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Effective Stresses of Non-fractured Elastic Rocks around a Horizontal Wellbore

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Abstract

For the case of porous rocks saturated with fluids under pressure, the real (effective) stress values from the mineral skeleton are generally different from the apparent values. A correct knowledge of the deformations of such rocks requires an exact evaluation of the effective stresses. A practical situation of this kind is encountered in the case of drilled rocks. Such situation is analysed in this paper based on a new model for the calculation of effective stresses in general.

Key words: *non-fractured elastic rocks, drilling, horizontal wellbore, well fluid pressure, pore pressure, apparent stress, effective stress, partition laws.*

Introduction

The drilled through rocks, placed around a wellbore filled with drilling fluid, are normally subject to various mechanical loads. Among these, more frequent and more important are: the weight of the strata above the considered one, the well fluid pressure and the pore pressure.

The first mathematical models used for the stress evaluation in the rocks close to an underground excavation did not take into account the presence of pores and pore pressure. The results obtained with these models are known as *apparent* stress values. The fact that such results were unsatisfactory for practical purposes led to the development of improved models which aimed at the calculation of the real (*effective*) values of the stresses from the mineral skeleton of the rocks and which unavoidably had to take into account the porosity and the pore pressure. Unfortunately, the calculation models known to that purpose give results which do not fully correspond to the real ones.

This paper presents the partition laws of the effective stresses around a horizontal wellbore, obtained on the basis of a new concept [1] of effective stress evaluation. Such concept takes simultaneously into account both the presence of pores and the pore pressure.

The partition of the effective stresses for other categories of rocks (inelastic, fractured) and the practical consequences of the new forms of partition will be presented in separate works to be soon published. The case of a vertical wellbore which goes through a massive of elastic non-fractured rocks has been presented in [2].

Stress Partition Laws

We consider a drilled wellbore, filled with fluid, which presents – at the depth H – a *horizontal* portion with the radius a (see fig. 1). The stratum that includes it is made from an *elastic non-fractured rock* with an approximately uniform distribution of the pores – as shape and size – in its entire volume.

The *natural* state of stress, also called *primary* state – existent in this stratum at the depth H before the excavation was made, is considered to be known and described by means of the normal vertical stress σ_v and the horizontal one σ_h. In addition, we assume that the vertical direction is one of principal stress directions, and the horizontal stress, for a given depth, has the same value whatever the position of a vertical sectioning plane passing through the considered material point N.

Once the excavation is made the primary state of stress of the rocks around it will change, the new stress values define the so-called *secondary state*, and the area around – in which the stress values are modified – is called the zone of the secondary state or, in brief, the *secondary zone*.

The secondary values of the *apparent* stresses in this zone are the result of the new mechanical equilibrium – obtained after the excavation is made – formed between the initial (primary) stress values of the rock and the mechanical load inside the excavation, that is the well fluid pressure, noted p_s , which is acting on its walls.

Due to their axial symmetry, the excavations having the shape of a circular cylinder, as the wellbores are considered, are preferred to be referred to the cylindrical coordinates system $Or\theta z$ with the Oz axis superposed to the wellbore axis and with the polar coordinates r and θ placed in the plane of its transversal section. With respect to this coordinates system, the *apparent stress tensor* – in an random material point M (r, θ), ($r = OM$, $r > a$) – has the following known form:

$$T_\sigma = \begin{pmatrix} \sigma_r & \tau_{r\theta} & \tau_{rz} \\ \tau_{\theta r} & \sigma_\theta & \tau_{\theta z} \\ \tau_{zr} & \tau_{z\theta} & \sigma_z \end{pmatrix}, \tag{1}$$

where σ_r, σ_θ, σ_z are the normal apparent stresses for the radial, circumferential and axial directions respectively, and $\tau_{r\theta}$, $\tau_{\theta z}$, τ_{zr} , … are the tangential apparent stresses from the planes defined by each two of these directions.

For the situation analysed, the tangential stresses – with the exception of $\tau_{r\theta}$, $\tau_{\theta r}$ ($\tau_{\theta r} = \tau_{r\theta}$) – are equal to zero. The partition laws of the apparent stresses which are not equal to zero are the ones known [3], i.e.:

$$\sigma_r = \frac{\sigma_v + \sigma_h}{2}\left[1 - \left(\frac{a}{r}\right)^2\right] + p_s\left(\frac{a}{r}\right)^2 - \frac{\sigma_v - \sigma_h}{2}\left[1 - 4\left(\frac{a}{r}\right)^2 + 3\left(\frac{a}{r}\right)^4\right]\cos 2\theta \ , \tag{2}$$

$$\sigma_\theta = \frac{\sigma_v + \sigma_h}{2}\left[1 + \left(\frac{a}{r}\right)^2\right] - p_s\left(\frac{a}{r}\right)^2 + \frac{\sigma_v - \sigma_h}{2}\left[1 + 3\left(\frac{a}{r}\right)^4\right]\cos 2\theta \ , \tag{3}$$

$$\sigma_z = \sigma_h + 4\mu\frac{\sigma_v - \sigma_h}{2}\left(\frac{a}{r}\right)^2 \cos 2\theta \ , \tag{4}$$

$$\tau_{r\theta} = \frac{\sigma_v - \sigma_h}{2}\left[1 + 2\left(\frac{a}{r}\right)^2 - 3\left(\frac{a}{r}\right)^4\right]\sin 2\theta \ . \tag{5}$$

As it can be observed, the apparent values of these stresses, for the rocks placed in the vicinity of the excavation (in the secondary zone), are function only of the mechanical loads exterior to this zone, expressed by means of the primary stress values, σ_v, σ_h, and the well fluid pressure, p_s, while they do not take into account the presence of pores and the pore pressure. In addition, the values of these stresses are influenced by the radius of the excavation, a, the position of the considered point with respect to the wellbore axis (the polar coordinates, r and θ), and the mechanical properties of the rock (the transversal contraction coefficient or *Poisson* coefficient, noted μ).

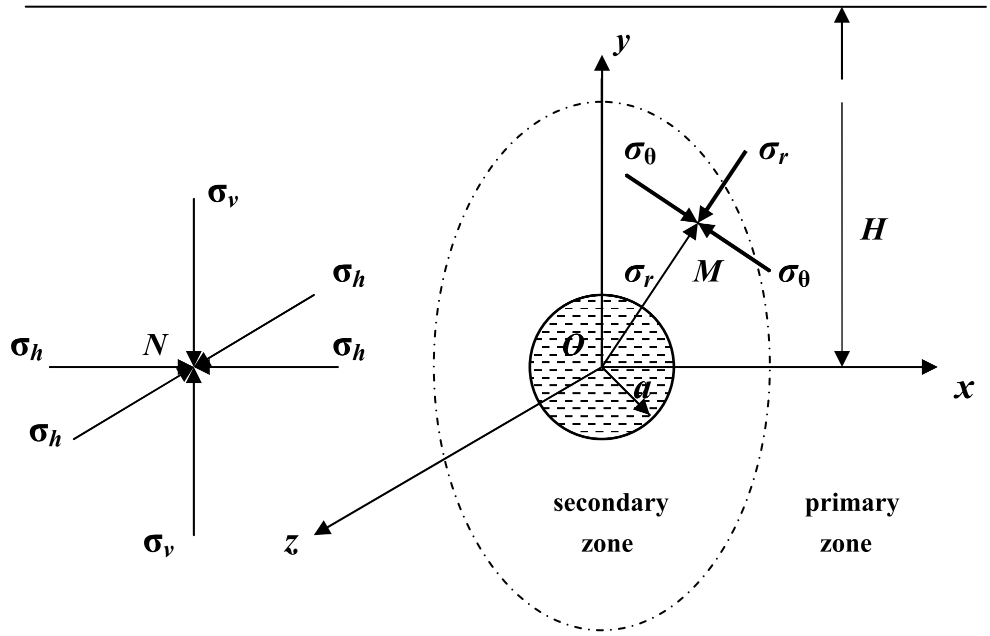

Fig. 1. Transversal sectioning plane through a horizontal wellbore

In the followings, in order to obtain the partition laws of the effective stresses in the considered rocks, a new calculation model [1] will be used, according to which the values σ', τ' of the normal and tangential effective stresses respectively are given by the equations:

$$\sigma' = \frac{1}{1-\phi}\sigma - \frac{\phi}{1-\phi}p \ , \tag{6}$$

$$\tau' = \frac{1}{1-\phi}\tau \ . \tag{7}$$

We have noted with ϕ the porosity of the rock, defined as the ratio between the volume of the pores and the total volume of the rock, with p the pore pressure, and with σ, τ the apparent values of the normal and tangential stresses respectively from the considered sectioning plane.

For a three-dimensional state of stress – as it is the case around a wellbore – equations (6), (7) can also be written using tensors, as follows:

$$T_{\sigma'} = \frac{1}{1-\phi}T_\sigma - \frac{\phi}{1-\phi}T_p \ , \tag{8}$$

where $T_{\sigma'}$ is the *effective stresses tensor*

$$T_{\sigma'} = \begin{pmatrix} \sigma'_r & \tau'_{r\theta} & \tau'_{rz} \\ \tau'_{\theta r} & \sigma'_\theta & \tau'_{\theta z} \\ \tau'_{zr} & \tau'_{z\theta} & \sigma'_z \end{pmatrix} , \tag{9}$$

and T_p is the *pore pressure tensor*

$$T_p = \begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix} . \tag{10}$$

The partition laws of the non-zero effective stresses in the vicinity of a horizontal wellbore are obtained from equations (6), (7), in which the apparent stresses are the ones given by equations (2) - (5); it results:

$$\sigma_r^{'} = \frac{1}{1-\phi}\left\{\frac{\sigma_v+\sigma_h}{2}\left[1-\left(\frac{a}{r}\right)^2\right]+p_s\left(\frac{a}{r}\right)^2-\phi\,p-\frac{\sigma_v-\sigma_h}{2}\left[1-4\left(\frac{a}{r}\right)^2+3\left(\frac{a}{r}\right)^4\right]\cos 2\theta\right\} \tag{11}$$

$$\sigma_\theta^{'} = \frac{1}{1-\phi}\left\{\frac{\sigma_v+\sigma_h}{2}\left[1+\left(\frac{a}{r}\right)^2\right]-p_s\left(\frac{a}{r}\right)^2-\phi\,p+\frac{\sigma_v-\sigma_h}{2}\left[1+3\left(\frac{a}{r}\right)^4\right]\cos 2\theta\right\} , \tag{12}$$

$$\sigma_z^{'} = \frac{1}{1-\phi}\left[\sigma_h-\phi\,p+4\mu\frac{\sigma_v-\sigma_h}{2}\left(\frac{a}{r}\right)^2\cos 2\theta\right] , \tag{13}$$

$$\tau_{r\theta}^{'} = \frac{1}{1-\phi}\frac{\sigma_v-\sigma_h}{2}\left[1+2\left(\frac{a}{r}\right)^2-3\left(\frac{a}{r}\right)^4\right]\sin 2\theta . \tag{14}$$

From the above equations, it can be noted that the effective values of the non-zero secondary stresses are increasing with the increase of the porosity, and – only those of the normal stresses – are decreasing with the increase of the pore pressure (the tangential stresses are not influenced by the variations of the pore pressure, but only by the porosity).

In this work we have used the *sign convention*, typical for the *Rocks Mechanics*, according to which the normal compression stresses are considered positive, and the traction stresses are considered negative.

In the followings, particularized forms of equations (11) - (14) are presented, for the material points occupying some distinct positions (the points placed on the contour of the excavation or the ones located on some radial directions), and for some limit situations ($p = 0$ or $\theta = 0$).

1. For the material points located on the *contour of the excavation*, characterised by $r = a$, equations (11) - (14) become:

$$\sigma_r^{'}(a) = \frac{1}{1-\phi}(p_s-\phi\,p) , \tag{15}$$

$$\sigma_\theta^{'}(a) = \frac{1}{1-\phi}\left[\sigma_v+\sigma_h-p_s-\phi\,p+2(\sigma_v-\sigma_h)\cos 2\theta\right] , \tag{16}$$

$$\sigma'_z(a) = \frac{1}{1-\phi}\left[\sigma_h - \phi\, p + 2\mu(\sigma_v - \sigma_h)\cos 2\theta\right] , \tag{17}$$

$$\tau'_{r\theta}(a) = 0 \ . \tag{18}$$

2. For the material points placed on the *horizontal direction* perpendicular to the well axis (*Ox* axis), corresponding to $\theta = 0^0$ and $\theta = 180^0$, the effective stress partitions are:

$$\sigma'_r\left(0^0\right) = \frac{1}{1-\phi}\left\{\frac{\sigma_v + \sigma_h}{2}\left[1-\left(\frac{a}{r}\right)^2\right] + p_s\left(\frac{a}{r}\right)^2 - \phi\, p - \frac{\sigma_v - \sigma_h}{2}\left[1-4\left(\frac{a}{r}\right)^2 + 3\left(\frac{a}{r}\right)^4\right]\right\} , \tag{19}$$

$$\sigma'_\theta\left(0^0\right) = \frac{1}{1-\phi}\left\{\frac{\sigma_v + \sigma_h}{2}\left[1+\left(\frac{a}{r}\right)^2\right] - p_s\left(\frac{a}{r}\right)^2 - \phi\, p + \frac{\sigma_v - \sigma_h}{2}\left[1+3\left(\frac{a}{r}\right)^4\right]\right\} , \tag{20}$$

$$\sigma'_z\left(0^0\right) = \frac{1}{1-\phi}\left[\sigma_h - \phi\, p + 4\mu\frac{\sigma_v - \sigma_h}{2}\left(\frac{a}{r}\right)^2\right] , \tag{21}$$

$$\tau'_{r\theta}\left(0^0\right) = 0 \ . \tag{22}$$

3. For the *vertical direction* perpendicular to the well axis (*Oy* axis), corresponding to $\theta = 90^0$ and $\theta = 270^0$, it results:

$$\sigma'_r\left(90^0\right) = \frac{1}{1-\phi}\left\{\frac{\sigma_v + \sigma_h}{2}\left[1-\left(\frac{a}{r}\right)^2\right] + p_s\left(\frac{a}{r}\right)^2 - \phi\, p + \frac{\sigma_v - \sigma_h}{2}\left[1-4\left(\frac{a}{r}\right)^2 + 3\left(\frac{a}{r}\right)^4\right]\right\}, \tag{23}$$

$$\sigma'_\theta\left(90^0\right) = \frac{1}{1-\phi}\left\{\frac{\sigma_v + \sigma_h}{2}\left[1+\left(\frac{a}{r}\right)^2\right] - p_s\left(\frac{a}{r}\right)^2 - \phi\, p - \frac{\sigma_v - \sigma_h}{2}\left[1+3\left(\frac{a}{r}\right)^4\right]\right\} , \tag{24}$$

$$\sigma'_z\left(90^0\right) = \frac{1}{1-\phi}\left[\sigma_h - \phi\, p - 4\mu\frac{\sigma_v - \sigma_h}{2}\left(\frac{a}{r}\right)^2\right] , \tag{25}$$

$$\tau'_{r\theta}\left(90^0\right) = 0 \ . \tag{26}$$

4. For the radial directions corresponding to $\theta = 45^0$ and $\theta = 225^0$, equations (11)-(14) become:

$$\sigma'_r\left(45^0\right) = \frac{1}{1-\phi}\left\{\frac{\sigma_v + \sigma_h}{2}\left[1-\left(\frac{a}{r}\right)^2\right] + p_s\left(\frac{a}{r}\right)^2 - \phi\, p\right\} , \tag{27}$$

$$\sigma'_\theta\left(45^0\right) = \frac{1}{1-\phi}\left\{\frac{\sigma_v + \sigma_h}{2}\left[1+\left(\frac{a}{r}\right)^2\right] - p_s\left(\frac{a}{r}\right)^2 - \phi\, p\right\} , \tag{28}$$

$$\sigma'_z\left(45^0\right) = \frac{1}{1-\phi}\left(\sigma_h - \phi\, p\right) , \tag{29}$$

$$\tau'_{r\theta}\left(45^0\right)=\frac{1}{1-\phi}\frac{\sigma_v-\sigma_h}{2}\left[1+2\left(\frac{a}{r}\right)^2-3\left(\frac{a}{r}\right)^4\right] . \tag{30}$$

5. For the radial directions corresponding to $\theta = 135^0$ and $\theta = 315^0$, equations (11)-(14) become:

$$\sigma'_r\left(135^0\right)=\frac{1}{1-\phi}\left\{\frac{\sigma_v+\sigma_h}{2}\left[1-\left(\frac{a}{r}\right)^2\right]+p_s\left(\frac{a}{r}\right)^2-\phi\, p\right\} , \tag{31}$$

$$\sigma'_\theta\left(135^0\right)=\frac{1}{1-\phi}\left\{\frac{\sigma_v+\sigma_h}{2}\left[1+\left(\frac{a}{r}\right)^2\right]-p_s\left(\frac{a}{r}\right)^2-\phi\, p\right\} , \tag{32}$$

$$\sigma'_z\left(135^0\right)=\frac{1}{1-\phi}\left(\sigma_h-\phi\, p\right) , \tag{33}$$

$$\tau'_{r\theta}\left(135^0\right)=-\frac{1}{1-\phi}\frac{\sigma_v-\sigma_h}{2}\left[1+2\left(\frac{a}{r}\right)^2-3\left(\frac{a}{r}\right)^4\right] . \tag{34}$$

From equations (27) - (34), it can be noted that, for the radial directions corresponding to $\theta = \pi/4 + i\pi/2$ ($i = 0, 1, 2, 3$), each of the non-zero effective stresses has the same partition, with the exception of the sign of the values for $\tau'_{r\theta}$.

6. For the material points outside the secondary zone (from the *primary zone*), the partitions of the effective values of the primary stresses – referred to the cylindrical coordinates system $Or\theta z$ and noted with the superscript P – result from equations (11) - (14) for $r \to\infty$, when the ratio $a/r \to 0$. Therefore:

$$\sigma_r^{P'}=\frac{1}{1-\phi}\left(\frac{\sigma_v+\sigma_h}{2}-\phi\, p-\frac{\sigma_v-\sigma_h}{2}\cos 2\theta\right) , \tag{35}$$

$$\sigma_\theta^{P'}=\frac{1}{1-\phi}\left(\frac{\sigma_v+\sigma_h}{2}-\phi\, p+\frac{\sigma_v-\sigma_h}{2}\cos 2\theta\right) , \tag{36}$$

$$\sigma_z^{P'}=\frac{1}{1-\phi}\left(\sigma_h-\phi\, p\right) , \tag{37}$$

$$\tau_{r\theta}^{P'}=\frac{1}{1-\phi}\frac{\sigma_v-\sigma_h}{2}\sin 2\theta . \tag{38}$$

It can be noted here that, for the primary zone, the influences of the pore pressure, p_s, and of the excavation radius, a, are disappearing.

7. A first limit situation described here is the one in which the pore pressure is zero ($p = 0$), when equations (11) - (14) become:

$$\sigma'_{r(p=0)}=\frac{1}{1-\phi}\left\{\frac{\sigma_v+\sigma_h}{2}\left[1-\left(\frac{a}{r}\right)^2\right]+p_s\left(\frac{a}{r}\right)^2-\frac{\sigma_v-\sigma_h}{2}\left[1-4\left(\frac{a}{r}\right)^2+3\left(\frac{a}{r}\right)^4\right]\cos 2\theta\right\} , \tag{39}$$

$$\sigma'_{\theta(p=0)} = \frac{1}{1-\phi}\left\{\frac{\sigma_v+\sigma_h}{2}\left[1+\left(\frac{a}{r}\right)^2\right] - p_s\left(\frac{a}{r}\right)^2 + \frac{\sigma_v-\sigma_h}{2}\left[1+3\left(\frac{a}{r}\right)^4\right]\cos 2\theta\right\} , \tag{40}$$

$$\sigma'_{z(p=0)} = \frac{1}{1-\phi}\left[\sigma_h + 4\mu\frac{\sigma_v-\sigma_h}{2}\left(\frac{a}{r}\right)^2\cos 2\theta\right] , \tag{41}$$

$$\tau'_{r\theta(p=0)} = \frac{1}{1-\phi}\frac{\sigma_v-\sigma_h}{2}\left[1+2\left(\frac{a}{r}\right)^2 - 3\left(\frac{a}{r}\right)^4\right]\sin 2\theta . \tag{42}$$

As mentined above, the pore pressure does not influence the tangential stresses and this is due to the fact that generally the fluids cannot sustain shear loads.

8. Another limit situation is the one of the rocks without pores ($\phi = 0$) for which equations (11) - (14) become:

$$\sigma'_{r(\phi=0)} = \sigma_r , \tag{43}$$

$$\sigma'_{\theta(\phi=0)} = \sigma_\theta , \tag{44}$$

$$\sigma'_{z(\phi=0)} = \sigma_z , \tag{45}$$

$$\tau'_{r\theta(\phi=0)} = \tau_{r\theta} . \tag{46}$$

It can therefore be noted that, while the porosity of a rock decreases towards zero, the effective values of all non-zero stresses approach the apparent values.

On the other hand, the results obtained for the cases $p = 0$ and $\phi = 0$ confirm the validity of equations (6) - (8), obviously for porous rocks without fractures and micro-fractures, the last ones being looked at as discontinuities of the cohesion of the mineral skeleton.

Regarding the partitions of the effective stresses around an underground excavation, the situations and the zones (material points) in which the normal effective stresses reach zero, in case of non-cohesive rocks, or negative values (traction), in case of cohesive rocks, are usually of interest. For such situations and zones, there exists the risk of occurrence of undesirable deformations – due to eventual consequences – like caving or fractures of the walls. In such context, analysing the equations above, it can be notred that: 1) – the effective stress $\tau'_{r\theta}$ presents non-zero values in the material points located on the contour of the excavation and on the horizontal and vertical directions; 2) – the effective stress σ'_r reaches minimum values in the material points placed on the horizontal radial direction, while stresses σ'_θ and σ'_z reach their minimum values on the vertical direction.

The negative values of σ'_r favour the walls caving while the negative values of σ'_θ and σ'_z favour the fractures; the fractures plane contains the well axis in the case they are generated by σ'_θ and is perpendicular to the well axis in the case of σ'_z. More details about the stability of the walls, together with a numerical analysis of the extreme values of the effective stresses, will be included in a future paper.

Conclusions

The effective stress values in a porous medium are important because they generally determine the type and size of deformations, together with the mechanical properties of the material. The factors which normally define these effective values are: the porosity, the internal mechanical loads (the pore pressure), and the external mechanical loads (the weight of the strata above and the well fluid pressure) expressed by means of the apparent stresses. The effective stress values in a rock differ from the apparent ones; the greater are the variations of the porosity and of the pore pressure, the greater are these differences.

For the case of the rocks around a horizontal wellbore, the extreme values of the effective stresses – but also the ones of the apparent stresses – are found on the contour of the wellbore and are influenced in the first place by the well fluid pressure. Other influent factors are: the primary (natural) stresses, the porosity, the pore pressure, the wellbore radius, the wellbore pressure, the distance from the well axis, the radial direction of the considered point, and the mechanical properties of the rock. Knowing the mechanical properties of the drilled rocks, the porosity and the pore pressure, by modifying the value of the wellbore pressure the type and size of the deformations in the rocks around can be controlled.

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Tensiunile efective ale rocilor elastice nefisurate din jurul unei găuri de sondă orizontale

Rezumat

În cazul rocilor poroase şi saturate cu fluide sub presiune valorile reale (efective) ale tensiunilor din scheletul mineral sunt, în general, diferite de valorile aparente. O cunoaştere corectă a deformaţiilor unei asemenea roci impune evaluarea exactă a tensiunilor efective. O situaţie practică de acest fel se întâlneşte la rocile traversate prin foraj – situaţie ce este analizată în prezenta lucrare plecând de la un model nou de calcul al tensiunilor efective, în general.