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RESEARCH CONCERNING THE DYNAMICS OF PLANE MECHANISMS

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ABSTRACT

In the paper is presented a method of studying the dynamics of plane mechanisms based on the determination of the variation of the balancing moment on the cinematic cycle. The components of the balancing moment due to the gravitational forces, forces and moments of inertia, as well as those corresponding to the technological resistances are highlighted. A series of results of the simulations performed in the case of a plane mechanism are finally presented.

Keywords: plane mechanism, dynamics, balancing moment

INTRODUCTION

The dynamic study of the mechanisms represents an essential stage in the evaluation of the performances during their operation and in terms of their optimal design [1-5]. Therefore, the identification and evaluation of the influence of all factors on the dynamics of the mechanisms is a field of study of great importance [6,7]. In this paper a method of studying the dynamics of plane mechanisms is presented. The method is based on the evaluation of the balancing moment by taking into account the influence of the gravitational forces, of the forces and of the moments of inertia, as well as of those corresponding to the technological resistances. Based on the presented method a computer program has been developed and the results of the simulations performed in the case of a plane mechanism are finally presented.

THEORETICAL CONSIDERATIONS AND SIMULATION RESULTS

The balancing moment is applied to the driving element of a plane mechanism and realizes together with all the categories of forces and moments that act on the components of the mechanism its dynamic movement balance. When the variation during a cinematic cycle of the technological forces and moments is known the values of the balancing moment corresponds to the values of the driving moment. From the expression of the dynamic balance in powers of all the categories of forces and moments that act on the n moving cinematic elements of a plane mechanism it results:

$$\overline{M}_e \cdot \overline{\omega}_1 + \sum_{j=1}^{n} \overline{G}_j \cdot \overline{v}_{C_j} + \sum_{j=1}^{n} \left(\overline{F}_{ij} \cdot \overline{v}_{C_j} + \overline{M}_{ij} \cdot \overline{\omega}_j \right) + \sum_{(j)} \overline{F}_{ru,j} \cdot \overline{v}_{ru,j} + \sum_{(j)} \overline{M}_{ru,j} \cdot \overline{\omega}_{ru,j} = 0 \qquad (1)$$

where: $\overline{M}_e$ is the balancing moment; $\overline{\omega}_1$ is the angular velocity of the driving element *1* of the mechanism; $\overline{G}_j = m_j \cdot \overline{g}$ is the weight of the component element *j* and $\overline{g}$ is the gravitational acceleration; $\overline{v}_{C_j}$ is the velocity of the mass centre of the element *j*; $\overline{F}_{ij} = -m_j \cdot \overline{a}_{C_j}$ is the inertia force corresponding to the *j* element and $\overline{a}_{C_j}$ is the acceleration of the mass centre of the element *j*; $\overline{M}_{ij} = -J_{C_j} \cdot \overline{\varepsilon}_j$ is the inertia moment corresponding to the *j* element and $J_{C_j}, \overline{\varepsilon}_j$ are the mass moment of inertia and the angular acceleration corresponding to the element *j*; $\overline{F}_{ru,j}$ represents the technological resistance force with the number *j* and $\overline{v}_{ru,j}$ is the velocity of the point where this force acts; $\overline{M}_{ru,j}$ is the technological resistance moment with the number *j* and $\overline{\omega}_{ru,j}$ is the angular velocity of the element on which this moment acts.

From equation (1) it follows:

$$M_e = M_e^G + M_e^{Fi} + M_e^{Mi} + M_e^{Fru} + M_e^{Mru} \tag{2}$$

where:

$$M_e^G = -\frac{1}{\omega_1} \cdot \sum_{j=1}^{n} \overline{G}_j \cdot \overline{v}_{C_j} = \frac{1}{\omega_1} \cdot \sum_{j=1}^{n} m_j \cdot g \cdot (v_{C_j})_y \tag{3}$$

$$M_e^{Fi} = -\frac{1}{\omega_1} \cdot \sum_{j=1}^{n} \overline{F}_{ij} \cdot \overline{v}_{C_j} = \frac{1}{\omega_1} \cdot \sum_{j=1}^{n} m_j \cdot \left((a_{C_j})_x \cdot (v_{C_j})_x + (a_{C_j})_y \cdot (v_{C_j})_y \right) \tag{4}$$

$$M_e^{Mi} = -\frac{1}{\omega_1} \cdot \sum_{j=1}^{n} \overline{M}_{ij} \cdot \overline{\omega}_j = \frac{1}{\omega_1} \cdot \sum_{j=1}^{n} J_{C_j} \cdot \varepsilon_j \cdot \omega_j \tag{5}$$

$$M_e^{Fru} = -\frac{1}{\omega_1} \cdot \sum_{(j)} \overline{F}_{ru,j} \cdot \overline{v}_{ru,j} \tag{6}$$

$$M_e^{Mru} = -\frac{1}{\omega_1} \cdot \sum_{(j)} \overline{M}_{ru,j} \cdot \overline{\omega}_{ru,j} \tag{7}$$

In the above relations it was considered that the plane of the motion of the mechanism is the plane (*Oxy*) and the gravitational acceleration $\overline{g}$ acts in the opposite direction to the axis (*Oy*). The projections on the (*Ox*) and (*Oy*) axes of the velocities and accelerations $\overline{v}_{C_j}$ and $\overline{a}_{C_j}$, as well as the angular velocities and accelerations ω_j and ε_j of the component elements of the mechanism can be determined with the relations:

$$\begin{cases} (v_{C_j})_x = \dot{x}_{C_j} = \dfrac{\mathrm{d}x_{C_j}}{\mathrm{d}\varphi_1} \cdot \dfrac{\mathrm{d}\varphi_1}{\mathrm{d}t} = \omega_1 \cdot \dfrac{\mathrm{d}x_{C_j}}{\mathrm{d}\varphi_1} \\ (v_{C_j})_y = \dot{y}_{C_j} = \dfrac{\mathrm{d}y_{C_j}}{\mathrm{d}\varphi_1} \cdot \dfrac{\mathrm{d}\varphi_1}{\mathrm{d}t} = \omega_1 \cdot \dfrac{\mathrm{d}y_{C_j}}{\mathrm{d}\varphi_1} \end{cases} \tag{8}$$

$$\begin{cases} (a_{C_j})_x = \dfrac{\mathrm{d}(v_{C_j})_x}{\mathrm{d}t} = \dfrac{\mathrm{d}(v_{C_j})_x}{\mathrm{d}\varphi_1} \cdot \dfrac{\mathrm{d}\varphi_1}{\mathrm{d}t} = \omega_1 \cdot \dfrac{\mathrm{d}(v_{C_j})_x}{\mathrm{d}\varphi_1} \\ (a_{C_j})_y = \dfrac{\mathrm{d}(v_{C_j})_y}{\mathrm{d}t} = \dfrac{\mathrm{d}(v_{C_j})_y}{\mathrm{d}\varphi_1} \cdot \dfrac{\mathrm{d}\varphi_1}{\mathrm{d}t} = \omega_1 \cdot \dfrac{\mathrm{d}(v_{C_j})_y}{\mathrm{d}\varphi_1} \end{cases} \tag{9}$$

$$\begin{cases} \omega_j = \dot{\varphi}_j = \dfrac{\mathrm{d}\varphi_j}{\mathrm{d}\varphi_1} \cdot \dfrac{\mathrm{d}\varphi_1}{\mathrm{d}t} = \omega_1 \cdot \dfrac{\mathrm{d}\varphi_j}{\mathrm{d}\varphi_1} \\ \varepsilon_j = \dot{\omega}_j = \dfrac{\mathrm{d}\omega_j}{\mathrm{d}\varphi_1} \cdot \dfrac{\mathrm{d}\varphi_1}{\mathrm{d}t} = \omega_1 \cdot \dfrac{\mathrm{d}\omega_j}{\mathrm{d}\varphi_1} \end{cases} \tag{10}$$

The x_{C_j} and y_{C_j} coordinates of the C_j centers of gravity of the component elements and the φ_j angles that appear in the previous relations are determined following the positional analysis of the mechanism.

The presented method was applied for the dynamic study of the mechanism in Figure 1.

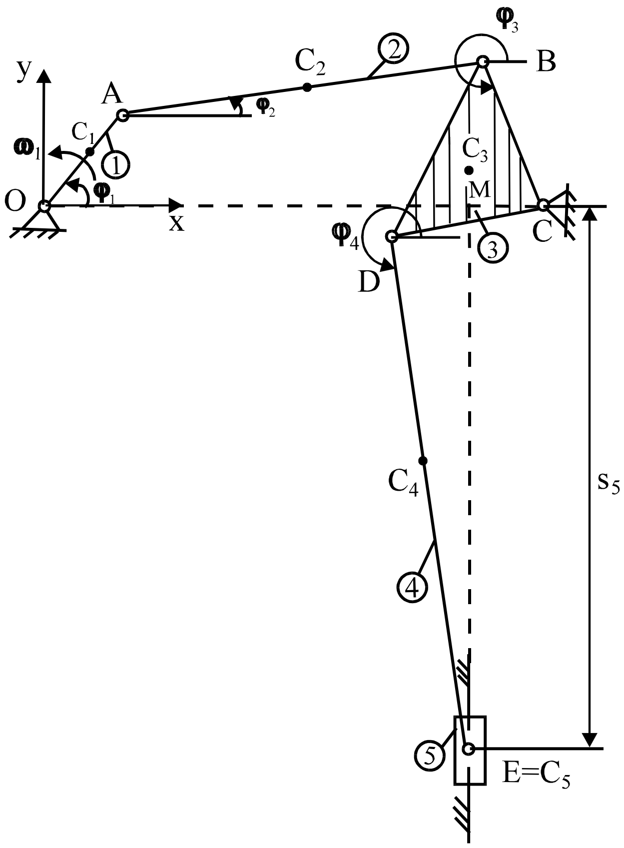

Figure 1. Plane mechanism

The dimensions of the mechanism components are:

$OA = 0.2\,\text{m}$; $AB = 0.5\,\text{m}$; $OC = 0.65\,\text{m}$; $BC = BD = CD = 0.45\,\text{m}$; $DE = 0.9\,\text{m}$; $CM = 0.15\,\text{m}$; $OC_1 = 0.1\,\text{m}$; $AC_2 = 0.25\,\text{m}$; $DC_4 = 0.45\,\text{m}$.

C_3 is located at the center of gravity of the triangle *BCD*. Elements *1*, *2* and *4* are made of steel bars with a round section of radius equal to 0.015 m. The thickness of the element *3* is equal to 0.01 m.

The element *5* mass is equal to 1 kg. On element *5* acts the $\overline{F}_{ru}$ technological force equal to: $\overline{F}_{ru} = -k_F \cdot \overline{v}_E$, where: $k_F = 300\dfrac{\text{N}\cdot\text{s}}{\text{m}}$.

The mechanism has two independent loops [8] in its component: *O-A-B-C-O* and *C-D-E-C*. The vector closing equations corresponding to the two loops are: $\overline{OA} + \overline{AB} + \overline{BC} + \overline{CO} = 0$ and $\overline{CD} + \overline{DE} + \overline{EC} = 0$. By projecting the two vector equations on the (*Ox*) and (*Oy*) axes, the following equation systems are obtained:

$$\begin{cases} l_1 \cdot \cos\varphi_1 + l_2 \cdot \cos\varphi_2 + BC \cdot \cos\varphi_3 - l_0 = 0 \\ l_1 \cdot \sin\varphi_1 + l_2 \cdot \sin\varphi_2 + BC \cdot \sin\varphi_3 = 0 \end{cases} \tag{11}$$

$$\begin{cases} CD \cdot \cos(\varphi_3 - 2\pi/3) + l_4 \cdot \cos\varphi_4 + CM = 0 \\ CD \cdot \sin(\varphi_3 - 2\pi/3) + l_4 \cdot \sin\varphi_4 + s_5 = 0 \end{cases} \tag{12}$$

where: $l_1 = OA$, $l_2 = AB$, $l_0 = OC$, $l_4 = DE$.

By solving the two systems of equations, the angles φ_2, φ_3 and φ_4 and the displacement s_5 may be calculated with the following relations:

$$\varphi_2 = \arcsin\left(\frac{C_2}{\sqrt{A_2^2 + B_2^2}}\right) - \text{ATAN2}\,(A_2, B_2) \tag{13}$$

where: $\text{ATAN}\,2(y, x)$ calculates arctan(*y/x*) by taking into account the signs of *x* and *y* and:

$$\begin{cases} A_2 = 2 \cdot l_1 \cdot l_2 \cdot \cos\varphi_1 - 2 \cdot l_0 \cdot l_2 \\ B_2 = 2 \cdot l_1 \cdot l_2 \cdot \sin\varphi_1 \\ C_2 = BC^2 - l_1^2 - l_2^2 - l_0^2 + 2 \cdot l_1 \cdot l_0 \cdot \cos\varphi_1 \end{cases} \tag{14}$$

$$\varphi_3 = \text{ATAN}\,2\left(-l_1 \cdot \sin\varphi_1 - l_2 \cdot \sin\varphi_2, -l_1 \cdot \cos\varphi_1 - l_2 \cdot \cos\varphi_2 + l_0\right) \tag{15}$$

$$s_5 = \frac{-B_{s5} + \sqrt{B_{s5}^2 - 4 \cdot A_{s5}}}{2} \tag{16}$$

where:

$$\begin{cases} A_{s5} = CD^2 + CM^2 - l_4^2 + 2 \cdot CD \cdot CM \cdot \cos(\varphi_3 - 2\pi/3) \\ B_{s5} = 2 \cdot CD \cdot \sin(\varphi_3 - 2\pi/3) \end{cases} \tag{17}$$

$$\varphi_4 = \mathrm{ATAN}\,2\left(-CD \cdot \sin(\varphi_3 - 2\pi/3) - s_5, -CD \cdot \cos(\varphi_3 - 2\pi/3) - CM\right) \tag{18}$$

The coordinates of the centers of gravity of the component elements can then be easily determined by projecting the corresponding position vectors on the (Ox) and (Oy) axes, using the angles φ_2, φ_3 and φ_4 calculated with the previous relations.

To dynamically analyze the mechanism a computer program has been developed using Maple programming language. With this computer program it has been analyzed the influence of the components $M_e^G, M_e^{Fi}, M_e^{Mi}$ and M_e^{Fru} on the variation of the balancing moment M_e by considering different values for the angular velocity ω_1 of the driving element *1* of the mechanism.

In Figures 2 and 3 are presented the variation during a cinematic cycle of the balancing moment M_e and of its components $M_e^G, M_e^{Fi}, M_e^{Mi}$ and M_e^{Fru} when the value of angular velocity ω_1 of the driving element *1* is equal to 20 rad/s.

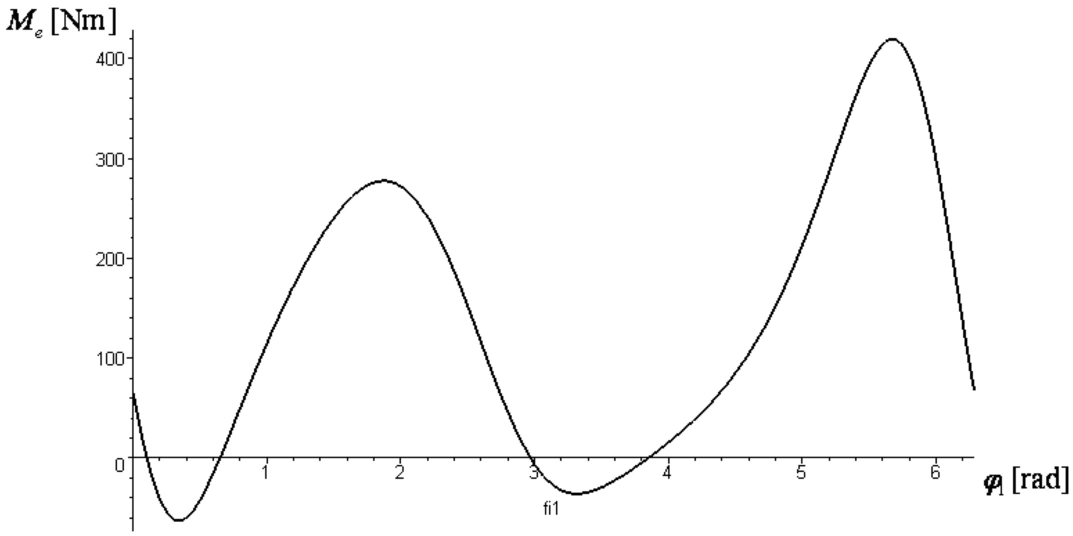

Figure 2. The variation of the balancing moment M_e when $\omega_1 = 20\,rad/s$

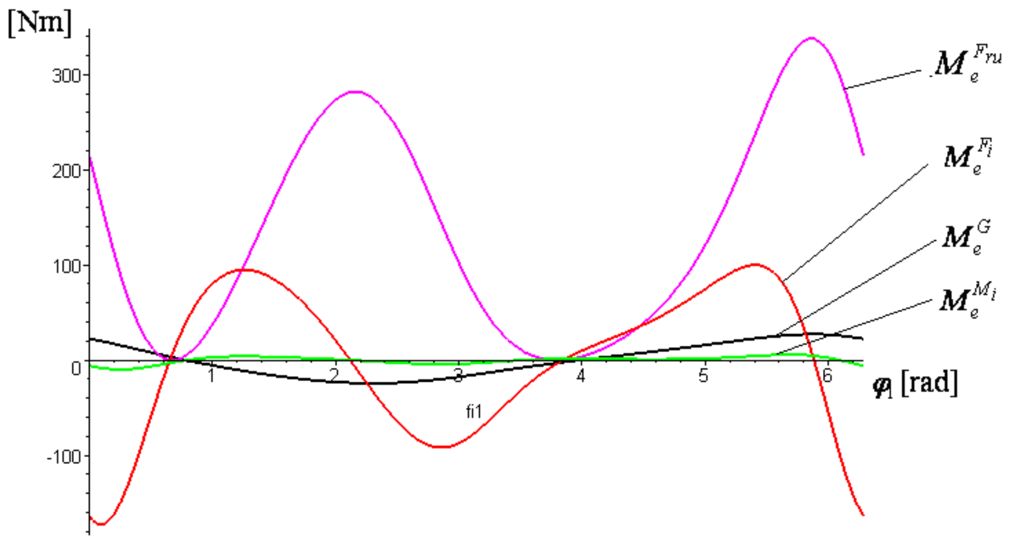

Figure 3. The variation of $M_e^G, M_e^{Fi}, M_e^{Mi}$ *and* M_e^{Fru} *when* $\omega_1 = 20\,rad/s$

In Figures 4 and 5 it is presented the variation during a cinematic cycle of the balancing moment M_e and of $M_e^G, M_e^{Fi}, M_e^{Mi}$ and M_e^{Fru} when the value of angular velocity ω_1 of the driving element *1* is equal to 10 rad/s.

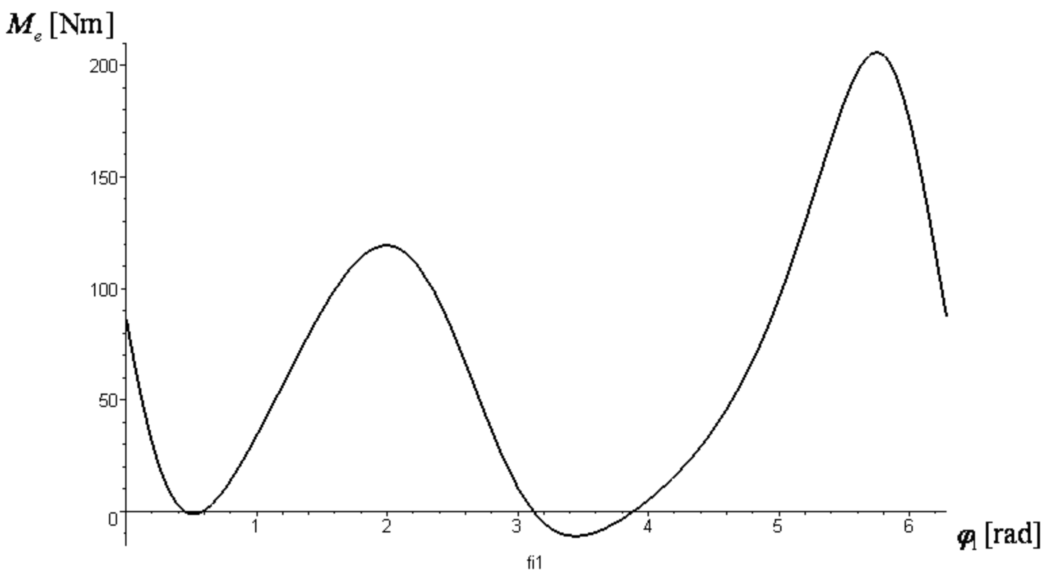

Figure 4. The variation of the balancing moment M_e *when* $\omega_1 = 10\,rad/s$

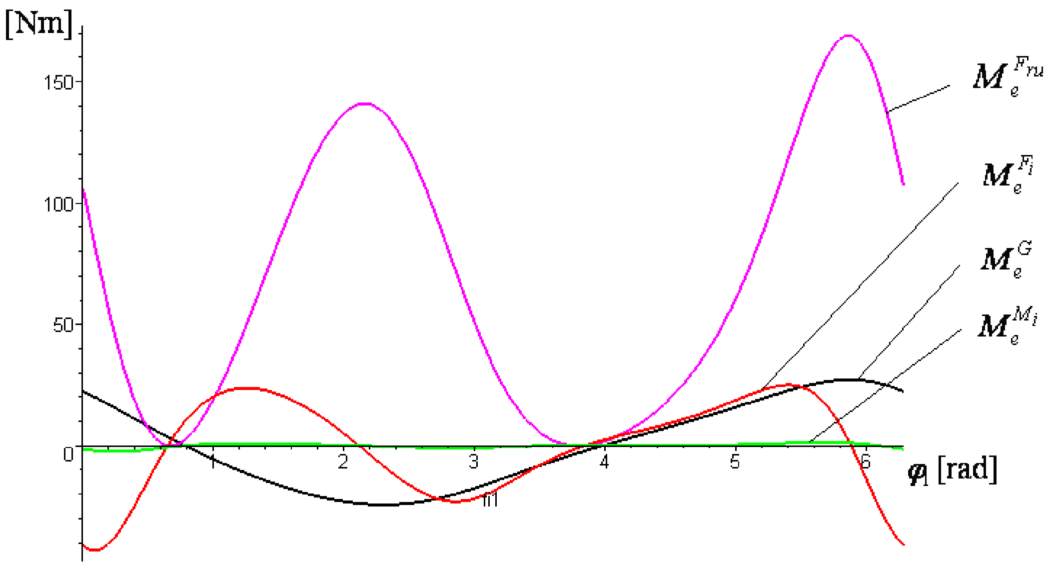

Figure 5. The variation of $M_e^G, M_e^{F_i}, M_e^{M_i}$ *and* $M_e^{F_{ru}}$ *when* $\omega_1 = 10\, rad/s$

The presented graphs show that the $M_e^{F_{ru}}$ component has the highest weight in terms of the values of the balancing moment M_e, while the contribution of the $M_e^{M_i}$ component is insignificant. Also, from the graphs presented in Figures 3 and 5 it can be seen that with the increase of the value of the angular velocity ω_1 the weight of the component $M_e^{F_i}$ also increases in terms of the values of the balancing moment M_e.

CONCLUSIONS

In this paper it was presented a method that allows studying the dynamics of plane mechanisms. The method based on the evaluation of the balancing moment has been transposed into a computer program and applied in the case of a plane mechanism having in the component two independent loops. The results obtained from the simulations highlighted the weight of the components due to the weight of the mechanism elements, to the forces and moments of inertia and to the technological resistances on the variation of the balancing moment.

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