodology of the numerical simulation flow diagram that was followed to generate the model. Figures 2 through 8 show the model results from creating the wellbore geometry till the stress distribution and rock displacement surrounding the wellbore for both vertical and inclined holes. Firstly, the geometry was created for both cases (Figures 2&3). The meshing was done by Ansys meshing tool. One important factor to highlight is that Ansys educational version is used throughout the experiment phase, and the maximum number of meshing elements allowed in the educational version is 512,000 elements. It is almost impossible to solve at once a whole problem domain unless it is divided into smaller pieces. A mesh can be explained as dividing a complete component into finite number of elements which their geometry and size varies depending on each problem (Figures 4&5). Then, the numerical simulation model was done based on 3 hypotheses which are:

1. Normal and shear stresses suitable sets applied to the surfaces of a 3D FE vertical wellbore model to convert it into a deviated wellbore model;
2. The same vertical wellbore model reused and converted into wellbore at various inclination and orientation;
3. The stability of the wellbore and stress distribution around an arbitrarily oriented and inclined wellbore analyzed using the 3D FE vertical wellbore model.

The boundary conditions applied to the model are illustrated in Figure 6 a&b. The external loads are the insitu normal and shear stresses calculated from the experimental data. These external loads have then equal magnitude but opposite internal loads and wellbore pressures kept the forces in equilibrium. The internal loads are principle stresses σ11, σ22, and σ33. Assuming undrained conditions, internal and external shear stresses will be τ13 but in opposite directions. The surfaces and direction in which these normal and shear stresses act upon are shown in Figure 6 a&b. Also, U1 = 0 Displacement in x-direction, U3 =0 Displacement in z-direction. Finally, the stress distribution contour and deformation plots are shown in Figures 7 and 8. It is clear that the stresses are gradually changed from normal distribution far away the wellbore to intensive and concentrated stresses around the wellbore which shown from the degree of colour and the meshing around the wellbore. Where going from the blue colour to red colour means moving stresses from well-balanced areas (blue areas) to unbalanced areas (red areas) (Figure 7). The situation is the same for rock deformation around the wellbore (Figure 8). Seriously deformation yielded near the wellbore wall. It means that the areas surrounding the wellbore are unstable zones. However, the existence of drilling fluids improve the stability of the wellbore as they reduce the stresses generated on the wellbore wall which cause failures. On the other hand, the rest or remaining surface (rest of the surface) of the geometry is suffering from instability problems (red areas) because of the non-existence of drilling mud (Figures 7 &8). Therefore, the suitable drilling fluids reduce the instability problems happen around the wellbore wall. It is good to know that the outer rectangular edge does not exist in reality but we took half-symmetric wellbore in order to see the stress concentration and rock deformation through the entire hole section.

CONCLUSIONS

The following conclusions and recommendations are drawn from the findings and analysis:

1. Finite-element 3D model built by Ansys FEM software package is successful for analysing isotropic elastic, impermeable rock formation of data.
2. The appropriate drilling mud is a key factor in improving the wellbore stability.
3. Intensive stresses means higher deformation around the wellbore.
4. Although Ansys static structure model has several assumptions, it visualizes the image of wellbore stability very well.
5. The 3D FE vertical wellbore models can be transformed into any arbitrarily oriented and deviated wellbore models

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