# MEASUREMENT UNCERTAINTY IN LEGAL METROLOGY 

Bogdan-Gabriel Duță ${ }^{1}$<br>Ionuț Banu ${ }^{1}$<br>Tania-Ioana Cîmpeanu ${ }^{1}$<br>Gabriela Bucur ${ }^{1}$<br>${ }^{1}$ Petroleum-Gas University of Ploiesti, Romania<br>email: bogdangabriel_duta@yahoo.com

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#### Abstract

In legal metrology, in addition to the elements of the measurement result, the measurement uncertainty is also required in order to establish the level of confidence or risk regarding compliance decisions.

Because of this, the uncertainty assessment considers the indicated uncertainty of the utilized measuring device, as well as the uncertainty linked to calibration and any other uncertainties pertaining to how the measuring instrument is utilized in practice. The uncertainty of these readings must also be taken into account if further measures, such as pressure and temperature, are required.

The basic concepts of measurement, legal metrology, measurement errors, measurement uncertainty, types of measurement uncertainties, the importance of the mixed models and two case studies in which we obtained different values for the uncertainty were examined in this article.

Both of the two case studies that are discussed are the general uncertainty with a backup strategy and the uncertainty for partially transferred source flows. Here, the installation's operating modes, the formulae used to estimate uncertainty, and the procedure for doing so were all described.


Keywords: measurement, measurement uncertainty, metrology, legal metrology, error

## INTRODUCTION

The International Committee on Measurements and Weights, the top metrology authority in the world, asked the International Bureau of Measures and Weights to solve this problem because there isn't a globally accepted standard for conveying measurement uncertainty. They were able to create the recommendations in 1978 with the assistance of the national calibration laboratories, creating an agreement.

Thanks to the International Bureau of Measures and Weights, a thorough questionnaire addressing the concerns of measurement uncertainty was developed and subsequently adopted by 32 national metrology institutes.

The most significant result was the combination of all the various uncertainty components into a single uncertainty after first identifying an internationally recognized method for expressing measurement uncertainty [2].

In the end, no agreement could be achieved. After being gathered by the International Bureau of Measures and Weights, the Working Group on the Statement of Uncertainties created the INC-1 (1980): Expressing Experimental Uncertainties suggestion. The Recommendation was accepted by the International Committee on Measurements and Weights in October 1981. In 1986, it was once again proven. The International Committee on Measures proposed to the International Organization for Standardization the creation of a comprehensive handbook based on the Working Group Recommendation [4].
Finally, the Working Group was set up, composed of experts appointed by BIPM, IEC, ISO and OIML. The Working Group had to put together a document based on different recommendations and to provide rules used in calibration, standardization and accreditation of laboratories.

In the current era of the global market, it is essential that measurements made in many nations be simple to compare and that the accuracy of the measurements be evaluated quantitatively everywhere using the same methodology.

The Guide to Expressing Uncertainty in Measurements was developed as an international agreement on how to describe the measurement quality.

## 1. BASIC CONCEPTS

## MEASUREMENT

Measurement refers to the set of experimental operations that compare the values of a measurement scale, expressed in units of measurement (um), to identify the value on the scale that is closest to the actual value of the measurement quantity in report form:
$N=\frac{x}{u m}$
that expresses the value equivalent of the measured quantity.
Performing a measurement involves the following operations [1]:

- Defining the size to be measured (measuring).
- Specifying the scale of measurement and the unit of measure adopted.
- Adoption and application of a measurement procedure / methods.
- Specifying the technical means (equipment) necessary to perform the measurement.
- Processing the primary results to obtain a result as accurate and in a form convenient to the user.
- Display / recording the measurement result.


## THE RESULT OF A MEASUREMENT

Simply put, it is an approximation or estimation of the value obtained as a result of the measurement process that is valid only when accompanied by the specification of the uncertainty [5].

## MEASUREMENT SPECIFICATION OR DEFINITION

The accuracy of the measurement has a significant impact on this concept. The value for all practical purposes associated with the measurement should be unique, so the measurement should be completely defined in accordance with the required accuracy.

## REPEATABILITY CONDITIONS

Most of the times, to reach the result of an accurate measurement it is required to perform a series of repeated observations, which can be replaced under any conditions. Obtaining different results following the measurement process may be caused due to the fact that the measured quantities are not actually maintained at a constant level, or due to other factors.

## SCALAR OR VECTOR MEASUREMENT

A generalized set of interdependent measurements can be generated through the replacement of the scalar measure and its variant with a vector measure and a covariance matrix [5].

## ERRORS

Generally speaking, a measurement is influenced by those imperfections that are directly modifying the error's value. Traditionally, an error has two components: a random one and a systematic one and because of that, error's value cannot be known exactly.

## 2. LEGAL METROLOGY

The analysis of measurements and their applications is realized by the science called metrology. All aspects, theoretical and practical, of measurements are included in metrology, regardless the measured quantity, the manner and purpose of their performance, the field in which they intervene, the level of accuracy [2].

Metrology is characterized by different aspects of the measurements such as: the quantities and units, the results of measurement, calibration, errors, uncertainty and accuracy. Besides that, the metrology also includes the conditions of a measurement and the characteristics of the measuring instruments.

Metrology aims to define internationally accepted units of measurement, corresponding to fundamental or derived physical quantities, to achieve units of measurement by scientific methods and to establish traceability chains to substantiate the accuracy of measurement.

Metrology is structured in several areas, grouped into three categories:

- scientific metrology, which deals with the realization, development, and conservation of standards of measurement units.
- industrial metrology, which ensures the proper functioning of the measuring instruments used in industry, both in production and in quality control.
- legal metrology, which ensures the accuracy and uniformity of measurements performed in areas of public interest.

Scientific metrology, together with those parts of legal and industrial metrology that involve scientific competence, constitute fundamental metrology, and mean the treatment of measurements with the highest level of accuracy in each field [2].

Fundamental metrology operates with 3 essential concepts for characterizing measurements. These are:

- measurement uncertainty is defined as the quality indication of the quality of the measurement result;
- accuracy of the used measurements methods and the results, characterized by the constancy and accuracy of the measurements;
- traceability is described as the ability of the measurements results or as the capability of a standard to refer to the established standards (national or international).


## 3. MEASUREMENT UNCERTAINTY

Measurement uncertainty is often referred to as absolute measurement uncertainty. As a result, the measurement uncertainty is stated using the same units as the measure. Measurement uncertainty is distinct from mistake in that it lacks a sign and does not represent a difference between two values. The measurement result cannot be corrected using the error since it has a sign, and the error cannot be estimated using the error.

The concepts of true value and mistake (random and systematic) are equally ethereal. The exact numbers are unknown. However, these ideas are helpful since it is possible to calculate and apply their estimations. The measured value is really a close approximation of the genuine value.

Uncertainty in measurement helps determine whether the difference between two results is insignificant due to uncertainty or significant due to a genuine change in the patient's condition, giving laboratories greater confidence in reported results.
The definitions of uncertainty in test and calibration operations are a little bit different. On the one hand, the word uncertainty is connected to the magnitude of the variability when addressing the measurement error in a result or the unexpected behaviour of an influence quantity. On the other hand, an uncertainty statement indicating a range of values thought to be likely to include the amount of interest is made when reporting findings. This is not a depiction of random behaviour, but rather a conclusion about the true value of a number. These two distinct terms, standard uncertainty and expanded uncertainty respectively, relate to these various interpretations of the word "uncertainty."

## 4. TYPES OF MEASUREMENT UNCERTAINTIES

The standard uncertainty is a highly biased measurement result expressed as a standard deviation [3].

Type A uncertainty assessment entails evaluating the uncertainty through statistical analysis of sequences of observations, which includes:

- a sequence of " n " measurements' arithmetic mean is calculated, and the results are $\mathrm{q}_{1} \ldots \mathrm{q}_{\mathrm{n}}$.
- the standard experimental deviation is calculated.

Type B uncertainty assessment - the uncertainty is evaluated using methods other than statistical analysis of a series of observations.

The estimated variance $\mathrm{U}^{2}\left(\mathrm{x}_{\mathrm{i}}\right)$ or the standard uncertainty $\mathrm{U}\left(\mathrm{x}_{\mathrm{i}}\right)$ associated with an estimate of an input quantity $\mathrm{X}_{\mathrm{i}}$ that was not obtained through repeated observations is computed scientifically using all available data. Manufacturer specifications, data specified in various types of certificates (calibration certificates or other), the uncertainty associated with the reference values taken from manuals, previous measurement results, basic knowledge about the properties and behaviour of the instruments and materials used, and manufacturer specifications are just a few examples of information that can be obtained [3].
The information obtained may include the following:

- previous measurement results;
- basic knowledge about the properties and behaviour of the instruments and materials utilised;
- manufacturer specifications;
- data specified in different types of certificates (calibration certificates or other);
- the uncertainty associated to the reference values taken from manuals.

For ease of use, type B variance and type B standard uncertainty are commonly used to refer to the variance $\mathrm{U}^{2}\left(\mathrm{x}_{\mathrm{i}}\right)$ and the uncertainty $\mathrm{U}\left(\mathrm{x}_{\mathrm{i}}\right)$ calculated in this manner, respectively.
Combining the standard uncertainties of the input estimations $\mathrm{X}_{1}, \mathrm{X}_{2}, \ldots, \mathrm{X}_{\mathrm{N}}$ yields the standard uncertainty of $y$. In this case, $y$ stands for the estimated value of the measure Y , while $\mathrm{X}_{1}, \mathrm{X}_{2}, \ldots, \mathrm{X}_{\mathrm{N}}$ are the input values that have an impact on the value of the Y output quantity.
Extended uncertainty U is the measure of the uncertainty that surrounds a measurement's results. A significant portion of the distribution of the values that may be correctly assigned to the measure and must be included within this period.
The following form can be used to express the measurement result: the formula $Y=y U$ can be understood as follows: y is the best estimate of the value ascribed to the Y measure, and the range from $y-U$ to $y+U$ is a range in which it can be said to make up a significant portion of the distribution of values that can be reliably attributed to $Y$, or $y-U \leq Y \leq y+U$.

## 5. MIXED MODELS

The indications of simple measuring instruments are frequently described using a simple linear observation equation involving both multiplicative and additive errors. Consider a meter that measures DC voltage and displays a value win in response to an applied voltage V. An observation equation is defined as:
$w=A_{\text {gain }} V+E_{\text {offset }}$

Because the meter is intended to measure the applied voltage, it is reasonable to assume that A gain is 1.0 and E offset is 0.0 V . These values correspond to a perfect meter, but we can use them as estimates with some uncertainty to account for the imperfect behaviour of a real meter.

Because the applied voltage V is of interest, the observation equation to obtain a measurement model can be rearranged
$V=\frac{w-E_{\text {off set }}}{A_{\text {gain }}}$
As a first step, the difference between the indication and the offset gives us an equation that the addition-subtraction rule can handle.
$V_{1}=w-E_{\text {offset }}$
This intermediate step could be viewed as a prototype for a 'offset-corrected-indication.' After that, dividing the 'offset-corrected-indication' by the gain factor yields an equation that can be solved using the multiplication-division rule.
$V_{2}=\frac{V_{1}}{A_{\text {gain }}}$
The quantity of interest $\left(\mathrm{V}=\mathrm{V}_{2}\right)$ is the result $\mathrm{V}_{2}$. As a result, two simpler models resulted.
Suppose now we have a reading
$w=10.015 \mathrm{~V}$
and the estimates
$a_{\text {gain }}=1.000$ and $e_{\text {offset }}=0.000 \mathrm{~V}$,
with a relative standard uncertainty in the gain of
$\frac{u\left(a_{\text {gain }}\right)}{a_{\text {gain }}}=0.005$
and a standard uncertainty in the offset of
$u\left(e_{o f f s e t}\right)=0.05 \mathrm{~V}$
These equations can be used to calculate the measured value of applied voltage. First, compute $\mathrm{V}_{1}$, which is an estimate of $\mathrm{V}_{1}$.
$v_{1}=w-e_{\text {offset }}=10.015-0.000=10.015 \mathrm{~V}$
This value's standard uncertainty as an estimate of $V_{1}$ is
$u\left(v_{1}\right)=\sqrt{u\left(e_{\text {offset }}\right)^{2}}=\sqrt{(0.05)^{2}}=0.05 \mathrm{~V}$
Second, compute $\mathrm{v}_{2}$, which is an estimate of $\mathrm{V}_{2}$.
$v_{2}=\frac{v_{1}}{a_{\text {gain }}}=\frac{10.015}{1.000}=10.015 \mathrm{~V}$
This estimate's relative standard uncertainty as a $V_{2}$ estimate is
$\frac{u\left(v_{2}\right)}{v_{2}}=\sqrt{\left(\frac{u\left(v_{1}\right)}{v_{1}}\right)^{2}+\left(\frac{u\left(a_{\text {gain }}\right)}{a_{\text {gain }}}\right)^{2}}=\sqrt{\left(\frac{0.05}{10.015}\right)^{2}+\left(\frac{0.005}{1.000}\right)^{2}}=0.00707$.

It should be noted that the results obtained in the first step are required in the second step.
The final result is:
$v=v_{2}=10.015 \mathrm{~V}$,
with a standard uncertainty of
$u(v)=10.015 \times 0.00707=0.071 \mathrm{~V}$.

## 6. CASE STUDIES

a) Uncertainty surrounding the connection between non-EU ETS installations and partially transferred source streams

It is possible that this quantity, measured by an internal sub-meter for the non-EU ETS part of the installation (uncertainty is $5 \%$ ), will be deducted from the source flow quantity when the installation is only partially covered by the EU ETS and not all parts of the installation are included in the scheme. measured by the primary meter that complies with the national metrological control standards (uncertainty is 2 percent) [4].
It is projected that the facility will need $500,000 \mathrm{Nm} 3$ of natural gas annually. The natural gas will be transferred and sold in quantities of $100,000 \mathrm{Nm} 3$ to a facility that does not follow EU ETS regulations. To determine the natural gas consumption for the EU ETS, the linked installation's natural gas consumption must be deducted from the installation's overall natural gas consumption. To assess how erratic the natural gas use under the EU ETS is:
$u_{\text {sourcestream }}=\frac{\sqrt{(2 \% * 500.000)^{2}+(5 \% * 100.000)^{2}}}{|500.000+(-100.000)|}=2.8 \%$
The primary gas meter under national metrological supervision shouldn't have its uncertainty evaluated. It is necessary to evaluate and confirm the uncertainty of the internal submeter that is not covered by national metrological control before establishing the uncertainty related to the source flow.
b) General ambiguity with a backup plan

A category plant solely used natural gas as fuel during the second trade period, resulting in 35,000 tCO2 in annual emissions. The uncertainty associated with the activity data can be as high as 2.0 percent considering the maximum inaccuracy permitted by applicable national legislation because this fuel is acquired through a commercial transaction subject to legal national metrological oversight [4].

The operator provides evidence that an uncertainty assessment for the source flow indicates an uncertainty of $18 \%$, in accordance with GUM ( 95 percent confidence interval). Annual source stream emissions are expected to be $12,000 \mathrm{t} \mathrm{CO} 2$. When implementing a backup strategy on a Category A installation, the operator must demonstrate that the emission uncertainty for the entire installation does not exceed 7.5 percent. The operator must apply the following to determine the uncertainty:
$E m_{\text {total }}=E m_{N G}+E m_{F B}$
$E m_{\text {total }}$ - the installation's overall emissions;
$\mathrm{Em}_{\mathrm{NG}}$ stands for emissions from burning natural gas ( $35,000 \mathrm{t}$ CO2);
Em ${ }_{\text {FB }}$ - emissions that flow through a fallback method from the monitored source $(12,000$ t CO2) [4].

The total uncertainty is determined as follows because the (relative) uncertainty of the total emissions can be viewed as the uncertainties of an amount:
$u_{\text {total }}=\frac{\sqrt{\left((2.0 \% * 35.000)^{2}+(18 \% * 12.000)^{2}\right)}}{|(35.000+12.000)|}=4.8 \%$
The overall installation's emission uncertainty is less than 7.5 percent. As a result, the suggested fallback strategy is appropriate.

## CONCLUSIONS

Separated from the working range, atmospheric conditions (wind, temperature variation, humidity, corrosive substances), working conditions (adhesion, density and viscosity variation, irregular flow), installation conditions, and long-term stability can be listed as some of the main influences of uncertainty.
Models are helpful in a variety of additional contexts in addition to data processing and uncertainty computation in measurement. They can be used as a specification during the development of data-analysis algorithms, as a point of reference when validating software, and they can both describe a measurement process and act as a tool for understanding, improving, and implementing that process. They can also shed light on the strengths and weaknesses of a specific measurement process and suggest ways to improve them.
Although uncertainty analysis is difficult, with practice and experience, one may become more skilled and confident in their ability to analyse measurement procedures. There is no "correct" or "wrong" way to go about it; you may use more or less information, use a different distribution to express influences, and develop the model over time.

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