BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LVIII No.4/2006	61-66	Seria Tehnică

Research Concerning the Design of the Primary Controller of the Industrial Robots Mechanisms

Dorin Bădoiu

Universitatea Petrol-Gaze din Ploieşti, Bd. Bucureşti 39, Ploieşti, Romania
e-mail: badoiu@mail.upg-ploiesti.ro

Abstract

The paper presents a method that permits the design of the primary controller of the robots mechanisms, that is, the calculus of the generalized torques produced by the joint actuators for controlling the position and velocity at the end-effector level. The method is applied in the case of a two degrees of freedom robot mechanism, when a linear trajectory is imposed for the manipulated object. Finally, some simulation results are presented.

Key words: *robot, joints, position, trajectory, torque*

Introduction

Industrial robots are characterized by an automatic and programmable running, their task being to substitute the human operator in different operations that are part of industrial process. In many cases, the analysis of the tasks that the robots have to do, requires the calculus of the torques produced by the joint actuators for controlling the position and velocity at the end-effector level. In control problems that is known as the design of the primary controller of the industrial robots mechanisms.

In this paper it is presented a method that allows the primary controller design. The general case of a spatial industrial robot mechanism, which has n mobile links and n active joints of fifth class (fig. 1) is considered.

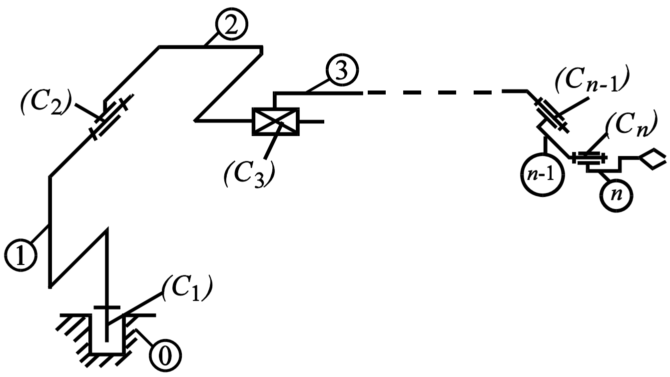

Fig. 1. Industrial robot mechanism

Theoretical Considerations

By considering the general case of an industrial robot mechanism, presented in fig. 1, a system of coordinates is attached to each component link $i, i = \overline{0,n}$ (*0* is the fixed link). Then, the homogeneous matrices ${}^{i}T_{i+1}$, corresponding to the relative position and orientation of the systems of coordinates $T_{i+1} = (O_{i+1}x_{i+1}y_{i+1}z_{i+1})$ and $T_i = (O_i x_i y_i z_i)$, $i = \overline{0,n-1}$, have to be calculated [1]:

$$ {}^{i}T_{i+1} = \begin{bmatrix} {}^{i}R_{i+1} & {}^{(i)}O_i O_{i+1} \\ 0 & 1 \end{bmatrix} \tag{1} $$

where: ${}^{i}R_{i+1}$ is the rotation matrix corresponding to the relative orientation of the systems of coordinates $T_{i+1} = (O_{i+1}x_{i+1}y_{i+1}z_{i+1})$ and $T_i = (O_i x_i y_i z_i)$.

The position of a certain point *P,* which belongs to a component link *i,* can be calculated with the following relation:

$$ \begin{bmatrix} {}^{(0)}O_0 P \\ 1 \end{bmatrix} = {}^{0}T_i \cdot \begin{bmatrix} {}^{(i)}O_i P \\ 1 \end{bmatrix} \tag{2} $$

where: the components of the position vector ${}^{(i)}O_i P$ are known and the homogeneous matrix ${}^{0}T_i$ is given by the following relation:

$$ {}^{0}T_i = {}^{0}T_1 \cdot {}^{1}T_2 \cdot \ldots \cdot {}^{i-1}T_i \tag{3} $$

The angular speeds $\overline{\omega}_i, i = \overline{1,n}$, of the component links and the speeds $\bar{v}_{O_i}, i = \overline{1,n}$, can be calculated in a recursive manner, using the following relations [1]:

$$ {}^{(i)}\omega_i = {}^{i}R_{i-1} \cdot {}^{(i-1)}\omega_{i-1} + {}^{(i)}\omega_{i,i-1} \tag{4} $$

$$ {}^{(i)}v_{O_i} = {}^{i}R_{i-1} \cdot ({}^{(i-1)}v_{O_{i-1}} + {}^{(i-1)}\overset{\mathrm{v}}{\omega}_{i-1} \cdot {}^{(i-1)}O_{i-1}O_i) + {}^{(i)}v_{i,i-1} \tag{5} $$

where: the rotation matrix ${}^{i}R_{i-1}$ is equal to ${}^{i-1}R_i^{\mathrm{T}}$, ${}^{(i-1)}\overset{\mathrm{v}}{\omega}_{i-1}$ is the matrix corresponding to the vector ${}^{(i-1)}\omega_{i-1}$ which is in a vector product with ${}^{(i-1)}O_{i-1}O_i$, and the relative speeds ${}^{(i)}\omega_{i,i-1}$ and ${}^{(i)}v_{i,i-1}$ have the following expressions:

$$ \begin{cases} {}^{(i)}\omega_{i,i-1} = \overline{\sigma}_i \cdot \dot{q}_i \cdot {}^{(i)}u_i \\ {}^{(i)}v_{i,i-1} = \sigma_i \cdot \dot{q}_i \cdot {}^{(i)}u_i \end{cases} \tag{6} $$

in which: ${}^{(i)}u_i$ is the unit vector of the axis of the joint (C_i), $\overline{\sigma}_i = 1 - \sigma_i$, where $\sigma_i = 0$, when (C_i) is a rotation joint, and $\sigma_i = 1$, when (C_i) is a translation joint.

The speed of a certain point *P,* which belongs to a component link *i,* can be calculated using the Euler formula for the speeds distribution into a rigid element.

By applying the Lagrange formalism, the movement equations can be obtained in the following compact matrix form [3]:

$$ A \cdot \ddot{q} + H(q, \dot{q}) = Q \tag{7} $$

where: A is the inertia matrix corresponding to the robot mechanism, $\ddot{q}$ is a vector that contains the accelerations $\ddot{q}_i, i=\overline{1,n}$, Q is the vector that has like components the generalized forces corresponding to the loads and operating forces and moments [3], and the vector H has the following form [3]:

$$H = B \cdot P + C \cdot R + G \tag{8}$$

where: B is a matrix whose elements can be calculated with the following relation:

$$B_{i,jk} = \frac{\partial A_{ij}}{\partial q_k} + \frac{\partial A_{ik}}{\partial q_j} - \frac{\partial A_{jk}}{\partial q_i}; \qquad 1 \le j < k \le n \tag{9}$$

P is a vector with $n(n-1)/2$ elements, which has the following expression:

$$P = \left[\dot{q}_1 \cdot \dot{q}_2 \;\; \dot{q}_1 \cdot \dot{q}_3 \; \dots \; \dot{q}_1 \cdot \dot{q}_n \;\; \dot{q}_2 \cdot \dot{q}_3 \dots \dot{q}_2 \cdot \dot{q}_n \dots \dot{q}_{n-1} \cdot \dot{q}_n\right]^{\mathrm{T}} \tag{10}$$

C is a matrix, whose elements can be calculated with the following relation:

$$C_{ij} = \frac{\partial A_{ij}}{\partial q_j} - \frac{1}{2} \cdot \frac{\partial A_{jj}}{\partial q_i}; \qquad 1 \le i, j \le n \tag{11}$$

R is a (nx1) vector, that has the following components:

$$R = \left[\dot{q}_1^2 \; \dot{q}_2^2 \; \dots \; \dot{q}_n^2\right]^{\mathrm{T}} \tag{12}$$

G is a (nx1) vector:

$$G = \left[\frac{\partial V}{\partial q_1} \, \frac{\partial V}{\partial q_2} \dots \frac{\partial V}{\partial q_n}\right]^{\mathrm{T}} \tag{13}$$

where: V is the total potential energy, corresponding to the weight of the component links and of the objects that are manipulated by the robot.

The inertia matrix A is symmetric and positive defined. The elements of this matrix can be determined by identification [2]. Thus, the elements $A_{ii}, i=\overline{1,n}$, will be equal to the coefficients of the elements $(\frac{1}{2} \cdot \dot{q}_i^2), i=\overline{1,n}$, in the expression of the total kinetic energy E_C, and the elements $A_{ij}, i \ne j, 1 \le i, j \le n$, will be equal to the coefficients of the elements $\dot{q}_i \cdot \dot{q}_j$. When the total kinetic energy is computed, the influence of the inertia of the manipulated objects and that of the operating motors must be taken into account.

When the trajectory and the variation of the speed of the manipulated object are imposed, the variation of generalized coordinates during the movement can be calculated by solving the inverse geometric model of the robot.

Simulation Results

The method is applied in the case of a two degrees of freedom manipulator (fig. 2). The following elements are known: the length of the component links: $l_1 = O_1O_2 = 0.5\mathrm{m}$, $l_2 = O_2O_s = 0{,}3\mathrm{m}$; the position of the weight centers of the links: $O_1C_1 = x_{C_1} = 0{,}25\mathrm{m}$, $O_2C_2 = x_{C_2} = 0{,}15\mathrm{m}$; the masses of the links and of the manipulated object: $m_1 = 2{,}8\mathrm{kg}$,

$m_2 = 1{,}15\,\mathrm{kg}$ and $m_s = 0{,}5\,\mathrm{kg}$ respectively; the values of the inertia moments corresponding to the links and to the manipulated object: $I_{z_1} = 0{,}06\,\mathrm{kg}\cdot\mathrm{m}^2$, $I_{z_2} = 0{,}01\,\mathrm{kg}\cdot\mathrm{m}^2$, $I_{z_s} = 0{,}623\cdot 10^{-4}\,\mathrm{kg}\cdot\mathrm{m}^2$; the inertia moments of the operating motors, that correspond to the axes $(O_i z_i), i = 1{,}2$: $I_{m1} = 1{,}8\cdot 10^{-2}\,\mathrm{kg}\cdot\mathrm{m}^2$ and $I_{m2} = 4{,}7605\cdot 10^{-2}\,\mathrm{kg}\cdot\mathrm{m}^2$.

The inertia moments: I_{z_1} and I_{z_2} correspond to the axes z of some systems of coordinates attached to the links in their weight centers. We assumed these systems of coordinates (that are not represented in fig. 2) have the axes parallel to those of the systems $T_i = (O_i x_i y_i z_i), i = 1{,}2$.

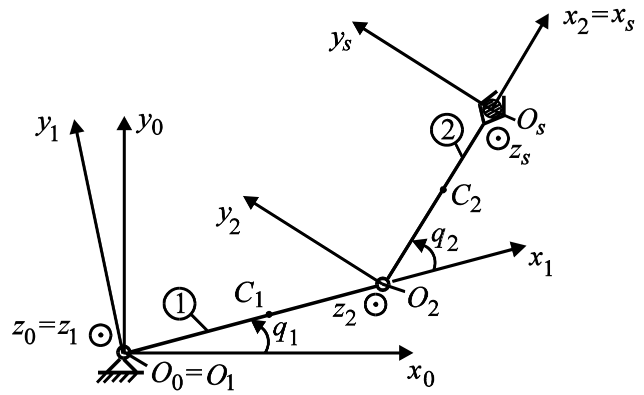

Fig. 2. Manipulator with two degrees of freedom

We consider the case when the point O_s moves along a linear trajectory given by:

$$\overline{O_0 O_s}(t) = \overline{O_0 O_{si}} + g(t)\cdot(\overline{O_0 O_{sf}} - \overline{O_0 O_{si}}) \tag{14}$$

where: the initial position O_{si} has the coordinates $(0.5\mathrm{m}, 0.3\mathrm{m})$ relative to the fixed system of coordinates and the coordinates of the final position O_{sf} are $(0.5\mathrm{m}, 0.5\mathrm{m})$; the function $g(t)$ has the following expression:

$$g(t) = 10\cdot\left(\frac{t}{t_f}\right)^3 - 15\cdot\left(\frac{t}{t_f}\right)^4 + 6\cdot\left(\frac{t}{t_f}\right)^5 \tag{15}$$

in which: t_f represents the duration of the movement. We assume: $t_f = 1{,}15\,\mathrm{s}$.

By applying the presented method we obtain for the total kinetic energy the following expression:

$$E_C = E_{C_1} + E_{C_2} + E_{C_S} \tag{16}$$

where: $E_{C_i}, i = 1{,}2$, correspond to the links and E_{C_S}, to the manipulated object and to the operating motors:

$$E_{C_1} = \frac{1}{2}\cdot\dot{q}_1^2\cdot(I_{z_1} + m_1\cdot x_{C_1}^2) \tag{17}$$

$$\begin{aligned} E_{C_2} = {} & \frac{1}{2}\cdot\dot{q}_1^2\cdot(I_{z_2} + m_2\cdot x_{C_2}^2 + m_2\cdot l_1^2 + 2\cdot m_2\cdot l_1\cdot x_{C_2}\cdot\cos q_2) + \\ & + \frac{1}{2}\cdot\dot{q}_2^2\cdot(I_{z_2} + m_2\cdot x_{C_2}^2) + \dot{q}_1\cdot\dot{q}_2\cdot(I_{z_2} + m_2\cdot x_{C_2}^2 + m_2\cdot l_1\cdot x_{C_2}\cdot\cos q_2) \end{aligned} \tag{18}$$

$$E_{C_s} = \frac{1}{2} \cdot \dot{q}_1^2 \cdot (I_{z_s} + I_{m_1} + m_s \cdot l_1^2 + m_s \cdot l_2^2 + 2 \cdot m_s \cdot l_1 \cdot l_2 \cdot \cos q_2) + \\ + \frac{1}{2} \cdot \dot{q}_2^2 \cdot (I_{z_s} + I_{m_2} + m_s \cdot l_2^2) + \dot{q}_1 \cdot \dot{q}_2 \cdot (I_{z_s} + m_s \cdot l_2^2 + m_s \cdot l_1 \cdot l_2 \cdot \cos q_2) \tag{19}$$

The following expressions for the elements of the inertia matrix A are obtained:

$$A_{11} = I_{z_1} + I_{z_2} + I_{z_s} + I_{m1} + m_1 \cdot x_{C_1}^2 + m_2 \cdot x_{C_2}^2 + m_2 \cdot l_1^2 + \\ + 2 \cdot m_2 \cdot l_1 \cdot x_{C_2} \cdot \cos q_2 + m_s \cdot l_1^2 + m_s \cdot l_2^2 + 2 \cdot m_s \cdot l_1 \cdot l_2 \cdot \cos q_2 \tag{20}$$

$$A_{12} = A_{21} = I_{z_2} + I_{z_s} + m_2 \cdot x_{C_2}^2 + m_2 \cdot l_1 \cdot x_{C_2} \cdot \cos q_2 + m_s \cdot l_2^2 + m_s \cdot l_1 \cdot l_2 \cdot \cos q_2 \tag{21}$$

$$A_{22} = I_{z_2} + I_{z_s} + I_{m2} + m_2 \cdot x_{C_2}^2 + m_s \cdot l_2^2 \tag{22}$$

The matrices B and C have the following components:

$$\begin{cases} B_{1,12} = -2 \cdot m_2 \cdot l_1 \cdot x_{C_2} \cdot \sin q_2 - 2 \cdot m_s \cdot l_1 \cdot l_2 \cdot \sin q_2 \\ B_{2,12} = 0 \end{cases} \tag{23}$$

$$\begin{cases} C_{11} = 0 \\ C_{12} = -m_2 \cdot l_1 \cdot x_{C_2} \cdot \sin q_2 - m_s \cdot l_1 \cdot l_2 \cdot \sin q_2 \\ C_{21} = m_2 \cdot l_1 \cdot x_{C_2} \cdot \sin q_2 + m_s \cdot l_1 \cdot l_2 \cdot \sin q_2 \\ C_{22} = 0 \end{cases} \tag{24}$$

The vector Q has the form: $Q = \begin{bmatrix} M_{1m} & M_{2m} \end{bmatrix}^{\mathrm{T}}$, where M_{1m} and M_{2m} are the moments developed by the operating motors. We, also, assumed that the movement of the manipulator is in the horizontal plane $(O_0 x_0 y_0)$. In this case $G = 0$.

In fig. 3 and 4, the variations of the operating moments M_{1m} and M_{2m}, corresponding to a solution of the inverse geometric model, are presented.

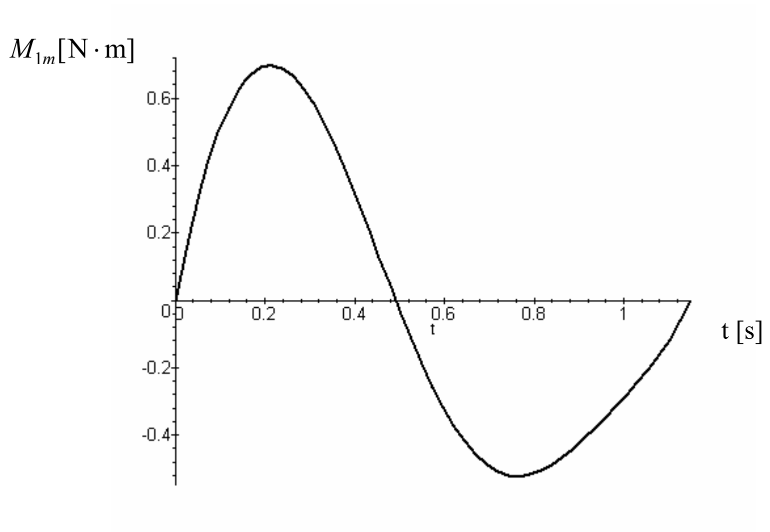

Fig. 3. The variation of the first operating moment

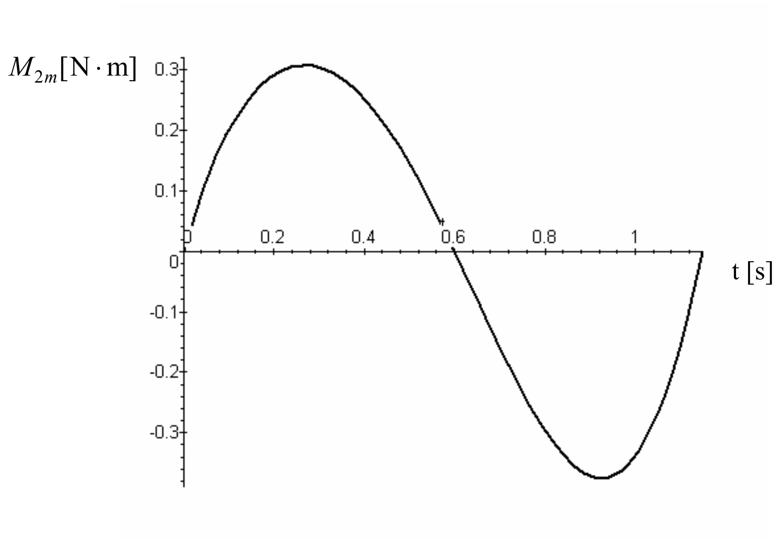

Fig. 4. The variation of the second operating moment

Conclusions

The paper presents some results concerning the positional, cinematic and dynamic analysis of the industrial robots mechanisms. The methodology presented, transposed into a computer program, can be used for an optimum design of the primary controller. Also, the simulator can be used for generating different trajectories in the corresponding work space.

References

1. Bădoiu, D. - *Analiza structurală şi cinematică a mecanismelor,* Editura Tehnică, 2001
2. Bădoiu, D. - *Analiza dinamică a mecanismelor şi maşinilor,* Editura Didactică şi Pedagogică, Bucureşti, 2003
3. Craig, J.J. - *Introduction to robotics: mechanics and control,* Addison-Wesley, 1986
4. Dombre, E., Khalil, W. - *Modélisation et commande des robots,* Ed. Hermès, Paris, 1988
5. Pelecudi, Chr. - *Precizia mecanismelor,* Editura Academiei Române, Bucureşti, 1975

Cercetări privind proiectarea controlerului primar al mecanismelor roboților industriali

Rezumat

Articolul prezină o metodă care permite proiectarea controlerului primar al mecanismelor de roboți, deci, a calculului cuplurilor motoare care permit controlul poziției şi vitezei la nivelul efectorului final. Metoda este aplicată în cazul unui manipulator cu două grade de libertate, când se impune o traiectorie lineară pentru obiectul manipulat. In final, sunt prezentate o serie de rezultate obținute în urma simulărilor.