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GAUSS-JACOBI'S ITERATION METHOD FOR NATURAL GAS WELLS PRODUCTION FORECAST USING DECLINE CURVE ANALYSIS

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ABSTRACT

This study seeks to perform production forecasts of gas wells using the concept of decline curve analysis (DCA) and regression analysis. The regression analysis was performed by fitting a polynomial curve fitting model to a set of production history data, so as to determine the initial gas flowrate. In addition, Gauss Jacobi method was used to provide a solution to the solved five-by-five matrix in order to compute values for the regression coefficients. Subsequently, a computer program "ROTEX" was developed to predict the efficacy of the proposed DCA and regression analysis method. Three wells (WELL Y1, Y2 and Y3) from a field in the Niger Delta region were evaluated with respect to forecasting future gas rates till abandonment. Consequently, Well Y1 was observed to have the potential to continue producing for the next 30 years, while Well Y2 and Y3 have the potential to produce for roughly five years before they reach abandonment. R-squared values were computed for each case, as a means to validate the integrity of the fitted regression curves to the production history data. All R-squared values were observed to be very close to unity and are thus considered reliable.

Keywords: analysis, flowrate, forecast, production, regression

INTRODUCTION

For decades, extrapolation of production history trends has been considered the most reliable method of estimating the remaining recoverable reserve from a well. Several methods have been developed so far to the earlier mentioned concern, ranging from the type-curve matching method to the frequently used decline curve analysis (DCA) method. The productivity of every well is dependent on its flow rate [1]. The future production rates of a gas well can be determined based on the rate at which the reservoir pressure is experiencing decline [2]. Basically, decline rate analysis involves extending the trend of certain reservoir or production parameters. Just by the use of production history data obtained over yester-periods (previous years), decline curves can forecast the future production rates of a well of interest with some level of precision [3]. Decline curve analysis (DCA) is also regarded as a method where production history data from a reservoir or well is used to predict the well future production [4]. The two primary goals of DCA are to estimate the remaining reserves and the remaining life down to a specified economic limit [5]. Predicting the production of a well is paramount in determining the value of a well. An over prediction of the well's performance is detrimental to the management teams of marginal field operators and international oil companies (IOC's). Decline curves are the most common means of forecasting production. Decline curves make use of data which are easy to obtain, they also yield results which can be on time basis, and most likely, the trends are deceptive to analyse [6]. A typical exponential model forecast of oil production rates can be seen in Fig. 1.

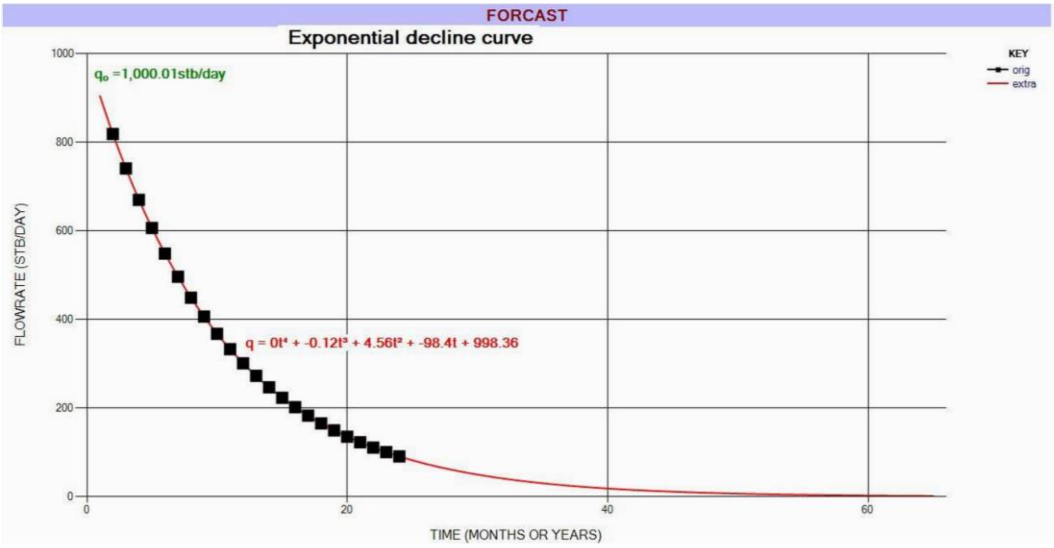


Fig. 1. *Typical DCA chart for an exponential model [7]*

Determining the initial flowrate of a well (i.e. the gas rate at time zero) can be tasking especially for a harmonic or hyperbolic trend. Nonetheless, existing methods involves the plot of production history data on spreadsheets before arriving at a solution, which is time consuming. More tedious method involves physically plotting gas production history data on a graph with the intent of obtaining the initial gas rate at time zero, which is even more tedious than using spreadsheets. It is therefore imperative to have a model that would study the behaviour of the gas production history for a period of years and predict future production rates as well as being informed on the project lifespan (the entire period of production to abandonment). The goal of decline curve analysis (DCA) is to predict future production and ultimate recovery for wells that have already produced. Accordingly, DCA can be used to predict future hydrocarbons (both oil and gas wells) production by studying past production [8]. Therefore, this study seeks to develop a model that can make future forecast of gas production rates using the concept of DCA and regression analysis. Once future gas production rates can be determined, it would be easy to estimate cumulative gas production for a specified period as well as the recoverable gas from different natural gas wells.

METHODOLOGY

The computer program (ROTEX) was developed using Microsoft C# programming language, incorporating the mathematical models presented herein. Field data from the Niger Delta region of Nigeria were used in this study with the intent of determining the future gas rates of the concerned wells. Three wells were analysed namely, WELL Y1, WELL Y2 and WELL Y3.

Mathematical models used for ROTEX development

In the course of making production rate forecasts, it was paramount that an appropriate model that best fits the gas production data was selected. To achieve this, there was a need to make some diagnostic plots either on a log-log scale or on a semi-log scale. The identified model was subsequently fitted into the gas production data for production rate forecast. The models used in this study are presented as follows:

a. Diagnostic models

The following procedures were observed in determining the appropriate decline model for the supposed gas history data;

i. A plot of log (q) against time(t) gives a steep straight-line trend for an exponential decline model

ii. A plot of log (q) against (t) – i.e. a semi-log plot – gives a slightly straight-line trend for a hyperbolic decline model. This can be expressed mathematically in Eq. 1 as follows;

$$\ln q = \mathrm{mt} + \mathrm{k} \tag{1}$$

b. Exponential decline curve model

The exponential models for decline curve analysis are presented in the following equations;

$$q_{0,t} = q_i e^{-bt} \tag{2}$$

$$b = \frac{1}{t_2 - t_1} In\left(\frac{q_1}{q_2}\right) \tag{3}$$

As presented in Eq. 2, it is evident that it is easy to forecast the future production rates of the producing once the initial production rate q_i is known. Moreover, the initial rate will be calculated in this study by incorporating polynomial models into the production data and extrapolating it to acquire the initial production rate.

c. Harmonic decline curve model

Harmonic model is shown in the following mathematical expressions;

$$q_{0,t} = \frac{q_i}{1+bt} \tag{4}$$

$$b = \frac{\frac{q_o}{q_1} - 1}{t_1} \tag{5}$$

Unlike exponential model, the initial rate for a harmonic model can be easily computed without using a regression model. This can be done by using the following mathematical relationship in Eq. 6;

$$q_i = q(1 + bt) \tag{6}$$

d. Hyperbolic decline curve model

The steps for fitting a hyperbolic curve into a given production data can be found in [3] and in the study on forecasting oil production rates for oil wells using decline curve analysis and polynomial regression model [7].

e. Polynomial regression model

Least square polynomial regression model as proposed in 2012 by Chapra [9] is presented in Eq. 7 below;

$$f(x) = a_o + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \ \ldots\ldots + a_nx^n + e \tag{7}$$

The above mathematical expression Eq. 7 was used to determine the initial gas flowrate, q_i, which is the gas rate at time zero. The expression is re-arranged to give Eq. 8 as follows

$$q(t) = a_o + a_1t + a_2t^2 + a_3t^3 + a_4t^4 + \ \ldots\ldots + a_nt^n + e \tag{8}$$

Where a_o, a_1, a_2, a_3 e.t.c. refers to the regression coefficients.

In this study, a fourth order polynomial was fitted so as to determine initial flowrate, q_i. However, the above coefficients can be obtained from the following matrix presented as Eq. 9.

$$\begin{bmatrix} n & \sum t & \sum t^2 & \sum t^3 & \sum t^4 \\ \sum t & \sum t^2 & \sum t^3 & \sum t^4 & \sum t^5 \\ \sum t^2 & \sum t^3 & \sum t^4 & \sum t^5 & \sum t^6 \\ \sum t^3 & \sum t^4 & \sum t^5 & \sum t^6 & \sum t^7 \\ \sum t^4 & \sum t^5 & \sum t^6 & \sum t^7 & \sum t^8 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} \sum q \\ \sum(tq) \\ \sum(t^2q) \\ \sum(t^3q) \\ \sum(t^4q) \end{bmatrix} \tag{9}$$

Invariably, the above mathematical expression can only be solved computationally, by the use of linearization models like Gaussian elimination model or Gauss-Jacobi algorithm [10]. This study however utilizes the Gauss-Jacobi method.

Coefficient of determination calculation procedure

Coefficient off determination R^2 was calculated to determine the level of fitness of the fitted polynomial regression curve. It is important to note that the polynomial regression was used to determine the initial gas flowrate of the concerned gas well. The mathematical relationship for R^2 is presented in Eq. 10 as follows;

$$R^2 = 1 - \frac{\sum(q - q_{st})^2}{\sum(q - \overline{q})^2} \tag{10}$$

Where q_{st} refers to the estimated value of q; q is the individual flowrate of the gas well and $\overline{q}$ refers to the mean flowrate which is obtained by dividing the total gas rates by the total number of variables “n”.

Computer Model Development

Prior to the use of selected decline curve model in making forecasts of production rates, it was imperative to first specify the value for initial production rate. Conventionally, this can be achieved by extrapolating the curve until it hits the q-axis at t=0. Nonetheless, in order to perform this procedure computationally, this study employs the use of a regression model as well as Gauss-Jacobi algorithm for the calculation of coefficients [10]. The subroutine flowchart developed in this study for Gauss-Jacobi is presented in Fig. 2. Meanwhile, detailed algorithm of Gauss-Jacobi algorithm is provided by Chapra (2012) in [9]. This algorithm was used to determine the regression coefficients for the polynomial regression. It is important to note that this regression was performed to determine the initial gas flowrate of the producing well prior to performing production forecast. The splash screen appears as shown in Fig. 3 and the flowchart for the developed computer model (ROTEX) is shown in Fig. 4, while Fig. 5 shows user interface dialog box for gas production history data import.

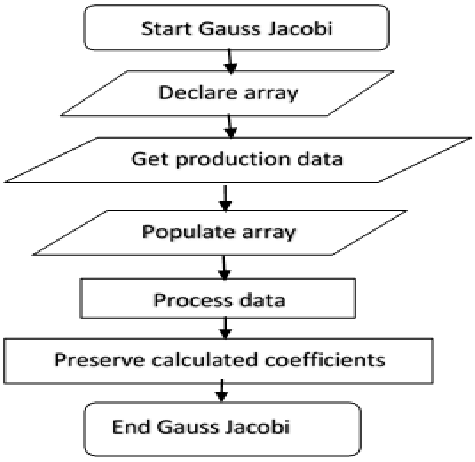


Fig. 2. *Procedure flowchart for Gauss-Jacobi computer algorithm*

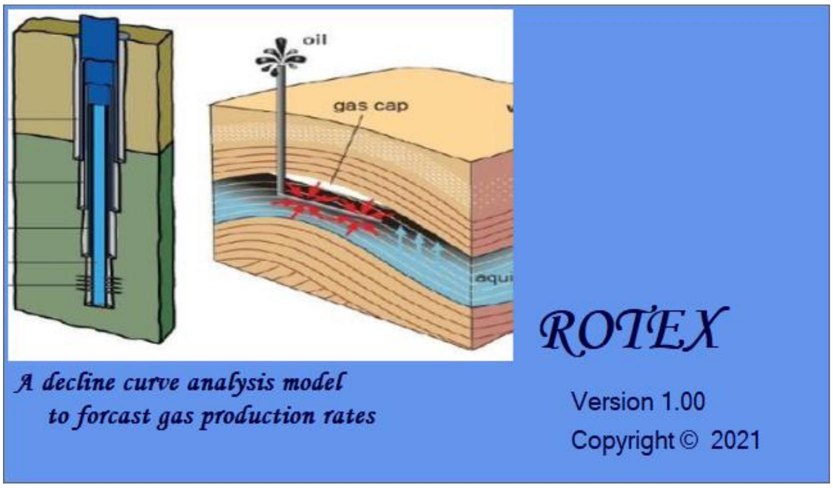


Fig. 3. *Splash screen of ROTEX computer program*

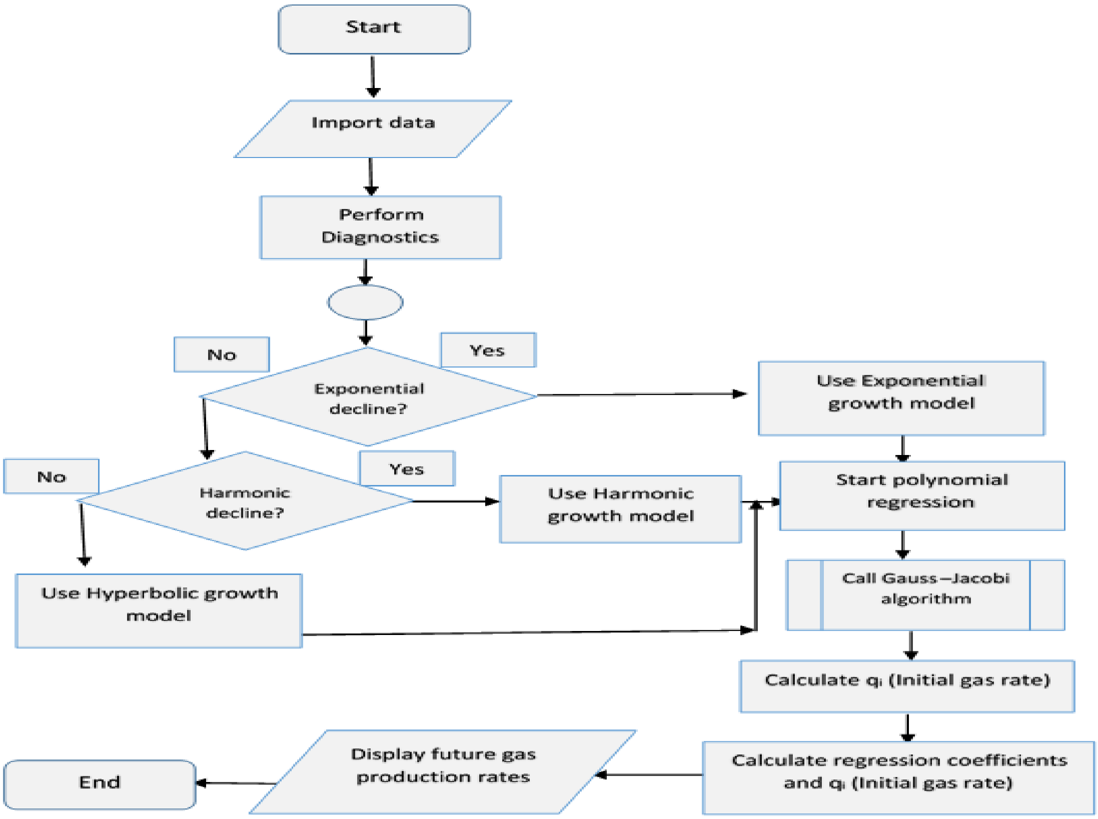
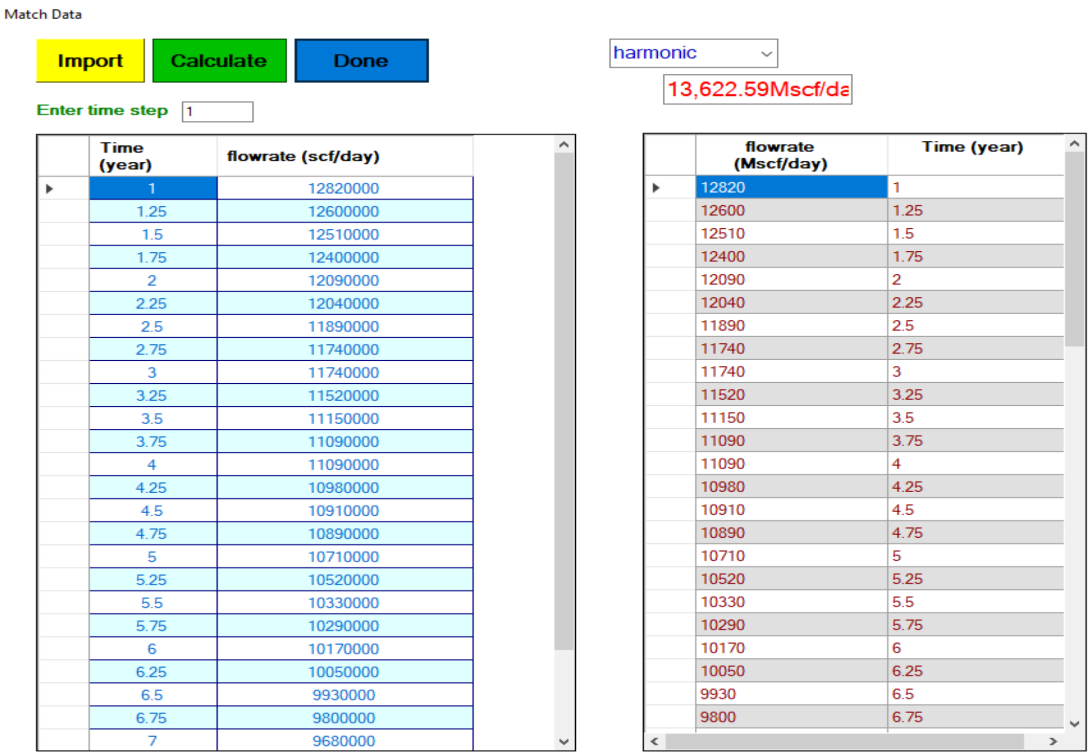


Fig. 4. *Overall flowchart for the developed computer model- ROTEX*



Match Data

Import | Calculate | Done

Enter time step: 1

harmonic

13,622.59Mscf/da

Time (year)	flowrate (scf/day)
1	12820000
1.25	12600000
1.5	12510000
1.75	12400000
2	12090000
2.25	12040000
2.5	11890000
2.75	11740000
3	11740000
3.25	11520000
3.5	11150000
3.75	11090000
4	11090000
4.25	10980000
4.5	10910000
4.75	10890000
5	10710000
5.25	10520000
5.5	10330000
5.75	10290000
6	10170000
6.25	10050000
6.5	9930000
6.75	9800000
7	9680000

flowrate (Mscf/day)	Time (year)
12820	1
12600	1.25
12510	1.5
12400	1.75
12090	2
12040	2.25
11890	2.5
11740	2.75
11740	3
11520	3.25
11150	3.5
11090	3.75
11090	4
10980	4.25
10910	4.5
10890	4.75
10710	5
10520	5.25
10330	5.5
10290	5.75
10170	6
10050	6.25
9930	6.5
9800	6.75

Fig. 5. *Match data form (ROTEX MODEL)*

RESULTS AND DISCUSSION

This section discusses the results obtained from the three wells (WELL-Y1, WELL-Y2 and WELL-Y3) that were analysed from the developed program (ROTEX).

Discussion of Results for WELL-Y1

First, a diagnostic plot was performed to determine the nature of the decline trend exhibited by the well (WELL-Y1). The semi-log plot was made with Gas flowrate as the ordinate on the logarithm axis and time as the abscissa on the cartesian axis.

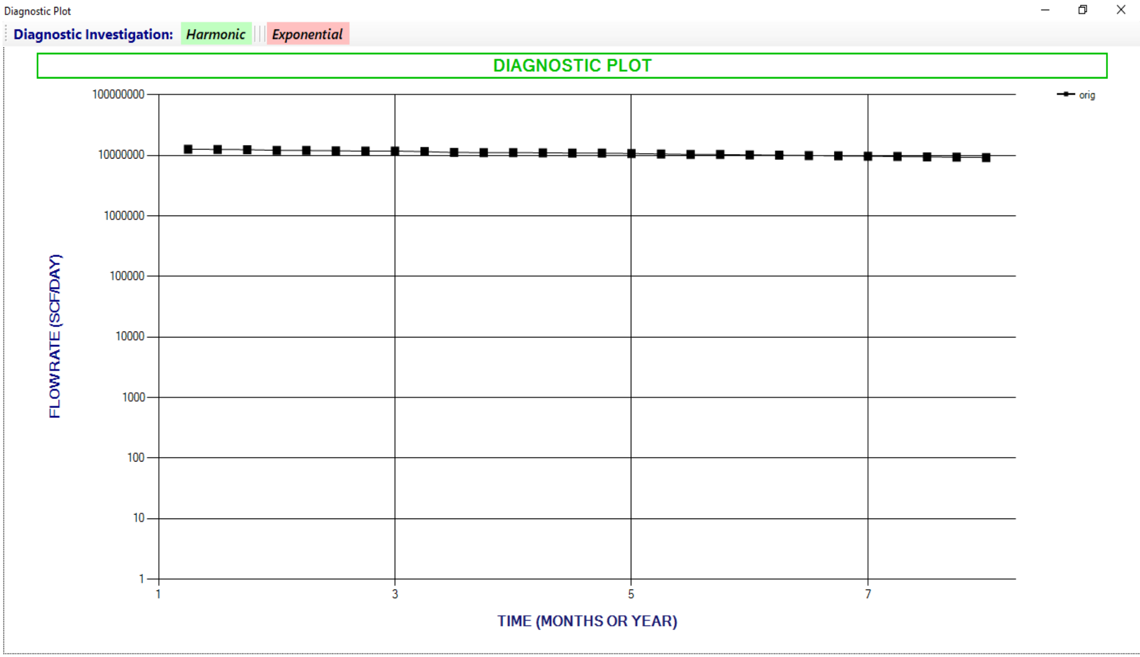


Fig. 6. *Diagnostic Plot for WELL-Y1*

As shown in Fig. 6, the plot showed a harmonic decline trend. Hence, a harmonic decline model was used by the computer program to perform forecast of future gas production rates. An initial gas rate was calculated as 13,622.59 mscf/day using the polynomial regression analysis of the gas production history data. The regression analysis on the production history data was fitted with an R-squared value of 0.996 which is an acceptable level of precision – since the value is very close to unity (See Fig. 7). The coefficients of the regression were computed using the Gauss-Jacobi method while the value of q_i was determined when t = 0. The decline trend was able to forecast a flow rate of 4000 mscf/day of gas production in the next thirty years of production. However, assuming an economic limit of 4000 mscf/day, WELL-Y1 only has roughly 31 years to be produced. This result is reflective of the fact that the reservoir pressure is still very active and would take many years for it to be depleted.

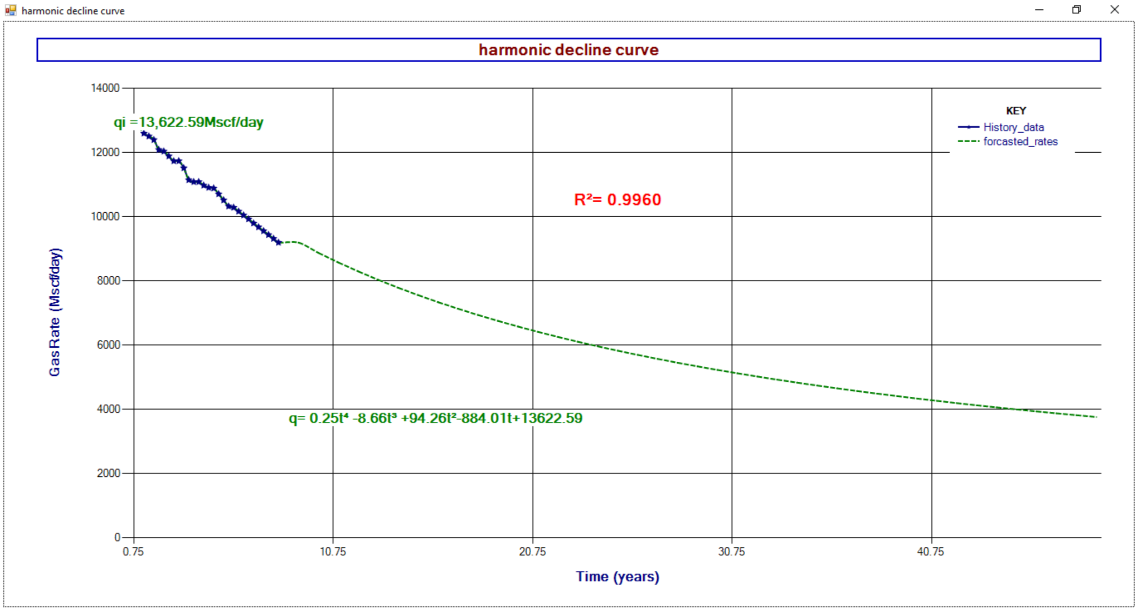


Fig. 7. *Harmonic decline forecast for WELL-Y1*

Discussion of Results for WELL-Y2

As shown in Fig. 8, the semi-log diagnostic plot was made to ascertain whether the decline trend is harmonic or hyperbolic.

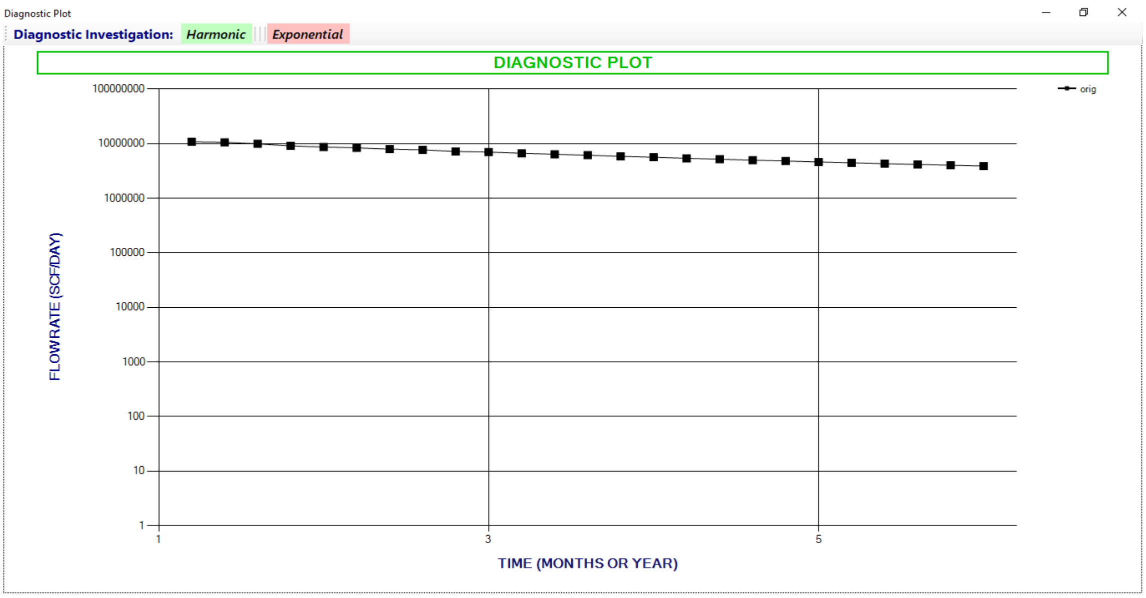


Fig. 8. *Diagnostic Plot for WELL-Y2*

The resulting plot did not give a straight line thus, confirming that the model cannot be harmonic. Nevertheless, it was confirmed to be a hyperbolic decline. More so, the initial gas flowrate was determined as 14,219.83 mscf/day and the R-squared value of the fitted polynomial regression was calculated as 0.9976. However, the gas rate forecast performed by the computer program, shows that the gas flowrate of WELL-Y2 will decrease from 3,880 mscf/day to 2001 mscf/day in the next 5 years (from the 6th year to the 11th year). Assuming the economic limit of the well is 2000 mscf/day, it means that the well only has five years until it reaches abandonment (see Fig. 9).

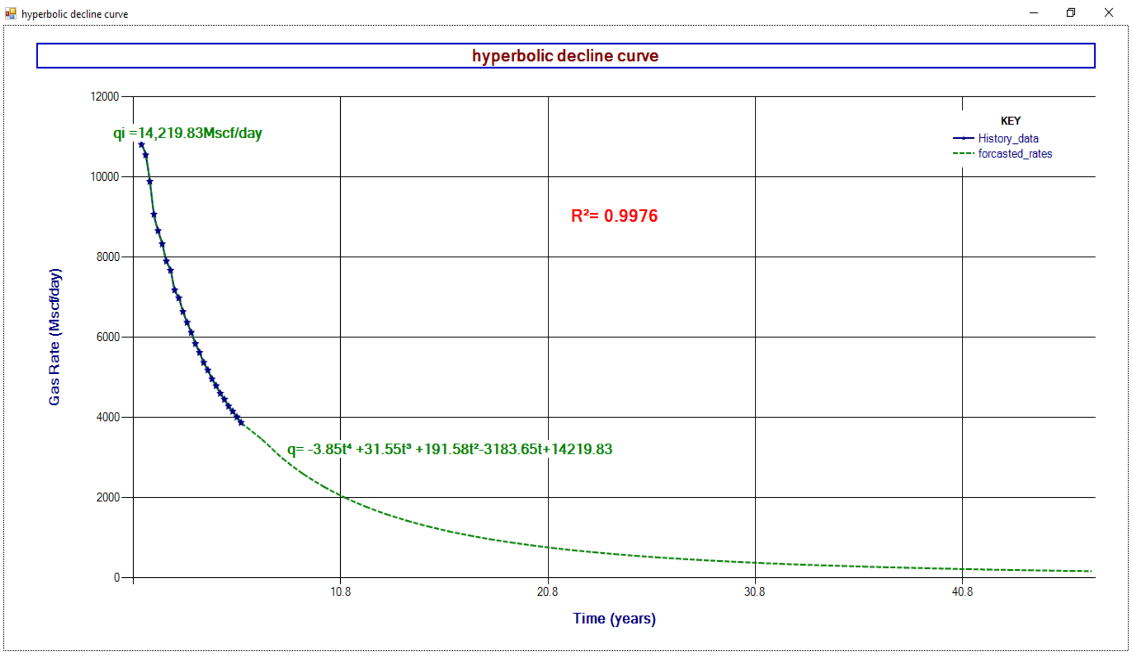


Fig. 9. *Hyperbolic decline forecast for WELL-Y2*

Discussion of Results for WELL-Y3

The semi-log diagnostic plot as shown in Fig. 10, was made to ascertain whether the decline trend is harmonic, exponential or hyperbolic.

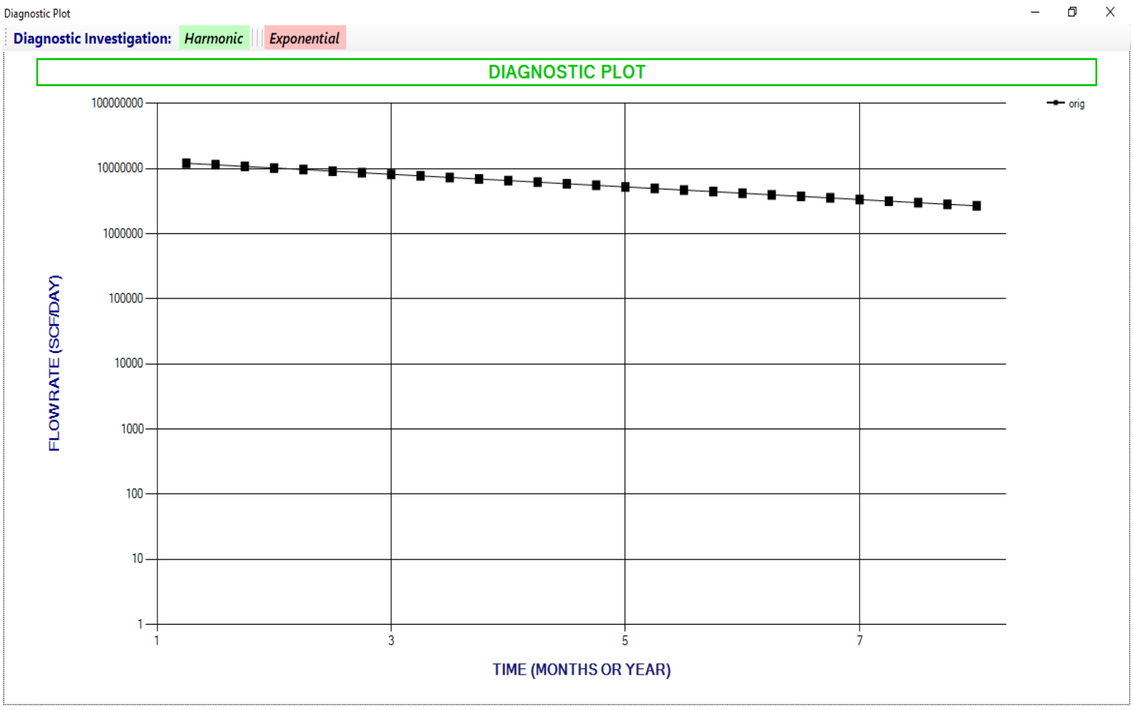


Fig. 10. *Diagnostic Plot for WELL-Y3*

The nature of the trend suggested either an exponential or hyperbolic decline. However, hyperbolic decline curve failed to fit the gas production history data. Meanwhile, an exponential model fitted perfectly as shown in Fig. 11. In addition, the initial gas flowrate was determined as 15,917.77 mscf/day and the R-squared value of the fitted polynomial regression was calculated as 1.0. Moreover, the gas rate forecast performed by the computer program, revealed that gas flowrate of WELL-Y3 will decrease from 2,680 mscf/day in the 8th year to 881.89 mscf/day in the 13th year (i.e. in the next 5 years). Invariably, assuming the economic limit of the well is 800 mscf/day, which implies that the well only has 5 years until it reaches abandonment (see Fig. 11).

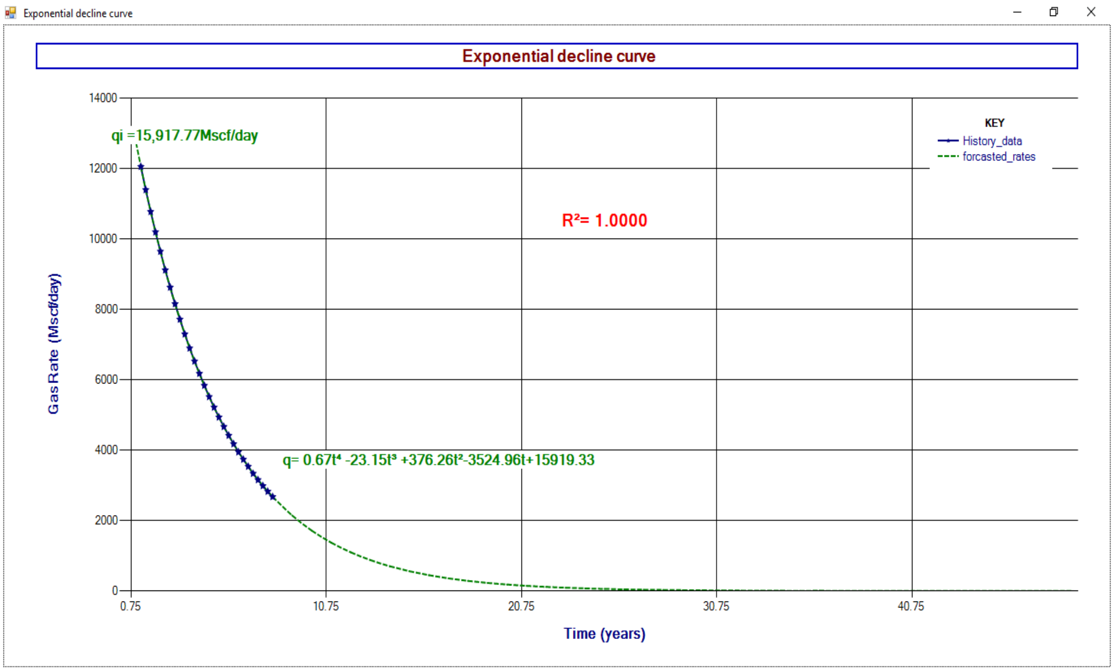


Fig. 11. *Exponential decline forecast for WELL-Y3*

CONCLUSION

This study has ascertained the use of polynomial regression model as a means to complement the conventional DCA method in the forecast of future production rates of natural gas wells. The proposed polynomial regression method is useful in the direct estimation of initial flowrate without having to plot charts as opposed to the conventional method. A computer program, ROTEX was developed in the course of this study, using Microsoft C# programming language. In addition, having developed the computer program, three wells (Wells Y1, Y2 and Y3) were evaluated with the help of the developed computer program. The proposed regression analysis performed adequately in calculating the initial flowrate of each of the gas wells that were studied. The coefficient of determination was also determined by the developed computer program, to ascertain the level of fitness of the polynomial regression while calculating the initial flowrate of the concerned gas wells at the time of zero. More so, all R-squared values were observed to be very close to unity hence, emphasizing the integrity of the polynomial curve fitting. Nevertheless, from the evaluation performed on the three wells as regards to forecasting future production rates, Well Y1 was observed to have the potential to continue producing for the next 30 years before it reaches abandonment, while Well Y2 and Y3 were observed to have the potential to produce for roughly five years before they reach abandonment.

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