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A Method of Calculation of the Tension and Displacements in an Extensible Cable Loaded with a Uniform External Force

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Abstract

In the paper is presented an exact method that allows the calculation of the tension and geometric parameters of an extensible cable supported at the ends and loaded with a uniform external force. The solution obtained contains the hyperbolic functions and may be applied for any types of cables and geometries of the suspensions.
The results obtained are analysed in a calculus example.

Keywords: *extensible cable, tension and suspension points*

General Equations

It is considered an extensible cable subjected to a uniform external force p (fig. 1). The cable has the l horizontal distance and the h vertical distance between the suspension points. The displacement of the cable at the half of the length is f.

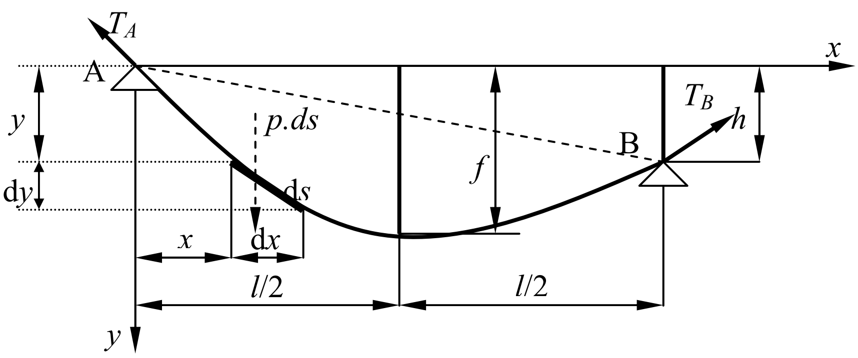

Fig. 1. The cable supported at the ends

The differential equations that describe the behaviour of the cable are [1] :

$$\frac{d}{ds}\left(T\frac{dx}{ds}\right)+p_x=0 \quad \text{a)} \qquad \frac{d}{ds}\left(T\frac{dy}{ds}\right)+p_y=0 \quad \text{b)} \qquad , \tag{1}$$

where T represents the current tension in the cable and p_x and $p_y = p$ the intensities of the distributed external forces on the length of the cable in the x and respectively y directions.
Because in case presented in the figure 1 $p_x=0$ and $p_y = p$, from (1a) it results :

$$T\frac{dx}{ds}=const.=H \tag{2}$$

The result obtained in (2) proves that the horizontal force in the cable is always constant. Replacing (2) in (1b) the differential equation of the displacement of the cable can be put under the form :

$$\frac{d^2y}{dx^2}=-\frac{p}{H}\frac{ds}{dx} \tag{3}$$

Taking into consideration that $ds=\sqrt{1+y'^2}\cdot dx$, the (3) differential equation can be written also as :

$$\frac{dy'}{\sqrt{1+y'^2}}=-\frac{p}{H}dx \tag{4}$$

The exact solution of the above differential equation is :

$$y'=sh\left(-\frac{p}{H}x+C_1\right) \tag{5}$$

Integrating (5) once again it results :

$$y=-\frac{H}{p}ch\left(-\frac{p}{H}x+C_1\right)+C_2 \tag{6}$$

The C_1 and C_2 constants can be obtained by solving the system of equations that results from the limit conditions :

$$\begin{aligned} &x=0 \Rightarrow y=0 \qquad \text{a)} \\ &x=l \Rightarrow y=h \qquad \text{b)} \end{aligned} \tag{7}$$

Replacing (6) in (7) it results the following system of equations :

$$0 = -\frac{H}{p} ch C_1 + C_2 \qquad \text{a)}$$
$$h = -\frac{H}{p} ch\left(-\frac{p}{H} l + C_1\right) + C_2 \qquad \text{b)} \tag{8}$$

Solving the above system the unknown constants can be written under the form :

$$C_1 = \ln\left[\frac{hp}{H}\frac{e^{\frac{pl}{H}}}{e^{\frac{pl}{H}} - 1} + \sqrt{\left(\frac{hp}{H}\frac{e^{\frac{pl}{H}}}{e^{\frac{pl}{H}} - 1}\right)^2 + e^{\frac{pl}{H}}}\right] \qquad \text{a)}$$
$$C_2 = \frac{H}{p} ch C_1 \qquad \text{b)} \tag{9}$$

In the case that at the middle of the horizontal distance between the suspension points the displacement of the cable is imposed to be f from (6) it results :

$$f = -\frac{H}{p} ch\left(-\frac{pl}{2H} + C_1\right) + C_2 \tag{10}$$

The above relation contains the unknown H that can be determined solving numerically (10).
After the horizontal force H is determined the difference between the final and initial length of the cable can be calculated with the relation [2] :

$$\Delta l = \frac{H}{E \cdot A} \cdot \int_0^l \left(1 + y'^2\right) dx \approx \frac{H \cdot l}{E \cdot A} \tag{11}$$

The final length of the cable can be calculated with :

$$L = \int_0^l \sqrt{1 + y'^2} \cdot dx \tag{12}$$

where y' is expressed in (5).
In the case that the variation of the temperature is neglected the initial length of the cable can be obtained as the difference between the (12) and (11) relations.

$$L_o = L - \Delta L \tag{13}$$

Calculus Example

It is considered an extensible cable made from *OL37* with cross sectional area $A = 3{,}36\ cm^2$, loaded with a uniform external force $p = 29\ N/m$. The cable is suspended at the ends in the points A and B. The horizontal distance between the suspension points is $l = 150\ m$ and the vertical distance is $h = 10\ m$. It is necessary to establish the initial length of the cable if the deflection at the middle of the horizontally distance is imposed to be $f = 40\ m$.

In the classical method it is supposed as a hypothesis that the weight of the cable is uniform distributed on the horizontal distance between the *A* and *B* suspension points. The main steps used in order to calculate the initial length of the cable are :

- it is determined the horizontal component of the force of the cable :

$$H = \frac{p \cdot l^2}{8 \cdot \left(f - \frac{h}{2} \right)} = \frac{29 \cdot \left(150^2\right)}{8 \cdot \left(40 - \frac{10}{2} \right)} = 2330{,}357 \;\; N$$

- the increasing length of the cable is calculated :

$$\Delta l = \frac{H \cdot l}{E \cdot A} = \frac{2330{,}357 \cdot 150}{2.1 \cdot 10^{11} \cdot 3.36 \cdot 10^{-4}} = 0{,}004954 \, m$$

- the final length of the cable is calculated :

$$L = l + \frac{2(2f - h)^2}{3l} + \frac{h^2}{2l} = 150 + \frac{2}{3} \frac{(2 \cdot 40 - 10)^2}{150} + \frac{10^2}{2 \cdot 150} = 172.11 \, m$$

- the initial length of the cable is calculated :

$$L_o = L - \Delta l = 172.11 - 0.004954 = 172{,}106 \, m$$

The exact method (presented in this paper) has the following steps :

- the numerically solving of the (10) equation in order to calculate the horizontal component of the force of the cable :

$$H = 2487 \; N$$

- the increasing in length of the cable is calculated :

$$\Delta l = \frac{H \cdot l}{E \cdot A} = \frac{2487 \cdot 150}{2.1 \cdot 10^{11} \cdot 3{,}36 \cdot 10^{-4}} = 0{,}05287 \; m$$

- the final length of the cable is calculated with the (13) relation :

$$L = 170{,}1596 \, m$$

- the initial length of the cable is calculated :

$$L_o = L - \Delta l = 170.1543\, m$$

It can be noticed that the method presented in this paper is easy to be used, the only difficulty being in the numerically solving of the (10) equation.

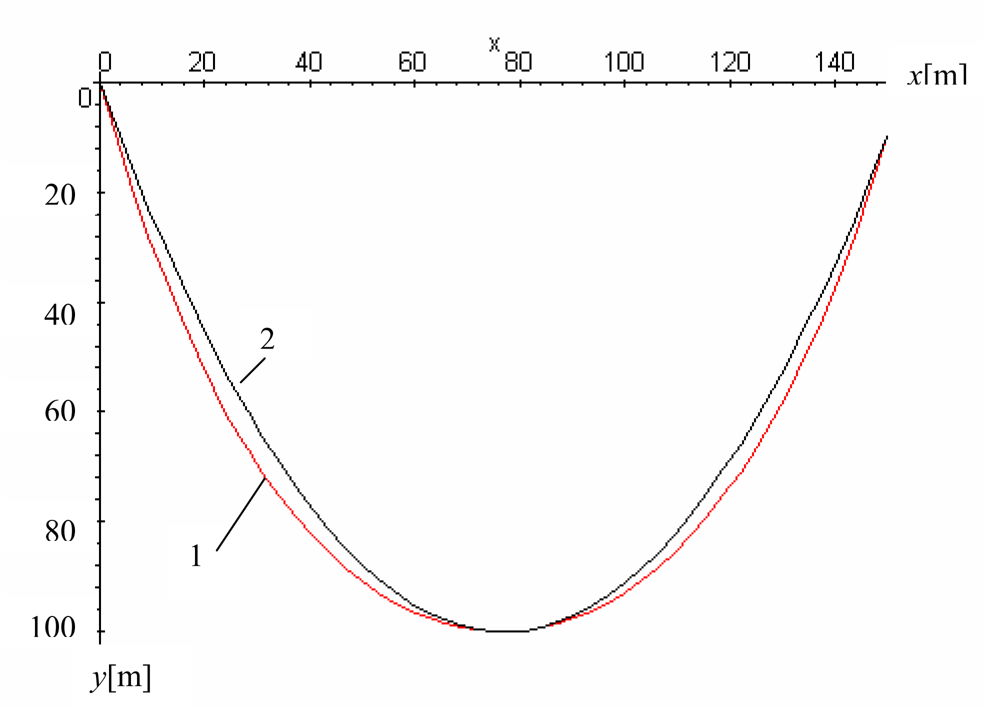

Fig.2. The deformed shapes of the cable

If the same calculus are repeated for the same cable with an imposed deflection at the middle $f = 100\, m$, the initial lengths of the cable are : $L_o = 310.77\, m$ (by classical method- curve 1) and $L_o = 257.342\, m$ (by the method presented in this paper-curve 2). The equilibrium curves of the cable are presented in the figure 2. It can be noticed that, in this case, the difference between the initial lengths of the cable is about 20%.

Conclusions

In the paper is presented an exact method of calculation in order to establish the tension force and displacements in an extensible cable. The result obtained are analysed in a calculus example. From the presented example it can be noticed that for a current distance between the suspension points (150 m), a vertical distance between the same points (10 m) and an imposed deflection in the cable displacement (40 m), the initial lengths of the cable (obtained by the classical and the above method) are less than 2 m, that means approximately 1.2%.

For the same cable, imposing a higher deflection (100 m), the difference between the initial lengths of the cable is higher than 20%, that can reach important economy of materials.
It can be appreciated that the presented method allow the calculation of an exact length of the cable that is smaller than the classical one, but the differences became considerable when the fraction f/l (maximum deflection/horizontal distance between the suspension points) is higher than 30%. The methodology that is presented in the paper can be applied in any situation, for any dimensions of the cable and any geometry of the suspension points.

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O metodă de calcul al tensiunii şi deplasărilor dintr-un cablu extensibil supus unei solicitări exterioare uniforme

Rezumat

În lucrare se prezintă o metodologie de determinare a tensiunii şi deplasărlor dintr-un cablu extensibil solicitat din exterior cu o forţă uniform distribuită pe deschiderea orizontală. Se utilizează soluţia exactă a ecuaţiei diferenţiale a unui astfel de cablu, care se integrează în funcţie de condiţiile la limită impuse de geometria suspendării. Rezultatele obţinute sunt analizate pe un exemplu de calcul în care punctele de suspensie sunt distanţate atât pe orizontală cât şi pe verticală şi este impusă o săgeată maximă la mijlocul deschiderii orizontale. Rezultatele obţinute sugerează faptul că pentru rapoarte dintre săgeata maximă şi deschiderea orizontală (f/l) mai mari de 30%, metoda exactă furnizează lungimi iniţiale şi deformate ale cablului diferite de cele obţinute prin metoda clasică. De aceea se recomandă ca, în astfel de situaţii, calculul să fie realizat după procedeul expus în această lucrare.