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Feedforward Control System Design by Limiting the Control Signal Magnitude Ratio

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Abstract

In this paper, we present a design procedure for a feedforward controller of a linear proportional process by limiting the magnitude ratio of the control signal, in order to diminish the noise amplification, the wear and tear of the plant, and the fuel and energy consumption, too. For practical applications, the magnitude ratio of the control signal, defined as the ratio between the maximum value and the final value of the control signal for a step input (disturbance or reference) must be limited to a value less than 20. Three feedforward control algorithms, some of them being well-known, are presented in this article: a pure proportional algorithm, a dedicated dynamic algorithm and a standard dynamic algorithm. In addition, a practical method for design and tuning a feedforward standard dynamic controller to a given proportional process with dead time is presented. Some simulation applications are given to point out and compare the control performance of the presented feedforward control algorithms.

Key words: *input-output model, feedforward control, magnitude ratio, static/dynamic compensation, semi/strict-proper algorithm.*

Introduction

In practical applications, the feedforward control is almost always used along with the feedback control because only by a feedback control the process output tracks the reference change and suppresses the unmeasured disturbance effect, which is always present in any real process [3, 5]. A pure feedforward controller adjusts his output (control variable) taking account of the present values of the measured disturbance and reference, but not of the controlled variable. In its action, the controller only relies on the process model accuracy. Therefore, if the controlled variable deviates from its reference there is no corrective action to eliminate this deviation (error), and this drawback makes the pure feedforward control system to be rarely used in industrial applications [1, 7].

By adding feedforward control to feedback control, a combined control system can significantly improve the control performance whenever there is a major measured disturbance which affects the process output (controlled variable). In fact, a combined control system is equivalent to a feedback control system without its most major disturbance. The feedback control can entirely eliminate the error between the reference and the controlled variable, while the feedforward control can provide this only in an ideal situation, when one uses an ideal process model and an ideal control algorithm. However, even when there are modeling errors and an imperfect control algorithm, the feedforward control can reduce the effect of the measured disturbance on the controlled variable. Since the compensating action of the controller is executed in parallel and simultaneous with the direct disturbing action, the feedforward control system can prevent the change of the controlled variable by the measured disturbance [2, 8].

In practice one use a combined feedforward plus feedback control system only if the control performance improved by feedforward control increases the product quality enough to justify the added costs of design, implementation and maintenance. A typical feedforward control loop is shown in the figure 1. From the control system equation [6]

$$Y(s)=C_0(s)P_0(s)\cdot R(s)+(P_1(s)+C_1(s)P_0(s))D_1(s)\,, \tag{1}$$

we get immediately the ideal controller transfer functions to have a perfect control:

$$C_0(s)=\frac{1}{P_0(s)}\,, \tag{2}$$

$$C_1(s)=\frac{-P_1(s)}{P_0(s)}\,. \tag{3}$$

Usually, the controller transfer function $C_0(s)$ given by (2) is improper (with negative dead time and/or relative order) or unstable, and then cannot be implemented in this form. Sometimes, the controller transfer function $C_1(s)$ given by (3) is also improper or unstable.

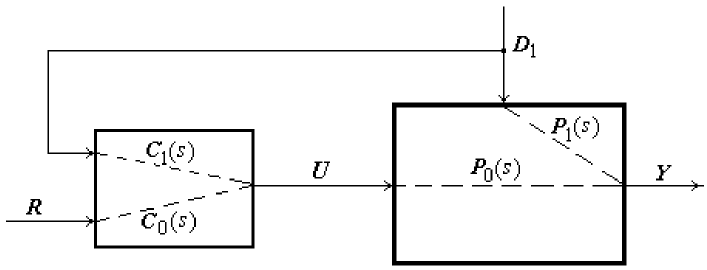

Fig. 1. Typical feedforward control structure.

Pure Proportional Controller

A feedforward control system with pure proportional controller is an average solution to achieve acceptable control performance with a low implementation cost. By using a pure proportional compensator, the control system does not perform a very well dynamic compensation to a step change in disturbance, but only a final compensation (at the steady-state behavior), which can be entirely realized for a perfect steady-state process model. How the controlled variable changes during the transient behavior to a step in disturbance or reference depends on dynamic characteristic of the process. For a stable proportional process, the pure proportional controller has the gains [1, 3]

$$K_{C0}=\frac{1}{P_0(0)}=\frac{1}{K_0}\,, \tag{4}$$

$$K_{C1}=\frac{-P_1(0)}{P_0(0)}=\frac{-K_1}{K_0}\,, \tag{5}$$

where K_0 and K_1 are the static gains of the process on the U-Y and D_1-Y channels, respectively.

In figure 2 are presented the control system response (y) and the process response (y_1) to a unit step disturbance for a process with

$$P_0(s) = \frac{3e^{-7s}}{(2s+1)(3s+1)(8s+1)}, \tag{6}$$

$$P_1(s) = \frac{2e^{-5s}}{(5s+1)(10s+1)}. \tag{7}$$

The control system response to a unit step reference is shown in figure 3. The graphs have been obtained in the Matlab/Simulink™ platform.

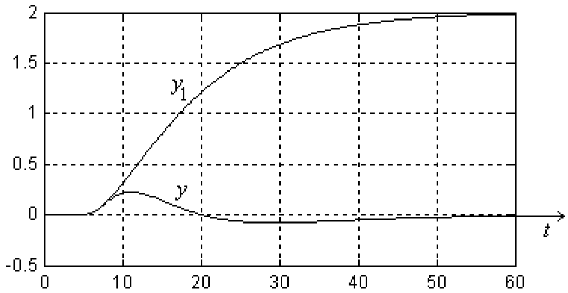

Fig. 2. Responses to a unit step disturbance, with and without pure proportional controller.

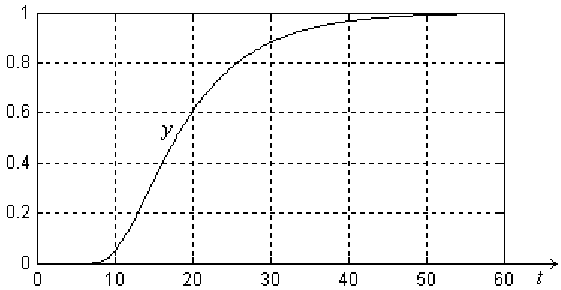

Fig. 3. Response to a unit step reference with pure proportional controller.

Dedicated Dynamic Controller

A feedforward control system with dedicated dynamic controller is an advanced solution to achieve very well control performance with a high implementation cost. Using such as controller one can achieve a very well compensation to a step change in disturbance, and a very well response to a step change in reference.

Let us consider a stable and minimum-phase process having the transfer functions [1]

$$P_0(s) = \frac{P_-(s)P_+(s)}{D(s)} e^{-\tau_0 s}, \tag{8}$$

$$P_1(s) = \frac{R_1(s)}{D_1(s)} e^{-\tau_1 s}, \tag{9}$$

where the polynomial $P_-(s)$ contains only left half zeros, while the polynomial $P_+(s)$ contains only right half zeros. According to the ideal relations (2) and (3), the integral square error optimal choice of the controller yields the following controller transfer functions

$$C_0(s)=\frac{D(s)}{P_-(s)P_+(-s)(T_0s+1)^{k_0}}\,, \tag{10}$$

$$C_1(s)=\tilde{C}_1(s)\mathrm{e}^{-\tau s}=\frac{-D(s)R_1(s)}{P_-(s)P_+(-s)D_1(s)(T_1s+1)^{k_1}}\,\mathrm{e}^{-\tau s}\,, \tag{11}$$

where

$$\tau=\begin{cases}\tau_1-\tau_0\,, & \tau_1>\tau_0\\ 0\quad, & \tau_1\le\tau_0\end{cases}\,, \tag{12}$$

T_0 and T_1 are filter time constants, and k_0 şi k_1 are positive integer numbers such that the rational functions $C_0(s)$ and $\tilde{C}_1(s)$ are semi-proper functions (with the same numbers of zeros and poles). In practical applications, the time constant T_0 and T_1 are chosen such that the magnitude ratio of the control variable u has a value less than 20 to avoid excessive noise amplification, to reduce the wear and tear of the plant, and to diminish the fuel and energy consumption. The magnitude ratio is defined as the ratio between the initial value and the final value of the control variable to a step input (reference or disturbance):

$$M=\frac{u(0_+)}{u(\infty)}\,. \tag{13}$$

After a convenient choice of the controller magnitude ratios M_0 and M_1, the filter time constants T_0 and T_1 are given by the relations

$$M_0=\frac{C_0(\infty)}{C_0(0)}\,,\quad M_1=\frac{\tilde{C}_1(\infty)}{\tilde{C}_1(0)}\,. \tag{14}$$

If the process response $y(t)$ to a step reference has an overshoot greater than 5 %, then we need to increase the value of the filter time constants T_0. The design of a feedforward control system with dedicated dynamic controller has two weak points: the control algorithm is based on a very accuracy process model and, on the other hand, the controller structure depends on the process transfer functions.

For the process transfer functions (6) and (7), we get a controller with the transfer functions

$$C_0(s)=\frac{(2s+1)(3s+1)(8s+1)}{3(T_0s+1)^3}\,, \tag{15}$$

$$C_1(s)=\frac{-2(2s+1)(3s+1)(8s+1)}{3(5s+1)(10s+1)(T_1s+1)}\,, \tag{16}$$

and the magnitude ratios

$$M_0 = \frac{48}{T_0^3}, \qquad M_1 = \frac{48}{50T_1}. \tag{17}$$

Choosing $M_0 = M_1 = 4$, we get $T_0 \cong 2.29$ and $T_1 \cong 0.24$. In figure 4 are presented the control system response y and the controller response u to a unit step disturbance. The control system response y and the controller response u to a unit step reference are shown in figure 5. Obviously, the control performance is much better than it was for the pure proportional controller.

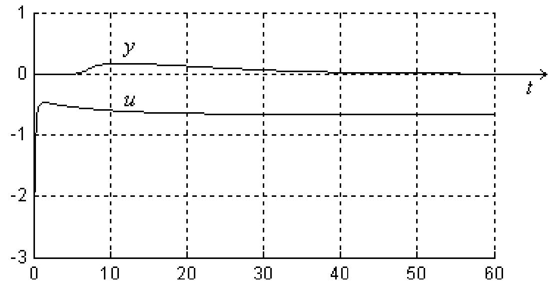

Fig. 4. Responses to a unit step disturbance with dedicated semi-proper dynamic controller.

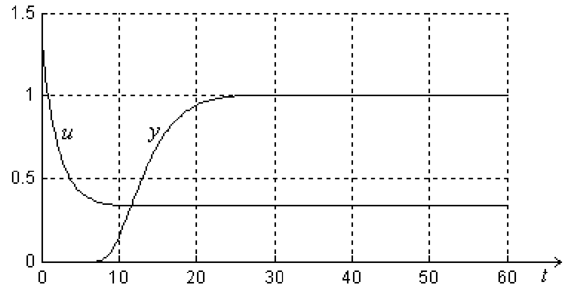

Fig. 5. Responses to a unit step reference with dedicated semi-proper dynamic controller.

Remark. In order to reduce the magnitude ratio of the control signal we can chose the positive integer power k_0 and k_1 such that the controller rational transfer functions $C_0(s)$ and $\tilde{C}_1(s)$ are strict-proper functions, with the pole-zero excess equal to 1. The magnitude ratio is smaller than in the previous case since the step input response of the controller have the initial value (at $t = 0+$) equal to zero. Note that the magnitude ratio of a strict-proper proportional system is defined as the ratio between the maximum value and the final value of the step response. For this reason, the time constants T_0 and T_1 of a strict-proper controller must be chosen less than those of a semi-proper controller.

In this strict-proper variant, the transfer functions (15) and (16) of the controller become

$$C_0(s) = \frac{(2s+1)(3s+1)(8s+1)}{3(T_0 s+1)^3}, \tag{18}$$

$$C_1(s) = \frac{-2(2s+1)(3s+1)(8s+1)}{3(5s+1)(10s+1)(T_1 s+1)^2}. \tag{19}$$

For $T_0 = 1.5$, in figure 6 are presented the control system response y and the strict-proper controller response u to a unit step disturbance. For $T_1 = 0.15$, the control system response y

and the strict-proper controller response u to a unit step reference are shown in figure 7. The control performance is comparable with that of the control system with semi-proper controller, but the control signal is a little smoother.

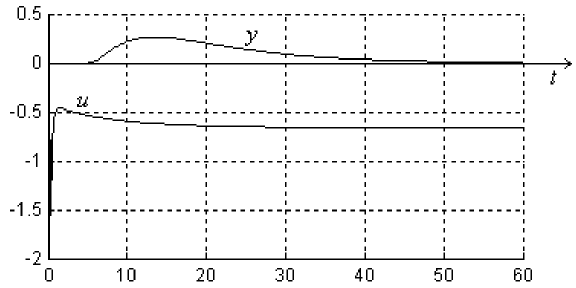

Fig. 6. Responses to a unit step disturbance with dedicated strict-proper dynamic controller.

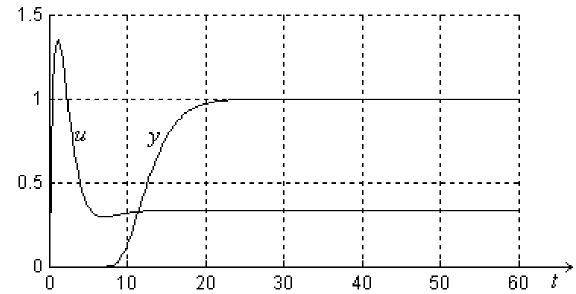

Fig. 7. Responses to a unit step reference with dedicated strict-proper dynamic controller.

Standard Dynamic Controller

In our opinion, a natural standard variant of feedforward controller is the following:

$$C_0(s) = \frac{K_{C0}(T_{20}s+1)}{T_{10}s+1}\ , \tag{18}$$

$$C_1(s) = \frac{K_{C1}(T_{21}s+1)e^{-\tau s}}{T_{11}s+1}\ . \tag{19}$$

The gains K_{C0} and K_{C1} are given by (4) and (5), while the dead time τ by (12). Let $(T_s)_0$ and $(T_s)_1$ be the process settling times (for an error band equal to 2 %) to a step change in disturbance and in control signal, respectively. We recommend choosing the lag time constant T_{11} and the lead time constant T_{21} of the controller C_1 as follows:

$$T_{11} \cong \frac{(T_s)_1}{4}, \qquad T_{21} \cong \frac{(T_s)_0}{4}\ . \tag{20}$$

These time constants can achieve almost perfect compensation of the disturbance effect if the both process channels are first order lag elements with the same dead time. Note that we can adjust the value of T_{11} by experimental way to have a minimal deviation of the controlled variable to a step change in disturbance. Also, we recommend choosing

$$T_{10} \cong \frac{(T_s)_0}{10}\,, \tag{21}$$

while the lead time constant T_{20} should be chosen by experimental way to have a small overshoot (about 3-5 %) of the process response $y(t)$ to a step reference.

For $\tau \cong mT$, where $m \in \mathbf{Z}$ and T is the sample period, the discrete equivalent of the compensating controller C_1 has the transfer function

$$C_1(z) = \frac{K_{C1}[\frac{T_{21}}{T_{11}} + (1 - \frac{T_{21}}{T_{11}} - p_{11})z^{-1}]z^{-m}}{1 - p_{11}z^{-1}}\,, \tag{22}$$

where $p_{11} = \mathrm{e}^{-T/T_{11}}$. Note that in the case $(T_s)_1 > (T_s)_0$, we can write the transfer function of the compensating controller C_1 in the reduced form

$$\tilde{C}_1(s) = \frac{K_{C1}\mathrm{e}^{-\tau s}}{\tilde{T}_{11}s + 1}\,, \tag{23}$$

where

$$\tilde{T}_{11} \cong \frac{(T_s)_1 - (T_s)_0}{4}\,. \tag{24}$$

For the process transfer functions (6) and (7), in figure 8 are presented the process responses y_0 and y_1 to a unit step change in control signal and in disturbance, respectively.

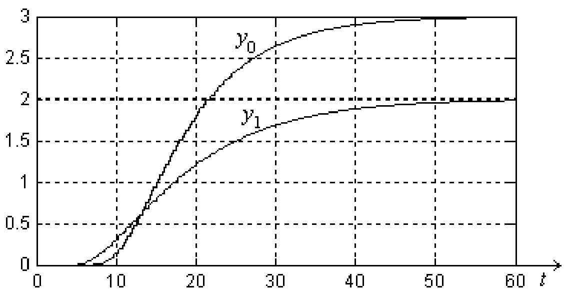

Fig. 8. Process responses to a unit step input.

It follows that $(T_s)_0 \cong 45$ and $(T_s)_1 \cong 48$, which provide $T_{11} = 11.5$, $T_{21} = 12$ and $T_{10} = 4.5$.

For $T_{20} = 9$, we get the standard dynamic controller

$$C_0(s) = \frac{9s+1}{3(4.5s+1)}\,,\quad C_1(s) = \frac{2(12s+1)}{3(11.5s+1)}\,. \tag{25}$$

In figure 9 and 10 are presented the control system responses to unit step disturbance and unit reference. Since $(T_s)_0 \cong (T_s)_1$, the step disturbance response is close to the response obtained by using a pure proportional controller. On the other hand, the step reference response is much better.

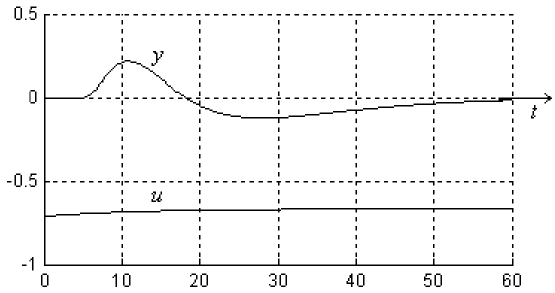

Fig. 9. Responses to a unit step disturbance using a standard dynamic controller.

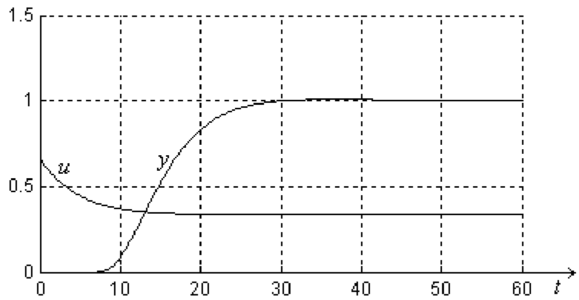

Fig. 10. Responses to a unit step reference using a standard dynamic controller.

Conclusions

Three feedforward control algorithms are presented in this article: a pure proportional algorithm, a dedicated dynamic algorithm and a standard dynamic algorithm. Moreover, two variants of dedicated dynamic algorithms are presented: with semi-proper and strict-proper controller. The variant with strict-proper controller is characterized by a control signal smoother than that in variant with semi-proper controller. In our opinion, the standard dynamic algorithm provides sufficiently well control performance, being more precise that the pure proportional algorithm and more practical than the dedicated dynamic algorithm. In addition, we presented a practical method for design and tuning a feedforward standard dynamic controller to a given proportional process with dead time.

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Proiectarea sistemelor de reglare după perturbaţie prin metoda limitării magnitudinii semnalului de comandă

Rezumat

În lucrare este prezentată o metodă de proiectare a regulatorului unui sistem de reglare după perturbaţie cu proces liniar de tip proporţional prin limitarea raportului de magnitudine al semnalului de comandă, în scopul reducerii amplificării zgomotului, a uzurii instalaţiei, a consumului de combustibil şi energie. Pentru aplicaţiile industriale, raportul de magnitudine al semnalului de comandă, definit ca raportul dintre valoarea maximă şi cea minimă a acestui semnal la intrare treaptă (perturbaţie sau referinţă), trebuie limitat la o valoare mai mică decât 20. Trei algoritmi de reglare după perturbaţie, unii dintre aceştia fiind cunoscuţi în literatura de specialitate, sunt prezentaţi comparativ: un algoritm de tip proporţional, un algoritm dedicat de tip dinamic şi un algoritm standard de tip dinamic. Algoritmul dedicat de tip dinamic este realizabil în două variante: varianta semi-proprie şi varianta strict-proprie. In cazul ultimei variante, răspunsul regulatorului este mai puţin agresiv, mai neted. De asemenea, este prezentată o metodă practică de proiectare şi acordare a regulatorului standard de tip dinamic. Considerăm că acest tip de regulator realizează performanţe de reglare suficient de bune, fiind mai precis decât regulatorul de tip pur proporţional şi mai practic decât regulatorul dedicat de tip dinamic. Sunt prezentate aplicaţii de simulare pentru a evidenţia şi compara caracteristicile şi performanţele celor trei tipuri de algoritmi de reglare după perturbaţie.

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