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RESEARCH ON ESTABLISHING THE DIFFERENTIAL MODEL OF THE ROBOTIC MECHANISMS

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ABSTRACT

The paper presents the stages of realization of a computer program that allows the establishment of the differential model of the active mechanisms from the component of industrial robots. The computer program was developed using the Maple programming language. Finally, a series of simulation results are presented in the case of an active mechanism of an industrial robot with six degrees of freedom.

Keywords*:* industrial robot, active mechanism, differential model

INTRODUCTION

Within the realization of various mechanisms, the development of computational algorithms based on the study models and their transposition into computer programs are extremely important stages in their design process [1-16]. In this context, the development of the computer programs for the simulation of the functioning of the industrial robots in order to increase their performances is part of a very topical issue in the field of Robotics [17-20]. The paper presents the stages of realization of a computer program that allows the establishment of the differential model of the active mechanisms from the component of industrial robots. The computer program was developed using the Maple programming language. A series of simulation results are presented in the case of an active mechanism of an industrial robot with six degrees of freedom.

THEORETICAL CONSIDERATIONS AND SIMULATION RESULTS

The differential model of the active mechanisms from the component of industrial robots is expressed by the following equation [17,18]:

$$\mathrm{d}\boldsymbol{x} = \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{q} \tag{1}$$

where $\boldsymbol{x}$ represents the vector that contains the operational coordinates:

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_{poz} \\ \boldsymbol{x}_{rot} \end{bmatrix} \tag{2}$$

$\boldsymbol{x}_{poz}$ and $\boldsymbol{x}_{rot}$ are the vectors that define the position and the orientation of the coordinate system $(O_T x_T y_T z_T)$ attached between the gripper fingers;

$$\boldsymbol{q} = [q_1 \quad q_2 \quad \dots \quad q_n]^{\mathrm{T}} \tag{3}$$

is the vector that contains the coordinates: $q_1, q_2, \dots q_n$ corresponding to the active joints of the robot;

$\boldsymbol{J}$ represents the Jacobean matrix which achieves the link between the variations of the operational coordinates and those of the coordinates corresponding to the active joints and may be calculated with the following relation [17]:

$$\boldsymbol{J} = \begin{bmatrix} \boldsymbol{R}_{poz} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}_{rot} \end{bmatrix} \cdot {}^{0}\boldsymbol{J}_T \tag{4}$$

where: $\boldsymbol{R}_{poz}$ depends on the type of coordinates used to define the position of the component modules. When cartesian coordinates are used: $\boldsymbol{R}_{poz} = \boldsymbol{I}_3$, where $\boldsymbol{I}_3$ is the unit matrix of rank three; $\boldsymbol{R}_{rot}$ depends on the type of coordinates used to define the orientation of the coordinate system $(O_T x_T y_T z_T)$;

$${}^{0}\boldsymbol{J}_T = \begin{bmatrix} {}^{0}\boldsymbol{R}_T & \boldsymbol{0} \\ \boldsymbol{0} & {}^{0}\boldsymbol{R}_T \end{bmatrix} \cdot {}^{T}\boldsymbol{J}_T \tag{5}$$

where: ${}^{0}\boldsymbol{R}_T$ is the rotation matrix corresponding to relative orientation between the coordinate system $(O_T x_T y_T z_T)$ and the fixed system of coordinates $(O_0 x_0 y_0 z_0)$.

$${}^{0}\boldsymbol{R}_T = {}^{0}\boldsymbol{R}_1 \cdot {}^{1}\boldsymbol{R}_2 \cdot \dots \cdot {}^{n-1}\boldsymbol{R}_n \cdot {}^{n}\boldsymbol{R}_T \tag{6}$$

where: ${}^{i}\boldsymbol{R}_{i+1}, i = \overline{0, n-1}$, are the rotation matrices corresponding to relative orientation between the systems of coordinates $(O_i x_i y_i z_i)$ and $(O_{i+1} x_{i+1} y_{i+1} z_{i+1})$ attached to the component modules i and $i+1$; ${}^{n}\boldsymbol{R}_T$ is the rotation matrix corresponding to the relative orientation between the systems of coordinates $(O_n x_n y_n z_n)$ attached to the last module of the robot and $(O_T x_T y_T z_T)$. The column k of the matrix ${}^{T}\boldsymbol{J}_T$ of dimensions (6xn) when a type of parameterization of the robot structure is used has the following expression [17]:

$$\begin{bmatrix} \sigma_k \cdot {}^{T}\boldsymbol{R}_k \cdot {}^{(k)}\boldsymbol{k}_k + \bar{\sigma}_k \cdot {}^{T}\boldsymbol{R}_k \cdot {}^{(k)}\boldsymbol{k}_k^{v} \cdot {}^{(k)}\boldsymbol{O}_k\boldsymbol{O}_T \\ \bar{\sigma}_k \cdot {}^{T}\boldsymbol{R}_k \cdot {}^{(k)}\boldsymbol{k}_k \end{bmatrix} \tag{7}$$

where: $\sigma_k = 0$ when the module k is of rotation and $\sigma_k = 1$ when the module k is of translation; $\bar{\sigma}_k = 1 - \sigma_k$; ${}^{T}\boldsymbol{R}_k = ({}^{k}\boldsymbol{R}_T)^{\mathrm{T}}$ is the rotation matrix corresponding to relative orientation between the systems of coordinates $(O_T x_T y_T z_T)$ and $(O_k x_k y_k z_k)$ attached to the module k of the robot;

$${}^{k}\boldsymbol{R}_T = {}^{k}\boldsymbol{R}_{k+1} \cdot {}^{k+1}\boldsymbol{R}_{k+2} \cdot \dots \cdot {}^{n-1}\boldsymbol{R}_n \cdot {}^{n}\boldsymbol{R}_T \tag{8}$$

$${}^{(k)}\boldsymbol{k}_k = [0 \quad 0 \quad 1]^{\mathrm{T}} \tag{9}$$

$$^{(k)}\boldsymbol{k}_k^v = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (10)$$

The vector $^{(k)}\boldsymbol{O}_k\boldsymbol{O}_T$ may be extracted from the homogeneous transformation matrix $^k\boldsymbol{T}_T$, where:

$$^k\boldsymbol{T}_T = \begin{bmatrix} ^k\boldsymbol{R}_T & ^{(k)}\boldsymbol{O}_k\boldsymbol{O}_T \\ \boldsymbol{0} & 1 \end{bmatrix} = {}^k\boldsymbol{T}_{k+1} \cdot {}^{k+1}\boldsymbol{T}_{k+2} \cdot \ldots \cdot {}^{n-1}\boldsymbol{T}_n \cdot {}^n\boldsymbol{T}_T \quad (11)$$

For realization the computer program that allows the establishment of the differential model of the active mechanisms from the component of industrial robots it was used the parameterization presented in [17,18]. In this case the parameters used to establish the relative position and orientation between the coordinates systems attached to the consecutive modules *i* and *i+1* are presented in Figure 1.

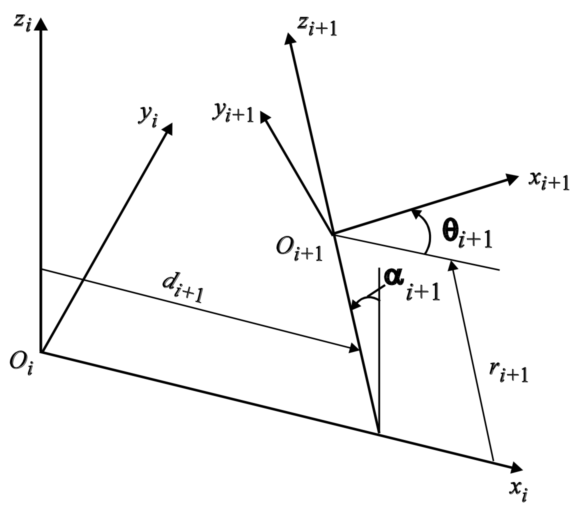

Figure 1. Parameters used to establish the relative position and orientation between the consecutive modules i and i+1 [17,18]

When using this parameterization the homogeneous transformation matrix $^i\boldsymbol{T}_{i+1}$ has the following expression [17,18]:

$$^i\boldsymbol{T}_{i+1} = \begin{bmatrix} ^i\boldsymbol{R}_{i+1} & ^{(i)}\boldsymbol{O}_i\boldsymbol{O}_{i+1} \\ \boldsymbol{0} & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \cos\theta_{i+1} & -\sin\theta_{i+1} & 0 & d_{i+1} \\ \cos\alpha_{i+1}\cdot\sin\theta_{i+1} & \cos\alpha_{i+1}\cdot\cos\theta_{i+1} & -\sin\alpha_{i+1} & -r_{i+1}\cdot\sin\alpha_{i+1} \\ \sin\alpha_{i+1}\cdot\sin\theta_{i+1} & \sin\alpha_{i+1}\cdot\cos\theta_{i+1} & \cos\alpha_{i+1} & r_{i+1}\cdot\cos\alpha_{i+1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

A computer program that allows the establishment of the differential model of the active mechanisms from the component of industrial robots was developed using the special capabilities of symbolic calculation of Maple programming language.

A series of simulation results are presented in the case of an active mechanism of an industrial robot with six degrees of freedom (Figure 2).

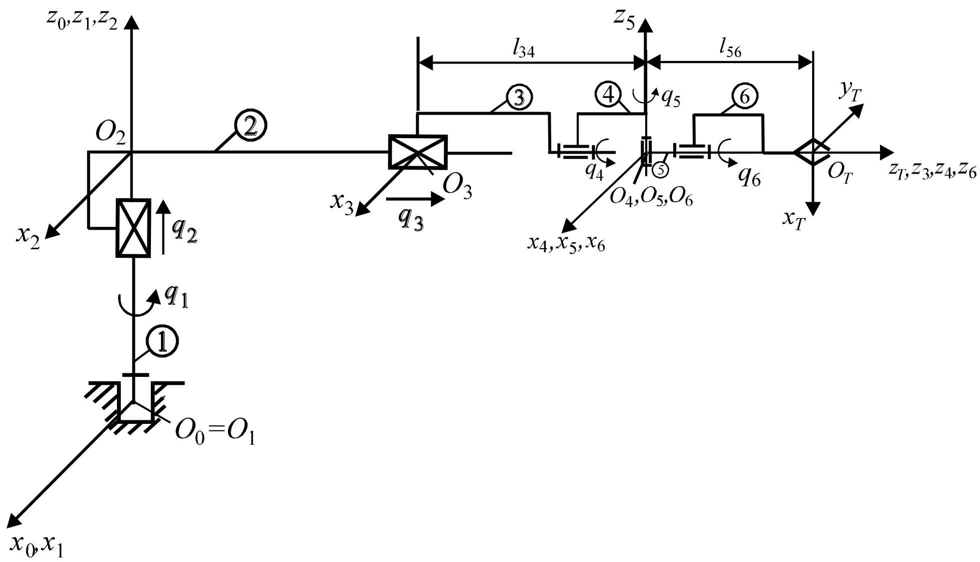

Figure 2. The mechanism of an industrial robot with six degrees of freedom

The values of the parameters that establish the relative position and orientation between the coordinates systems attached to the component modules are given in Table 1.

Table 1. *The values of the parameters*

i	α_i	d_i	θ_i	r_i
1	0	0	q_1	0
2	0	0	0	q_2
3	-90°	0	0	q_3
4	0	0	q_4	l_{34}
5	90°	0	q_5	0
6	-90°	0	q_6	0

In the sequel are presented the expressions obtained with the computer program for the elements of all six columns of the matrix ${}^{T}\boldsymbol{J}_T$:

$$col1 := \begin{bmatrix} -\sin(q_5)^2\cos(q_6)\cos(q_4)\,l56 + (-\cos(q_4)\cos(q_5)\cos(q_6) + \sin(q_4)\sin(q_6))(\cos(q_5)\,l56 + l34 + q_3) \\ \sin(q_5)^2\sin(q_6)\cos(q_4)\,l56 + (\cos(q_4)\cos(q_5)\sin(q_6) + \sin(q_4)\cos(q_6))(\cos(q_5)\,l56 + l34 + q_3) \\ -\cos(q_5)\cos(q_4)\sin(q_5)\,l56 + \cos(q_4)\sin(q_5)(\cos(q_5)\,l56 + l34 + q_3) \\ -\sin(q_4)\cos(q_5)\cos(q_6) - \cos(q_4)\sin(q_6) \\ \sin(q_4)\cos(q_5)\sin(q_6) - \cos(q_4)\cos(q_6) \\ \sin(q_4)\sin(q_5) \end{bmatrix}$$

$$col2 := \begin{bmatrix} -\sin(q_4)\cos(q_5)\cos(q_6) - \cos(q_4)\sin(q_6) \\ \sin(q_4)\cos(q_5)\sin(q_6) - \cos(q_4)\cos(q_6) \\ \sin(q_4)\sin(q_5) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$col3 := \begin{bmatrix} \sin(q_5)\cos(q_6) \\ -\sin(q_5)\sin(q_6) \\ \cos(q_5) \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad col4 := \begin{bmatrix} -\sin(q_6)\sin(q_5)\,l56 \\ -\cos(q_6)\sin(q_5)\,l56 \\ 0 \\ \sin(q_5)\cos(q_6) \\ -\sin(q_5)\sin(q_6) \\ \cos(q_5) \end{bmatrix} \quad col5 := \begin{bmatrix} -\cos(q_6)\,l56 \\ \sin(q_6)\,l56 \\ 0 \\ -\sin(q_6) \\ -\cos(q_6) \\ 0 \end{bmatrix} \quad col6 := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

CONCLUSIONS

In the paper a method for achieving the differential model of the active mechanisms from the component of industrial robots has been presented. In this scope, a computer program was developed using the special capabilities of symbolic calculation offered by Maple programming language. A series of simulation results have been presented in the case of an active mechanism of an industrial robot with six degrees of freedom.

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