BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LXI No. 3/2009	93 - 98	Seria Tehnică

An Adjusted Mathematical Model for Realistic Road Traffic Simulation

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Abstract

Modelling and simulation, as one of the most used tools in processes investigation, are successfully applied for road traffic dynamic studies. As shown in the open literature, such a system with complex behavior is characterized by strong interactions between traffic participants, transport infrastructure and traffic controls, having a serious environmental impact – even deeper than other fields of human activity [1]. This paper addresses a modern modelling approach, originally adapted and included in the already announced software framework for controlled traffic investigation [3], as mathematical core-engine for independent lanes dynamic behavior description.

Key words: *road traffic, dynamic modelling.*

Introduction

Recently, the author of this paper has started a research project focused on road traffic mathematical modeling techniques, embedded within a modern framework which allows an easy traffic simulators implementation – presented in [3]. As the cited paper presents the project overview, from general aspects (like general/standard modeling and simulation approaches) to specific solved problems when building-up the software framework, this work offers a more complex look inside the mathematical model which is the core-engine of the application.

Modeling the traffic actors – a new approach

Since each mobile entity acts accordingly with its neighbors' behavior and (own) established rules, this work adopts a microscopic representation technique which may become the mathematical core of a traffic cellular automaton. This approach naturally leads to a significant flexibility in numerically defining a wide range of behavioral entities, which can be easily used for simulation and/or analysis purposes [2].

In the current representation, an independent traffic actor is determined by its *passive properties* (seen as model constant parameters: car length l, maximal acceleration a_+, maximal deceleration a_-, driver reaction time t_{react} and sensitivity S) and *active properties* (its allocated state variables: position x, actual speed v and acceleration a – updated with each simulation step Δt). Acceleration is considered as the main state variable, because it strongly depends on the environment and, more, v and x can be easily calculated from a. The only macroscopic parameter, seen as *traffic scene property*, is the maximum allowed speed value v_{max}.

Figure 1 shows the simplest case of a one-way road with only two cars, where vehicle 1 (in the back) behavior is described via the proposed algorithm, while vehicle 2 (in the front) is controlled by directly specifying its acceleration values over the entire simulation time horizon.

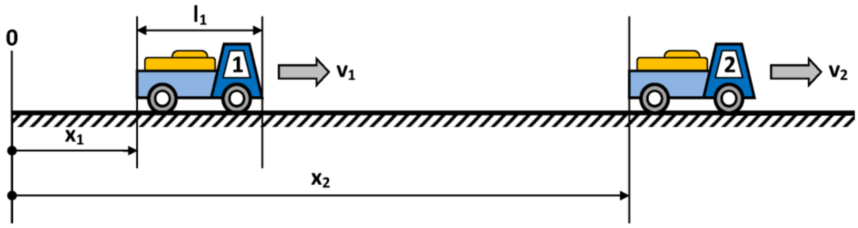

Fig. 1. Traffic scene: one-way road with two vehicles.

So, focusing on car 1 only, the first simplifying assumption is to have a constant acceleration value for each Δt time horizon, the a_1 value (positive for acceleration, negative for deceleration) being directly influenced by the driver's actions on gas pedal. Considering also that a_1 should tend to its extreme values (a_{1+} or a_{1-}), the following equations in the model gives the vehicle 1 acceleration:

$$a_1(t) = \begin{cases} a_+ \tanh(S_1 \times \varepsilon_1(t)), \text{ if } \varepsilon \geq 0, \\ a_- \tanh(S_1 \times \varepsilon_1(t)), \text{ otherwise,} \end{cases} \tag{1}$$

where

$$\varepsilon_1(t) = \begin{cases} \Delta v_1(t) \times \left(x_2(t) - x_1(t) - l_1 - v_1(t) \times t_{1\,react} + \frac{\left(v_1(t)-v_2(t)\right)^2}{2a_{1-}} \right), & \text{if } \Delta dist_{12} < 0, \\ \Delta v_1(t) \times \left(x_2(t) - x_1(t) - l_1 - v_1(t) \times t_{1\,react} + \frac{\left(v_1(t)-v_2(t)\right)^2}{2a_{1+}} \right), & \text{otherwise.} \end{cases} \tag{2}$$

$\Delta dist_{12}$ represents the tendency of inter-vehicles distance variation, directly observed by driver 1. It takes into account the current time step (t) and the previous one ($t - \Delta t$), having negative values when $v_1 > v_2$ or non-negative values otherwise:

$$\Delta dist_{12} = \left(x_2(t) - x_1(t)\right) - \left(x_2(t - \Delta t) - x_1(t - \Delta t)\right). \tag{3}$$

$\Delta v_1(t)$ is the relative deviation between current vehicle 1 speed and its maximum allowed speed, v_{max}, calculated as

$$\Delta v_1(t) = \left(v_{max} - v_1(t)\right)/v_{max}\,. \tag{4}$$

Equation (1) establishes a direct dependency between acceleration a and ε which defines the deviation between *ideal* traffic conditions (free road, no maximum speed limit) and *real* ones. The author of this work propose a modified ε definition (in comparison with other classical approaches in the open literature – [2, 3]), which now *simultaneously* takes into account both restrictions (obstacles presence and speed limitations).

As shown in [3], for an independent traffic actor, *the fixed obstacles* (traffic lights, stopped cars) or *mobile* ones (moving vehicles on the same pathway) need a permanent state evaluation. But, regardless the obstacles type, the general *safety arrival distance* rule applies; it correlates the driver's actions (changes in a) with current traffic conditions, in a way allowing obstacles approaching, but never touching them. Considering vehicle 2 as the only (mobile) obstacle, the term $\left(x_2(t) - x_1(t) - l_1 - v_1(t) \times t_{1\,react} + \left(v_1(t) - v_2(t)\right)^2/2a_{1-}\right)$ in equation (2) estimates, at each time step, if car 1 can be safely slowed down when $\Delta dist_{12} < 0$ and a_1 hypothetically becomes a_{1-}. Greater this term is, safer its current situation becomes, while a negative value indicates the crashing danger; zero represents the critical limit when car 1 touches vehicle 2 exactly when v_1 becomes v_2 (so there will be no true collision after).

Of course, the same principle may be considered when evaluating the safety arrival distance rule for any fixed obstacle, $v_2(t)$ being replaced with zero in the term above, as shown in [3]. On the other hand, in equation (2) – after many experimental studies – the author of this work proposes a symmetric term in ε expression when $\Delta dist_{12} \geq 0$, finally leading to a true realistic vehicle behavior.

The speed limits, imposed by local traffic rules, road state and direction changes for instance, are taken into account by the term $\Delta v_1(t)$ in ε definition. Considering another simplifying assumption ($v_1(0) \leq v_{max}$, which is in fact absolutely normal), $\Delta v_1(t)$ is always positive and only slightly adjust the ε value when v_1 is close to v_{max}, until a_1 becomes zero. As time as the vehicle 1 speed value for the next step may be calculated with

$$v_1(t + \Delta t) = max(0, v_1(t) + a_1(t) \times \Delta t), \tag{5}$$

it is easy to demonstrate that, after several number of time steps Δt, v_1 will equal v_{max} whenever there is a safe distance between considered vehicles, proving a good adapting feature for the model (when new limitations – shown by changes in v_{max} – happen to occur). This approach can also be successfully applied to all dynamic changes in traffic regime, like traffic lights color switches and concurrence with vehicles having higher priority (when an additional decision structure completes the so-called *gap acceptance algorithm*) [2, 3].

Regarding the car 1 position, it is given by

$$x_1(t + \Delta t) = max\left(x_1(t),\ x_1(t) + v_1(t) \times \Delta t + \frac{a_1(t)\times(\Delta t)^2}{2}\right). \tag{6}$$

One can observe that equations (5) and (6) do not allow any negative values for v, respectively any x_1 decreasing tendency (meaning no turning back for the considered vehicle 1).

As remark, the positive or negative value of a is directly influenced only by ε, as all other terms in equation (1) are strictly greater than zero. Then, it can be observed that (1) brings a realistic representation of a depending on ε value by using the hyperbolic tangent operator, denoting a stronger driver's reaction on the gas pedal as the deviation (positive or negative) has a bigger absolute value [2].

Simulation results

For this paper, four simulation scenarios were selected, in order to prove the modified model adequacy in describing a two-vehicle traffic situation, where the car in front (2) is freely controlled (by directly specifying its acceleration $a_2(t)$ value(s) during simulation horizon, initial speed $v_2(0)$ and position $x_2(0)$), while the following car (1) behavior is modeled by the cinematic laws above presented. In all cases, vehicle 2 is characterized by $x_2(0)$ = 100m, $v_2(0) = v_{max}$ = 19.46m/s (70 km/h) and the same acceleration profile. Both vehicles have $a_{1+} = a_{2+}$ = 1.7m/s^2 and $a_{1-} = a_{2-}$ = –5m/s^2.

Scenario 1: $v_1(0)$ = 0m/s, sensitivity factor S = 2.5 (normal driving style)

Figure 2 presents how vehicle 1 reacts when starting with zero speed (at t = 0). The sensitivity factor value may be considered as medium/normal for this traffic case. First, the driver pushes completely the gas pedal ($a_1 = a_+$ = 1.7m/s^2) during the first 9 seconds. As consequence, v_1 rapidly increases from 0 to 16m/s, close to the maximum allowed speed v_{max} until (at $t \cong$ 10s), the brake is seriously hit ($a_1 \cong$ –3.2m/s^2) for a short time in order to prevent an imminent collision with vehicle 2. For the next 15s a_1 moderately increases, reaching again its maximum allowed value (1.7m/s^2) because there is no collision risk anymore. Since at t = 16s v_1 = v_2 = 3m/s, during the next time interval (t > 18s) it is expected that driver 1 will try to adapt its actions in order to keep v_1 as close as possible to v_2, maintaining this way an approximately constant safety gap ($x_2 - x_1$). One can see in figure 2 that the proposed algorithm successfully satisfies the *car following principle* above mentioned, for the chosen sensitivity value (2.5), the collision state being constantly kept at “0” (meaning car 1 never touches car 2, even when at t = 60s both vehicles are stopped).

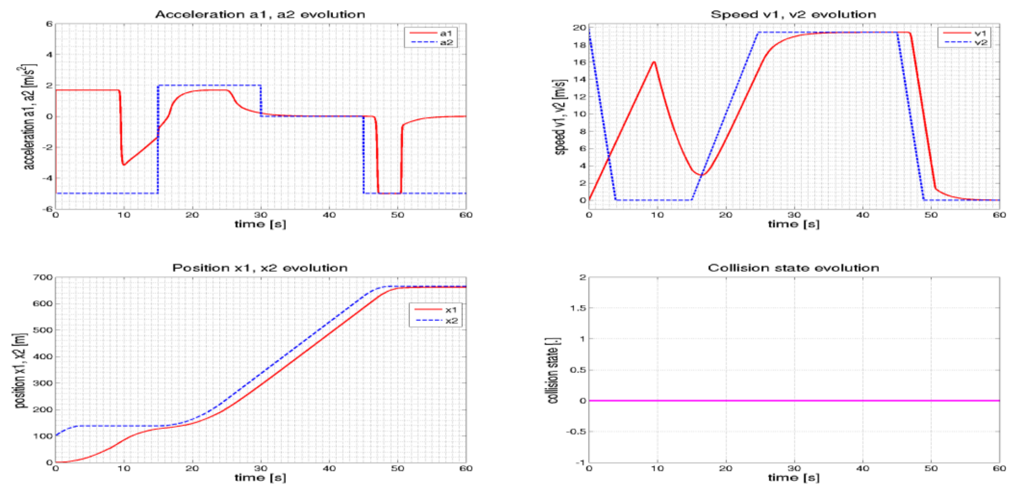

Fig. 2. Simulation results for scenario 1.

Scenario 2: $v_1(0)$ = 0m/s, sensitivity factor S = 0.1 (“lazy” driving style)

This new scenario differs from the first one only by intentionally considering a (very) low sensitivity factor value. As the good sense tells and figure 3 shows, the effect of a calmer action on the gas and brake pedals consists in a much slower speed variation, with lower amplitude (on corresponding time values) than in previous case. But, by analyzing the collision state evolution, it can be seen that vehicle 1 hits the car in front in two situations, at $t \cong$ 16s and $t \cong$ 51s (when collision state becomes “1”). In this case, the driver cannot keep a safe distance as it reacts too slowly when vehicle 1 suddenly stops (in about 4 seconds), because $a_2 = a_- = -5$m/s^2 at t = 0s and t = 45s. One can see in figure 3 how the v_1 profile is right-shifted from the previous case, meaning v_1 is adapted to v_2 with a serious delay, leading to this unwanted crashing situations.

Scenario 3: $v_1(0)$ = 0m/s, sensitivity factor S = 20.0 (“aggressive” driving style)

The third scenario illustrates the effect of a high sensitivity factor value, characterizing a sporty or nervous driver, on the controlled car (2) behavior. Such a driver over-estimates as potential dangers what all other drivers call “normal traffic situations” (i.e. a car in front quick speed decreasing, but still in the safe limits). On the other hand, the sporty/nervous driver usually hits the gas pedal shortly after he sees the distance to followed vehicle increases.

The proposed model successfully addresses this aggressive driving style simulation. As figure 4 depicts, by keeping the same behavior for vehicle 2, as well as other parameters for car 1 controlling algorithm (except the sensitivity factor), two false-critical time intervals can be identified (at t = 15s, for one second, and at t = 52s, for about 8 seconds), when vehicle 1 seriously approaches car 2. During these periods, driver 1 seems to nervously hit the brake, until it appreciates the “critical” situation ended. As remark, figure 4 presents only the acceleration evolution (with a zoomed vicinity of t = 16s), because all other diagrams look identical.

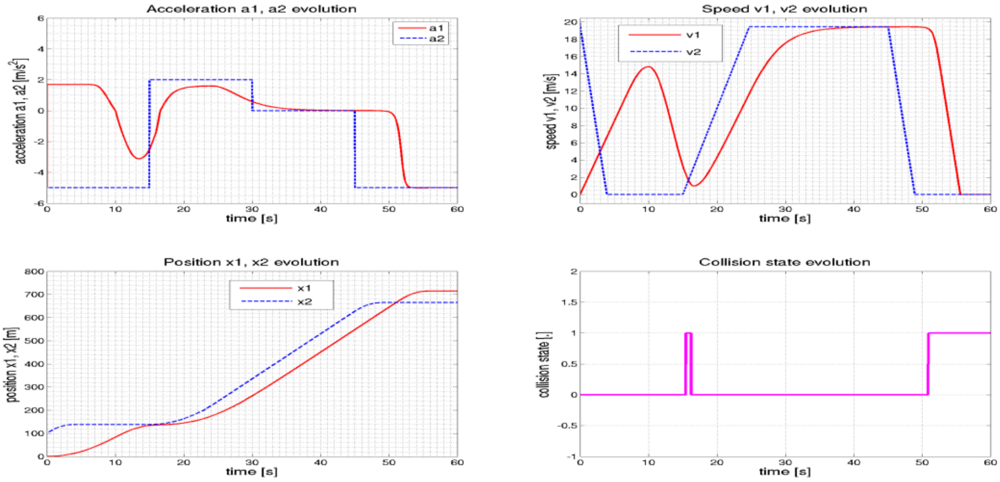

Fig. 3. Simulation results for scenario 2.

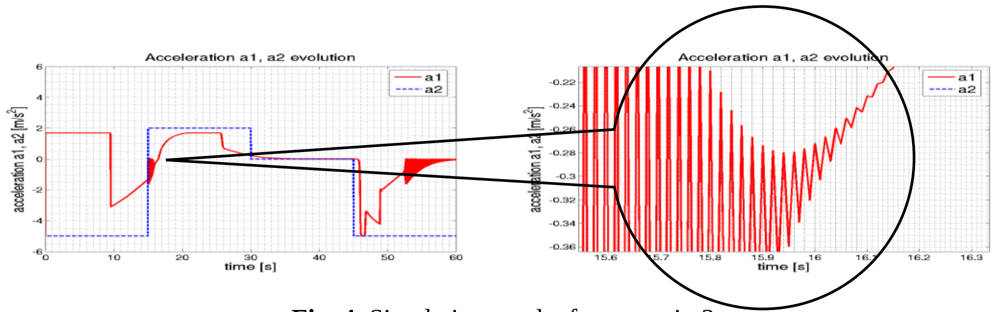

Fig. 4. Simulation results for scenario 3.

Scenario 4: $v_1(0)$ = 19.46m/s, sensitivity factor S = 10.0 (increased sensitivity)

Last chosen scenario represents another traffic situation, when vehicle 1 initial speed ($v_1(0)$) has the maximum allowed value, 19.46m/s, being the same as $v_2(0)$.

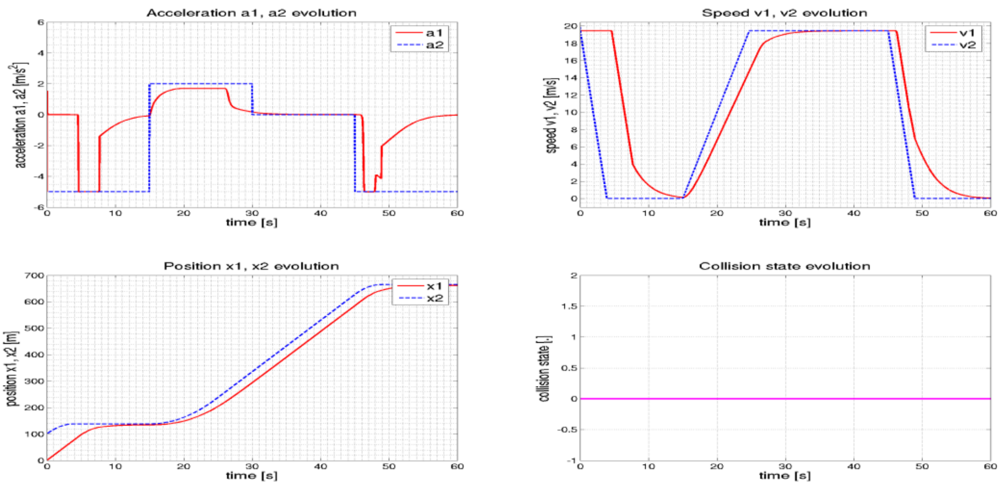

Fig. 5. Simulation results for scenario 4.

Although the results are not presented here, a sensitivity factor of 2.5 (like in scenario 1) proved not to be adequate anymore, as the high initial value of v_1 combined with a stiff situation ($a_2(0) = a_- = -3.2\text{m/s}^2$) imposes a different driver 1 attitude in order to slow down the vehicle within a safe time interval (meaning $x_2 - x_1 \geq l_1$ when $v_1 = v_2$). A test sensitivity value of 10.0 was used instead, the results depicted by figure 5 showing no collision for the entire simulation horizon.

Conclusions

This paper offers a more complex image on the mathematical model as the core-engine of a modern software framework (previously announced in [3]) allowing an easy traffic simulators design and implementation. Two changes in the model (introducing driver's sensitivity factor and fine acceleration tuning when approaching the maximum legal speed) were tested through simulation, with extremely promising results. In future research, the sensitivity must not have a constant value (as it is now), because traffic conditions are subject to serious variations from one scenario to another. The author will try to find an adaptive variation law for the sensitivity factor, where the main idea is to increase/decrease it until car in the back approaches the front car, and then revert it to a standard value (i.e. something between 2.5 and 10.0). As starting example, scenario 4 has to be considered: when $t > 16$s, sensitivity may be decreased because both vehicles start again with $v_1 = v_2 = 0$, somewhere at about 140m from the x-axis origin.

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Un model matematic adaptat pentru simularea fidelă a traficului auto

Rezumat

Modelarea și simularea, ca instrumente puternice asociate studiului sistemelor complexe, sunt aplicate cu succes în investigarea dinamicii traficului auto. Așa cum literatura menționează, un astfel de proces este caracterizat de puternice interacțiuni între entitățile participante, infrastructura rutieră și regulile de gestiune a circulației, având și un deosebit impact asupra mediului (ce poate depăși depășind chiar pe cel al industriilor productive [1]). Această lucrare prezintă o abordare modernă și originală a modelării matematice a traficului, ce se constituie în motorul platformei de simulare descrisă în lucrarea [3].