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Optimization of Flux and Pressure Distribution along a Horizontal Well Using Interval Control Valves

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Abstract

This paper presents a novel mathematical model which can be used to get an uniform flux and pressure distribution along a horizontal well using Interval Control Valves.

Key words: *horizontal well, interval control valve, fluid flow through pipes, friction pressure loss.*

Introduction

A very important element in horizontal well production evaluations is played by the flux and pressure distribution along the horizontal section which is highly influenced by the friction pressure loss due to the fluid flow between the heel and the toe.

The fluid flow and the pressure distribution along the horizontal section can be controlled by using Interval Control Valves (ICV) as illustrated in figure 1. In this case the horizontal section completion includes an extended stinger with intermediate packers dividing the horizontal section into few intervals produced separately by ICV devices.

The complexity of ICV devices is depending on purpose and price, the highest price being about half million US$ for a complex ICV measuring pressure and temperature with a continuous flow rate control. Cheaper devices are on/off valves or presenting few discrete steps for flow rate control. All of these ICV types can be electrically or hydraulically controlled.

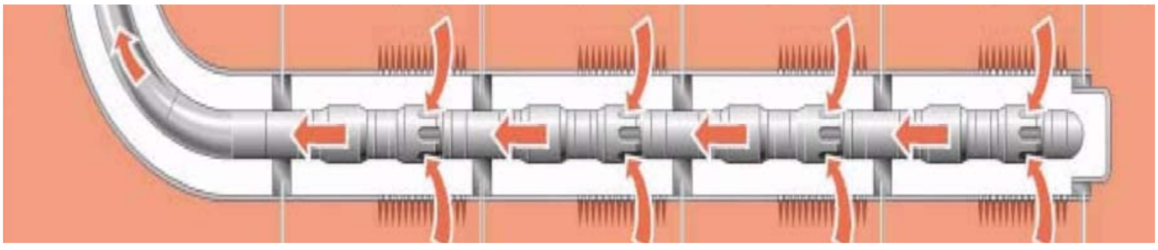

Fig. 1. Horizontal well section completed with four Interval Control Valves (ICV).

Flow Rate Equations

The flow of an incompressible and homogeneous fluid through a slightly inclined pipe with a constant cross section A is described by the Bernoulli equation, while the friction pressure losses can be approximated the Weisbach–Darcy equation as:

$$p_1 + \gamma h_1 = p_2 + \gamma h_2 + \Delta p_f \; ; \; \Delta p_f = \lambda \frac{w^2}{2g} \frac{L}{d} \gamma \, , \tag{1}$$

where p and γh represent the pressure and the potential energy respectively, λ – hydraulic resistance coefficient, w – average velocity of the fluid through the pipe, g – gravity acceleration, L – pipe length, d – pipe diameter, and γ – fluid specific gravity. The pressure loss due to the friction forces may be expressed as well as:

$$Q = A\, w \, ,$$

where

$$w = \frac{4Q}{\pi d^2} \, .$$

Thus, the fundamental flow equation becomes:

$$\Delta p_f = \frac{\gamma \sqrt{\pi}}{4g} \frac{Q^2 L \lambda}{A^{5/2}} \, , \tag{2}$$

$$p_1 + \gamma h_1 = p_2 + \gamma h_2 + \frac{\gamma \sqrt{\pi}}{4g} \frac{Q^2 L \lambda}{A^{5/2}} \, . \tag{3}$$

For an elementary interval $x_{j-1} \ldots x_j$, the above relations can be written as:

$$\begin{cases} p_{j-1} - p_j = \dfrac{\gamma \sqrt{\pi}}{4g} \dfrac{Q_{j-1}^2 \left(x_{j-1} - x_j\right) \lambda_{j-1}}{A_{j-1}^{5/2}} + \gamma \left(h_{j-1} - h_j\right), \\ Q_j = Q_{j-1} + q_j \left(x_{j-1} - x_j\right), \end{cases} \tag{4}$$

where h is the isometric depth. Using the differential form, the system becomes:

$$\begin{cases} \mathrm{d}p(x) = \dfrac{\gamma \sqrt{\pi}}{4g} \dfrac{Q^2(x) \lambda(x) \mathrm{d}x}{A^{5/2}(x)} + \gamma \, \mathrm{d}h(x) \, , \\ -\mathrm{d}Q(x) = q(x) \mathrm{d}x \, , \end{cases} \tag{5}$$

The second relationship (5) represents the material balance equation, where the rate of fluid flowing through the pipe equals the reservoir specific flow.

For simplicity reasons, we consider $A(x) = A = \text{ct.}$, $A_i(x) = A_i = \text{ct.}$, $\lambda(x) = \lambda[Q(x)]$ and $\lambda_i(x) = \lambda[Q_i(x)]$, where $A(x)$ and $A_i(x)$ represent the cross section area of the tubing and of the inner space existing between the tubing and the casing, respectively. Accordingly, $Q(x)$ and $Q_i(x)$ represent the fluid flow rates through the tubing and through the inner space existing between the tubing and the casing, respectively.

Flux, Flow Rate and Pressure Loss into the Inner Space Evaluation

For the inner space the above system of equations can be written as:

$$q(x)=i_p\left[p_c-p_d(x)\right], \text{q} \tag{6}$$

$$-\frac{\mathrm{d}Q_i(x)}{\mathrm{d}x}=q(x), \tag{7}$$

$$\frac{\mathrm{d}p_d(x)}{\mathrm{d}x}=\frac{\gamma\sqrt{\pi}}{4g}\frac{Q_i^2(x)\lambda_i(x)\mathrm{d}x}{A_i^{5/2}}+\gamma\frac{\mathrm{d}h(x)}{\mathrm{d}x}. \tag{8}$$

The first equation describes the fluid flux produced by the reservoir, where i_p is the specific productivity index, p_c – reservoir pressure, and $p_d(x)$ – well-bore pressure corresponding to the x cross section within the analyzed horizontal interval. Using an alternate form, it can be written:

$$-\dot{Q}_i(x)=i_p\left[p_c-p_d(x)\right],\ -\ddot{Q}_i(x)=i_p\,\ddot{p}_d(x), \tag{9}$$

$$\dot{p}_d(x)=\frac{\gamma\sqrt{\pi}}{4g}\frac{1}{A_i^{5/2}}Q_i^2(x)\lambda_i\left[Q_i(x)\right]+\gamma\,\dot{h}(x), \tag{10}$$

$$\ddot{Q}_i(x)=i_p\frac{\gamma\sqrt{\pi}}{4g}\frac{1}{A_i^{5/2}}Q_i^2(x)\lambda_i\left[Q_i(x)\right]+i_p\,\gamma\,\dot{h}(x). \tag{11}$$

It has to be noted that the hydraulic resistance factor $\lambda=\lambda(Q)$ should be evaluated as function of Reynolds number:

$$\mathrm{Re}=\frac{wd}{\nu}=\frac{\rho wd}{\mu},\ \ \mu=\rho\nu, \tag{12}$$

where μ, ν are the cinematic, respectively the dynamic fluid viscosity. Or:

$$\mathrm{Re}=\frac{2\rho Q}{\mu\sqrt{A\pi}}. \tag{13}$$

Considering the laminar flow through the pipe, it results:

$$\lambda=\lambda(Q)=\frac{64}{\mathrm{Re}}=\frac{32\mu\sqrt{A\pi}}{\rho Q},\ \ \gamma=\rho g, \tag{14}$$

$$\ddot{Q}_i(x)=i_p\frac{\gamma\sqrt{\pi}}{4g}\frac{1}{A_i^{5/2}}Q_i^2(x)\frac{32\mu\sqrt{A\pi}}{\rho Q(x)}+i_p\,\gamma\,\dot{h}(x), \tag{15}$$

$$\ddot{Q}_i(x)=i_p\,8\pi\mu\frac{1}{A_i^2}Q_i(x)+i_p\,\gamma\,\dot{h}(x). \tag{16}$$

By changing the variables: $\dot{Q}_i(x)=Y\left[Q_i(x)\right]$, where $Q_i(x)$ is independent variable:

$$\ddot{Q}_i(x)=\frac{\mathrm{d}}{\mathrm{d}x}\dot{Q}_i(x)=\frac{\mathrm{d}Y(x)}{\mathrm{d}x}=\frac{\mathrm{d}Y(x)}{\mathrm{d}Q_i(x)}\frac{\mathrm{d}Q_i(x)}{\mathrm{d}x}=\frac{\mathrm{d}Y(x)}{\mathrm{d}Q_i(x)}\dot{Q}_i(x)=Y(x)\frac{\mathrm{d}Y(x)}{\mathrm{d}Q_i(x)}, \tag{17}$$

the equation (16) becomes:

$$Y(x)\frac{\mathrm{d}Y(x)}{\mathrm{d}Q_i(x)} = i_p\, 8\pi\mu \frac{1}{A_i^2} Q_i(x) + i_p\, \gamma\, \dot{h}(x). \tag{18}$$

Separating variables, results:

$$Y(x)\mathrm{d}Y(x) = i_p\, 8\pi\mu \frac{1}{A_i^2} Q_i(x)\mathrm{d}Q_i(x) + i_p\, \gamma\, \dot{h}(x)\mathrm{d}Q_i(x). \tag{19}$$

Integrating equation (19) for a subinterval k, x becomes $x–x_{p(k–1)}$ and then:

$$\int_{Y_{p(k-1)}}^{Y(x-x_{p(k-1)})} Y\left(x - x_{p(k-1)}\right)\mathrm{d}Y\left(x - x_{p(k-1)}\right) = i_p\, 8\pi\mu \frac{1}{A_i^2} \int_{Q_{i\,p(k-1)}}^{Q_i(x-x_{p(k-1)})} Q_i\left(x - x_{p(k-1)}\right)\mathrm{d}Q_i\left(x - x_{p(k-1)}\right) +$$

$$+ i_p\, \gamma \int_{Q_{i\,p(k-1)}}^{Q_i(x-x_{p(k-1)})} \dot{h}\left(x - x_{p(k-1)}\right)\mathrm{d}Q_i\left(x - x_{p(k-1)}\right), \tag{20}$$

where $x_{p\,(k–1)}$ represents the distance measured from the beginning of the interval up to the first packer allocated to the interval. Considering that $\dot{h}(x) = m_k$, where m_k represents the pipe slope for the subinterval $x_{p\,k} – x_{p(k–1)}$, after integration it results:

$$Y^2\left(x - x_{p(k-1)}\right) = i_p\, 8\pi\mu \frac{1}{A_i^2} Q_i^2\left(x - x_{p(k-1)}\right) + 2\, i_p\, \gamma\, m_k\, Q_i\left(x - x_{p(k-1)}\right) + C_{1k}\,. \tag{21}$$

If we note: $\sqrt{i_p\, 8\lambda\, m}\,\frac{1}{A_i} = a$ and $\sqrt{i_p \frac{1}{8\pi\mu}}\,\gamma\, m_k\, A_i = b_k$; if $C_{1k} = b_k^2$, then

$$Y^2\left(x - x_{p(k-1)}\right) = \left[a\, Q_i\left(x - x_{p(k-1)}\right) + b_k\right]^2,$$

wherefrom:

$$Y\left(x - x_{p(k-1)}\right) = \dot{Q}_i\left(x - x_{p(k-1)}\right) = \pm\left[a\, Q_i\left(x - x_{p(k-1)}\right) + b_k\right]. \tag{22}$$

Two first order linear differential equations are resulting:

$$\dot{Q}_i\left(x - x_{p(k-1)}\right) - a\, Q_i\left(x - x_{p(k-1)}\right) - b_k = 0\,, \text{ for } \dot{Q}_i\left(x - x_{p(k-1)}\right) \le 0\,, \tag{23}$$

$$\dot{Q}_i\left(x - x_{p(k-1)}\right) + a\, Q_i\left(x - x_{p(k-1)}\right) + b_k = 0\,, \text{ for } \dot{Q}_i\left(x - x_{p(k-1)}\right) > 0\,, \tag{24}$$

For a given interval, the fluid flow rate through the inner space is then described by the following equations:

$$Q_i\left(x - x_{p(k-1)}\right) = \frac{b_k}{a}\exp\left[a\left(x - x_{p(k-1)}\right)\right] - \frac{b_k}{a}\,, \text{ for } x_{p\,(k-1)} \le x \le x_{v\,k}\,, \tag{25}$$

$$Q_i\left(x - x_{p(k-1)}\right) = \frac{b_k}{a}\exp\left[a\left(x_{p\,k} - x\right)\right] - \frac{b_k}{a}\,, \text{ for } x_{v\,k} \le x \le x_{p\,k}\,. \tag{26}$$

For the same interval, the distribution of the fluid flux exchanged between the well bore and the reservoir is given by:

$$q\left(x - x_{p(k-1)}\right) = -b_k \exp\left[a\left(x - x_{p(k-1)}\right)\right], \text{ for } x_{p\,(k-1)} \le x \le x_{v\,k}\,, \quad (27)$$

$$q\left(x - x_{p(k-1)}\right) = b_k \exp\left[a\left(x_{p\,k} - x\right)\right], \text{ for } x_{v\,k} \le x \le x_{p\,k}\,. \quad (28)$$

Using equation (6), the well bore pressure is given by

$$p_d(x) = p_c + \frac{\dot{Q}_i(x)}{i_p}\,,$$

and thus:

$$p_d(x) = p_c + \frac{b_k}{i_p} \exp\left[a\left(x - x_{p(k-1)}\right)\right], \text{ for } x_{p\,(k-1)} \le x \le x_{v\,k}\,, \quad (29)$$

$$p_d(x) = p_c - \frac{b_k}{i_p} \exp\left[a\left(x_{p\,k} - x\right)\right], \text{ for } x_{v\,k} \le x \le x_{p\,k}\,. \quad (30)$$

Tubing Well Bore Pressure Distribution

Considering n production intervals and using equation (4) it results:

$$p_n - p_0 = \frac{\gamma\sqrt{\pi}}{4g} \frac{1}{A^{5/2}} \sum_{k=1}^{n} Q_k\, l_k\, \lambda(Q_k) + \gamma(h_n - h_0)\,, \quad (31)$$

where p_n is the tubing pressure corresponding to the n^{th} ICV, Q_k – flow rate of the k^{th} ICV, and l_k – distance from the beginning of the k^{th} interval to the k^{th} ICV.

Conclusions

The design of a horizontal well using ICV devices is made possible using the equations derived in this paper. The elaborated mathematical model is very useful for the optimization of the process in order to get uniform flux and pressure distributions along the horizontal well.

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Optimizarea distribuției de flux şi presiune într-o sondă orizontală folosind valve de control al intervalului

Rezumat

În lucrare se prezintă un nou model matematic deosebit de util pentru proiectarea amplasării valvelor de control al intervalului (ICV-uri) în lungul unei sonde orizontale în scopul obținerii unei distribuții uniforme a fluxului şi a presiunii pe toată lungimea drenei.