BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LVIII No.4/2006	73-78	Seria Tehnică

Kinematical Exact Analysis of an Inverse Schematic Pumping Unit

Liliana Rusu, Ion Eparu

Universitatea Petrol – Gaze din Ploieşti, Bd. Bucureşti 39, Ploieşti, Romania
e-mail:lianarusu8@yahoo.com

Abstract

The closed vector contour method is used to determin the kinematic of reverse schematic pumping units. The computer program written in Mathcad 2001 having this mathematic model as base is flexible, allows any input parameter and throws exact outputs that can be used by pumping unit designers.

Key words: *pumping unit, kinetic parameter, reverse schematic.*

General Considerations

The vectorial closed contour method for conventional schematic pumping units, used in [1], can easily be adapted to the reverse schematic pumping unit (as seen in Fig.1) whose kinematical schematic is illustrated in fig.2. For the inverse schematic's vectorial contour we must choose an anti-clockwise sense. We attach a xOy coordinate system and we obtain the angles between the normal Ox sense and the sense of each contour vector. We measure the angles in a trigonometrycal sense. The vectorial equation has the same expression as in [1]

$$\bar{l}_1 + \bar{l}_2 + \bar{l}_3 = \bar{l}_0 \tag{1}$$

The constructive components are depicted in the same fig. 2. These are $OA=l_1$, $AB=l_2$, $BC=l_3$, $CD=l_4$, $OC=l_0$.

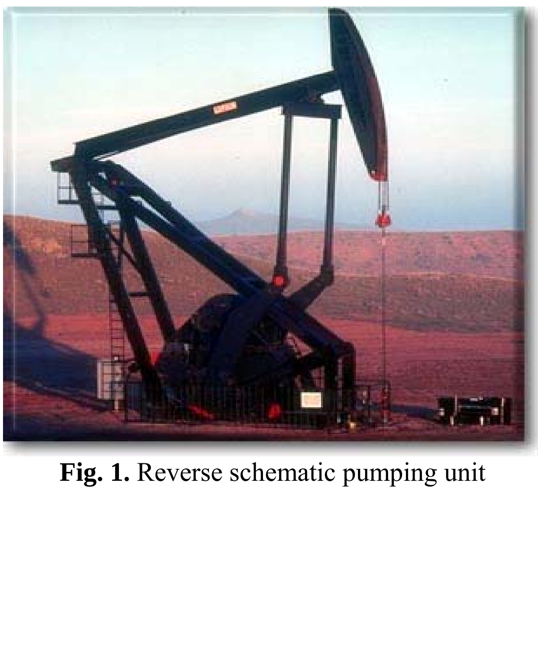

Fig. 1. Reverse schematic pumping unit

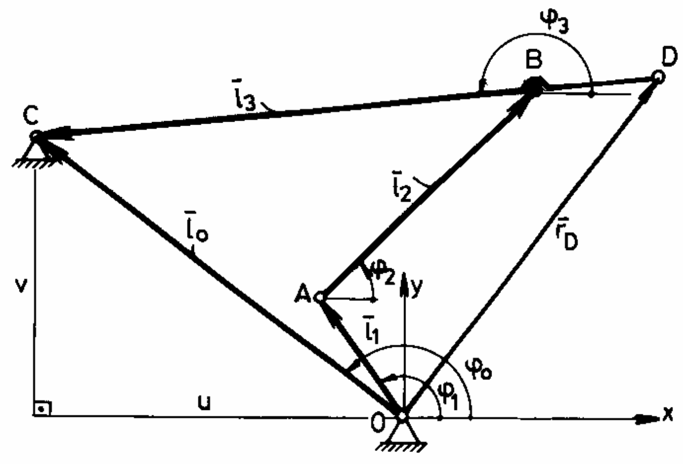

Fig. 2. Kinematical schematic for the reverse schematic pumping unit – notations for the closed vectorial contour method.

The closed vectorial contour method has the advantage that the only change that must be done is to replace the angle $\varphi_0 = arctg\frac{v}{u}$ from [1] with $\varphi_0 = \pi - arctg\frac{v}{u}$, where u and v are illustrated in fig. 2. We obtain for φ_2 and φ_3 the expressions:

$$\varphi_2 = -arctg\frac{b_2}{a_2} - (-1)^k \arcsin\frac{(-1)^k c_2}{\sqrt{a_2^2 + b_2^2}} + k\pi \tag{2}$$

$$\varphi_3 = -arctg\frac{b_3}{a_3} + (-1)^t \arcsin\frac{(-1)^t c_3}{\sqrt{a_3^2 + b_3^2}} + (t-2)\pi\,, \tag{3}$$

where: $k(\varphi_1) = 0$ and $t(\varphi_1) = 3$

$$a_2 = 2l_1l_2\sin\varphi_1 - 2l_2l_0\sin\varphi_0 \tag{4.a}$$

$$b_2 = 2l_1l_2\cos\varphi_1 - 2l_2l_0\cos\varphi_0 \tag{4.b}$$

$$c_2 = 2l_0l_1\cos\varphi_1\cos\varphi_0 + 2l_0l_1\sin\varphi_1\sin\varphi_0 + l_3^2 - l_1^2 - l_2^2 - l_0^2 \tag{4.c}$$

$$a_3 = 2l_1l_3\sin\varphi_1 - 2l_3l_0\sin\varphi_0 \tag{4.d}$$

$$b_3 = 2l_1l_3\cos\varphi_1 - 2l_3l_0\cos\varphi_0 \tag{4.e}$$

$$c_3 = 2l_0l_1\cos\varphi_1\cos\varphi_0 + 2l_0l_1\sin\varphi_1\sin\varphi_0 + l_2^2 - l_1^2 - l_3^2 - l_0^2 \tag{4.f}$$

for the angular velocity of the crank and beam the relations:

$$\omega_2 = \frac{l_1}{l_2}\cdot\frac{\sin(\varphi_3 - \varphi_1)}{\sin(\varphi_2 - \varphi_3)}\cdot\omega_1 \tag{5}$$

$$\omega_3 = \frac{l_1}{l_3}\cdot\frac{\sin(\varphi_1 - \varphi_2)}{\sin(\varphi_2 - \varphi_3)}\cdot\omega_1 \tag{6}$$

and for the angular accelerations of the crank and beam the relations:

$$\varepsilon_2 = \frac{\Delta_2}{\Delta} \tag{7}$$

$$\varepsilon_3 = \frac{\Delta_3}{\Delta} \tag{8}$$

where:

$$\Delta = \begin{vmatrix} AB \\ CD \end{vmatrix} = A \cdot D - B \cdot C \,,\ \Delta_2 = \begin{vmatrix} EB \\ FD \end{vmatrix} = E \cdot D - B \cdot F \,,\ \Delta_3 = \begin{vmatrix} AE \\ CF \end{vmatrix} = A \cdot F - C \cdot E \tag{9.a}$$

$$A = l_2 \sin\varphi_2 \tag{9.b}$$

$$B = l_3 \sin\varphi_3 \tag{9.c}$$

$$C = l_2 \cos\varphi_2 \tag{9.d}$$

$$D = l_3 \cos\varphi_3 \tag{9.e}$$

$$E = -l_1\omega_1^2 \cos\varphi_1 - l_2\omega_2^2 \cos\varphi_2 - l_3\omega_3^2 \cos\varphi_3 \tag{9.f}$$

$$F = l_1\omega_1^2 \sin\varphi_1 + l_2\omega_2^2 \sin\varphi_2 + l_3\omega_3^2 \sin\varphi_3 \tag{9.g}$$

Calculus Examples

We will now present the graphics of kinematical parameters variation that we calculated before, for a reverse schematic beam pumping unit (API Spec. 11E, IIIrd class - VULCAN) with the following constructive characteristics:

OA=1,381m, AB=4.2m, BC=6.57m, CD=7.92m, OC=6.27m, ω_1*=0.94* s^{-1}*, u=4.74m, v=4.1m,. s=3.66m.*

The stroke is considered to start at the lower position D, so for the angle φ_1=4.05 rad = 232 degrees. For φ_2 and φ_3 we obtain a variation illustrated in the fig 3 graphic, for angular speeds ω_2 and ω_3 the graphics in fig 4 and for the angular accelerations ε_2 and ε_3, the graphic in fig 5.

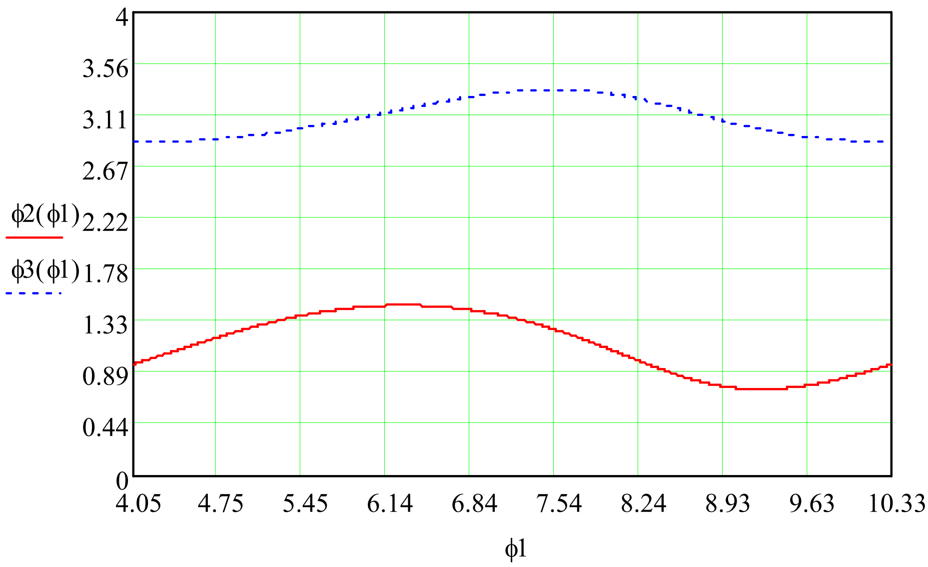

Fig.3. Graphic of the crank and beam position variation

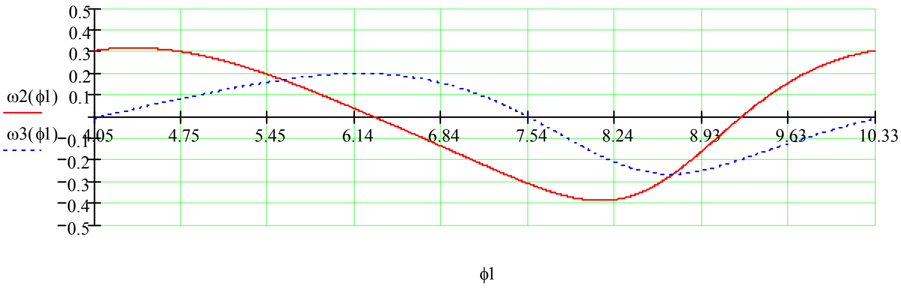

Fig. 4. Graphic for the crank and beam angular velocity variation

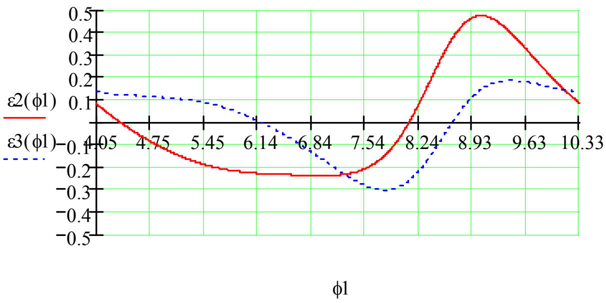

Fig.5. Graphic for the crank and beam angular acceleration variation

The stroke is determined with the relation: $s_d = l_4(\varphi_3 - \pi)$ (10)

The velocity of the point D is calculated according to: $v_d = l_4 \cdot \omega_3$ (11)

The acceleration of point D, with the relation: $a_d = l_4 \cdot \varepsilon_3$ (12)

The graphic in figure 6 allows the simultaneous analysis of the 3 parameters, strokee, speed and acceleration of the polished rod

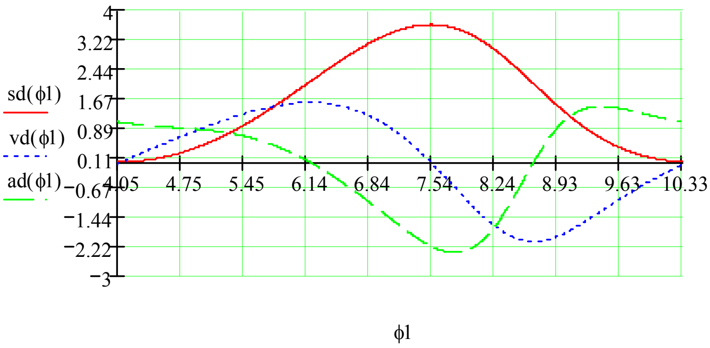

Fig. 6. Curves of variation for stroke, speed and acceleration of the polished rod

We can observe that:

- the upstroke takes longer than the downstroke (as seen in fig 3), the crank's handle rotates 200.2 degrees during the ascendent stroke and 159.8 degrees during the descendent one, thus improving the work regime of the gear reducer. Ascension begins at the crank angle $\varphi_1 = 4.05 rad$ and ends at $\varphi_1 = 7.54 rad$ - then descending starts, ending at $\varphi_1 = 10.33 rad$.

- velocity and acceleration for the ascending curve (when the load is maximal) are reduced compared to the conventional units, as shown by fig. 4 and 5. Therefore, the maximum load for the polished rod is reduced. For the descending stroke (when load is minimal) velocity and acceleration increase, but their impact is less harmful.

A comparison will be made between two units, with a conventional and reverse schematic, having the same angular crank velocity ω_1=0.94 s^{-1} and the same polished rod stroke s=3.66m. Using the *p* and *t* reports for velocity , *r* and *q* for acceleration we get:

$$p = \frac{v_{urcare}^{\max,schdirecta}}{v_{urcare}^{\max,schemainversa}} = \frac{1.791}{1.572} = 1.14\ ;$$

$$t = \frac{v_{coborare}^{\max,schdirecta}}{v_{coborare}^{\max,schemainversa}} = \frac{1.786}{2.104} = 0.85\ ;$$

$$r = \frac{a_{urcare}^{\max,schdirecta}}{a_{urcare}^{\max,schemainversa}} = \frac{2.293}{1.2} = 1.91\ ;$$

$$q = \frac{a_{coborare}^{\max,schdirecta}}{a_{coborare}^{\max,schemainversa}} = \frac{1.506}{2.398} = 0.65$$

Graphics in fig 7-10 underline these conclusions.

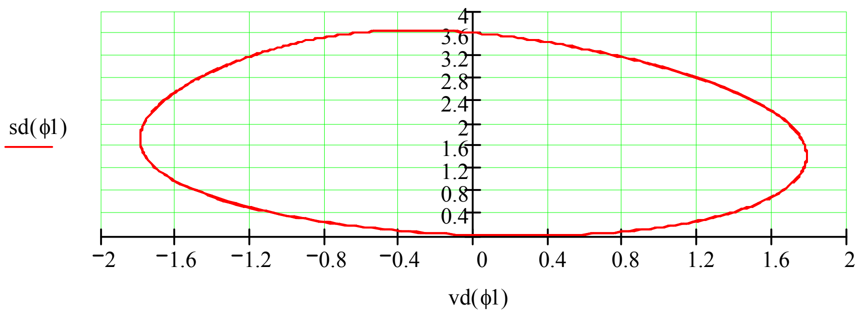

Fig. 7. Variation curve of stroke depending on polished rod velocity for conventional schematic

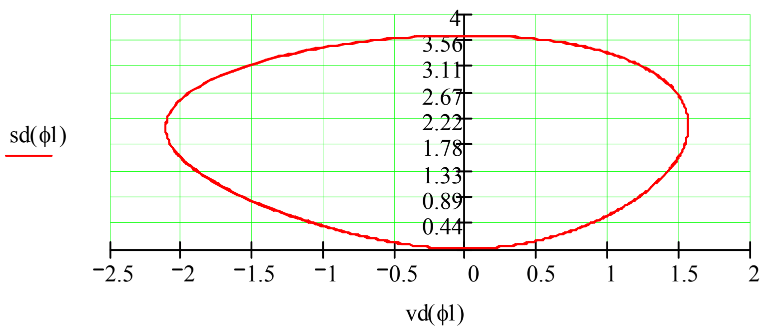

Fig. 8. Variation curve of stroke depending on polished rod velocity for reverse schematic

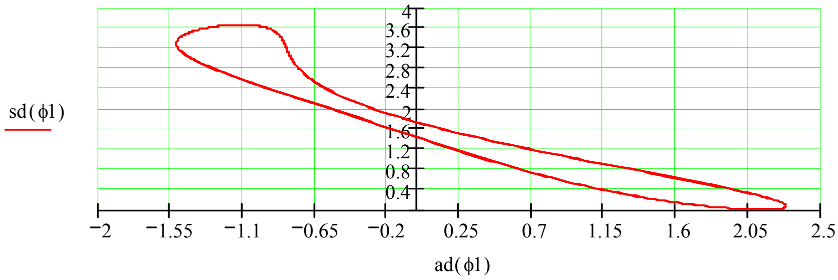

Fig.9 Variation curve of stroke depending on polished rod acceleration for conventional schematic

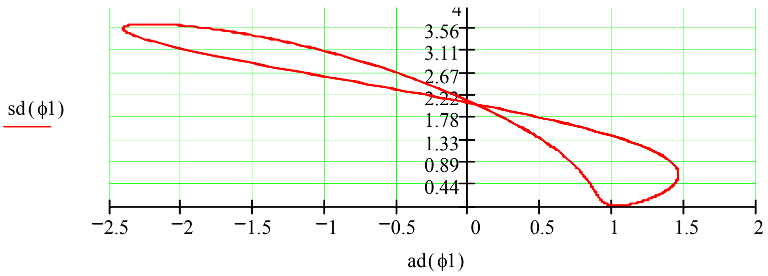

Fig.10. Variation curve of stroke depending on polished rod acceleration for reverse schematic

Conclusions

The method proposed leads to a quick way of calculating kinematical parameters for pumping units. Both for conventional and reversed schematics, sizes can be determined for any chosen geometry and charts for all parameters, angles, velocities and angular accelerations of any point on the kinematical schematic can be drawn.The computer program written in Mathcad 2001 having this mathematic model as base is flexible, allows any input parameter and throws exact outputs that can be used by pumping unit designers.

References

1. Rusu, L., Florea I., *Metodă analitică pentru calculul cinematic al unităţilor de pompare*, Bul. U.P.G.-Ploieşti, Vol.LVII, 4/2005.
2. Popovici, Al,. *Utilaj pentru exploatarea sondelor de petrol*, Ed. Tehnică, 1989
3. Panait, Gh**.,** *Teoria mecanismelor şi maşinilor, vol.I*, Ed. Universal Cartfil, Ploieşti, 1998

Analiza cinematică exactă a unei unităţi de pompare cu schemă inversă

Rezumat

Este utilizată metoda conturului vectorial închis,, pentru determinarea cinematicii unităţilor de pompare cu schemă inversă. Programul de calculator conceput în Mathcad 2001, pe baza modelului matematic prezentat este flexibil, permite utilizarea pentru orice variabilă de intrare şi duce la rezultate exacte ce pot fi utilizate de proiectanţii unităţilor de pompare