BULETINUL	Vol. LXI	105 - 110	Seria Tehnică
Universității Petrol – Gaze din Ploiești	No. 3/2009		

# Analysis of a Two-Coordinate Driving System Aiming at Performance Improvement

# Mikho Mikhov, Ivan Tatarov

Technical University of Sofia, Faculty of Automatics, 8 Kliment Ohridski Blv., 1797 Sofia, Bulgaria e-mail: mikhov@tu-sofia.bg

## Abstract

The performance of a two-coordinate hybrid stepping motor system developed for precise position applications is discussed in this paper. Load influence on motion trajectories is shown and the respective acceleration and deceleration profiles are derived. Microstepping control is applied and drive behavior is analyzed. The results obtained could be used in the design of such types of positioning systems.

Key words: stepping motor drive, position control, microstepping operation mode.

# Introduction

A number of industrial applications require two-coordinate drive systems with high positioning accuracy. Such performance can be ensured by stepping motors because they have several advantages, such as good position resolution, wide speed range, simple and robust construction, ability to operate in open-loop driving systems and easy compatibility with microprocessor controllers [1, 2].

Hybrid stepping motors combine the best characteristics of both variable reluctance and permanent magnet motors. They are appropriate in applications requiring small step length and high torque within restricted working space. The microstepping control can provide some additional improvements such as higher position accuracy and resolution, vibrations and noise reduction, as well as elimination or simplification of the gearboxes [3, 4].

The performance of a two-coordinate driving system developed for precise position applications is presented in this paper. Load influence on motion trajectories is shown and the respective acceleration and deceleration speed profiles are derived. The drive system behavior in micro-stepping operation mode is analyzed to determine the necessary micro-step length in compliance with the desired accuracy.

# **Features of the Driving System**

The simplified block diagram of the developed two-coordinate driving system is shown in Fig. 1, where the notations are as follows: KB - keyboard; MC - microprocessor controller; C1, C2 – power converters; DP - display; M1, M2 – hybrid stepping motors; E1, E2 – position

encoders; G1, G2 – mechanical gears; L1, L2 – loads applied to the respective motor shafts;  $\theta_1$ ,  $\theta_2$  – rotor positions;  $T_1$ ,  $T_2$  – motor torques;  $T_{l_1}$ ,  $T_{l_2}$  – load torques.

The hybrid stepping motors used consist of a slotted stator with two phases and a permanent magnet rotor. It is axially magnetized to produce the respective pole pairs. Both phases can be excited with positive and negative currents. When the windings are unexcited the magnet flux produces a small detent torque, which retains the rotor at the step position.



Fig. 1. Block diagram of the twocoordinate driving system under consideration.

The electrical and mechanical equations representing one coordinate of the developed stepping motor drive are as follows:

$$v_a = L \frac{di_a}{dt} + Ri_a - K_t \omega \sin\left(p\theta\right); \tag{1}$$

$$v_b = L \frac{di_b}{dt} + Ri_b + K_t \omega \cos(p\theta);$$
<sup>(2)</sup>

$$T = -K_t i_a \sin(p\theta) + K_t i_b \cos(p\theta); \qquad (3)$$

$$\omega = \frac{d\theta}{dt}; \tag{4}$$

$$T = J \frac{d\omega}{dt} + T_l \,, \tag{5}$$

where:  $v_a$  and  $v_b$  are phase voltages;  $i_a$  and  $i_b$  – phase currents;  $\omega$  – angular velocity;  $K_t$  – torque constant; R – winding resistance; L – winding inductance; p – number of the rotor teeth; J – total inertia referred to the motor shaft.

Using the MATLAB/SIMULINK software package a number of computer simulation models have been developed to analyze the behavior of such a driving system in the respective dynamic and static regimes.

#### **Formation of Motion Trajectories**

Generally, there are two operating regions of the stepping motors, namely a start/stop and a slew one (Fig. 2). They are defined by pull-in and pull-out torque curves, respectively.

To operate the stepping motor at higher speeds, it is necessary to start at a frequency within the first area and then accelerate the motor into the second range. While the motor is stopped, it should be decelerated until the start-stop zone, before the impulse sequence is terminated. Otherwise, synchronization in rotor positioning will be lost.

Acceleration and deceleration of stepping drives depend on both load torque and total inertia referred to the motor shaft. To ensure maximum speed of the positioning system, control should be applied in accordance with the pull-out curve.



The angular velocity and rotor position can be expressed as follows:

$$\omega = \alpha f ; \tag{6}$$

$$\theta = \alpha N , \qquad (7)$$

where:  $\alpha$  is mechanical step; f – frequency of the clock pulses; N – number of the steps.

Reading Eqs. (6) and (7), to achieve the desired trajectory of  $\omega(\theta)$ , the respective f(N) can be programmed. Fig. 3 shows such a motion trajectory for one coordinate, during execution of an assigned position cycle.

Taking into consideration Eq. (6), Eq. (5) becomes as follows:

$$J\alpha \frac{df}{dt} + T_l = T(f).$$
(8)

The acceleration and deceleration as functions of frequency are obtained in accordance with the next expressions:

$$\left[\frac{df}{dt}(f)\right]_{a} = \dot{f}_{a}(f) = \frac{T(f) - T_{l}}{J\alpha};$$
(9)

$$\left[\frac{df}{dt}(f)\right]_{d} = \dot{f}_{d}(f) = -\frac{T(f) + T_{l}}{J\alpha}.$$
(10)

The impulse frequency/step number relations can be determined from the following equations:

$$N = \int_{f_s}^{f} \frac{f}{\dot{f}_a(f)} df \to [f(N)]_a; \qquad (11)$$

$$N = \int_{f_r}^{f} \frac{f}{\dot{f}_d(f)} df \to [f(N)]_d, \qquad (12)$$

where  $f_s$  and  $f_r$  are the starting and the reference frequency values respectively.

# **Positioning Accuracy Improvement**

To improve the position accuracy in the developed driving system, microstepping control has been applied. In this operating mode, the full step length is divided electronically into small increments of rotor motion:

$$\alpha_{\mu} = \alpha/n \,, \tag{13}$$

where: *n* is the number of micro-steps;  $\alpha_{\mu}$  – the angular displacement of each micro-step.

Subdivision of the basic motor step is possible by proportioning the phase currents in the two windings (Fig. 4). Current magnitudes vary and the number of current levels depends on the desired micro-step size.



Fig. 4. Current waveforms in microstepping operation mode.

Air gap flux is proportional to the vector sum of the currents in the resultant vector direction. To achieve a required rotating flux, the phase currents' magnitudes are calculated as follows:

$$i_a(N_j) = I_{\rm rat} \cos(ja_\mu); \tag{14}$$

$$i_b(N_j) = I_{\text{rat}} \sin(ja_\mu), \tag{15}$$

where:  $I_{\text{rat}}$  is the rated current value;  $N_j$  – the number of current levels; j = 0, 1, 2, ..., 4n - 1.

The resultant stator current represents the phase currents vector sum:

$$I = \sqrt{\left[I_{\text{rat}}\cos\left(j\alpha_{\mu}\right)\right]^{2} + \left[I_{\text{rat}}\sin\left(j\alpha_{\mu}\right)\right]^{2}} = I_{\text{rat}}.$$
 (16)

Eq. (16) shows that the resultant current remains constant and equal to the rated value. Therefore, by correct combinations of phase current levels it is possible to obtain constant resultant current and smooth movement of the stepping motor shaft.

#### **Practical Implementation**

Theoretical and experimental research has been carried out for two-phase hybrid step motors with basic parameters as follows: rated voltage  $V_{\text{rat}} = 5 \text{ V}$ ; rated current  $I_{\text{rat}} = 1 \text{ A}$ ; number of rotor teeth p = 50; full motor step  $\alpha = 1.8^{\circ}$ .

The formation of motion trajectories for both coordinate is carried out in a sequence as follows:

- 1. The pull-out torque curve  $T_i(f_i)$  should be specified experimentally (i = 1, 2).
- 2. An appropriate approximation for the obtained curve is carried out.
- 3. In compliance with Eqs. (9) and (10) the  $f_{ai}(f_i)$  and  $f_{di}(f_i)$  relationships are determined.
- 4. Through Eqs. (11) and (12) acceleration and deceleration profiles  $[f(N)]_{ai}$  and  $[f(N)]_{di}$  are obtained.
- 5. The respective motion profiles are programmed for the desired position cycle  $f_i(N_i)$ .

Applying this methodology for the acceleration, the following expressions have been derived:

$$\dot{f}_a = \dot{f}_{a0}(1 - \frac{f}{f_0});$$
(17)

$$N = \frac{f_0}{\dot{f}_{a0}} (f_0 \ln \frac{f_0}{f_0 - f} - f).$$
(18)

Analogous dependencies have also been obtained about deceleration. The control can be simplified if the breaking process is executed with motion trajectories symmetrical to the starting ones.



Fig. 5. Current levels when one full step is divided into 8 sub-steps.

For each driving position coordinate, tuning of the microprocessor controller is carried out in the following sequence:

- 1. The appropriate micro-step  $\alpha_{\mu}$  is calculated in accordance with the desired position resolution  $\Delta S$ .
- 2. The number of micro-steps *n* is defined on the basis of the  $\alpha_{\mu}$  value.
- 3. The necessary phase current levels are calculated in compliance with Eq. (14) and Eq. (15).
- 4. A respective lookup table for the microstepping mode of operation is compiled.



Fig. 6. Position responses obtained in the respective control modes.

The current levels calculated for obtaining constant motor torque are given in Fig. 5. In this case one full step is divided into eight micro-steps and the currents are represented in relative units:

$$i_a^* = i_a / I_{\text{rat}}; i_b^* = i_b / I_{\text{rat}}.$$
 (19)

Fig. 6 a shows the position response obtained in full step operation mode when the two phases are excited simultaneously ( $\alpha = 1.8^{\circ} \approx 0.0314$  rad).

The microstepping control applied is illustrated by Fig. 6 b. The positioning resolution is 1600 micro-steps per revolution ( $\alpha_{\mu} = \alpha/8 \approx 0.00393$ rad).

Analysis of the driving system behavior shows that the applied method for microstepping control allows any sub-step length. However there is a practical limit on how small micro-steps can become until the rotor and its mechanical load cease to react adequately. Limitations can be

induced by the following factors: static friction in the mechanical system; non-sinusoidal character of the torque versus rotor position curves; current quantization resolution.

# Conclusion

The behavior of a two-coordinate hybrid stepping motor drive system has been analyzed and discussed in this paper.

Load influence on motion trajectories is shown and appropriate acceleration and deceleration profiles are derived. Microstepping control is applied which improves the positioning resolution, reduces shaft vibrations and ensures maximum torque at both low and high frequencies. The respective tuning sequence is presented.

The research carried out as well as the results obtained could be used in the design of such types of two-coordinate precise positioning systems.

# Acknowledgement

The Technical University of Sofia, Project No 091NI142-08/2009, has supported this research.

# References

- 1. Kenjo, T., Sugawara, A. Stepping motors and their microprocessor controls. Clarendon Press, Oxford, 1995.
- 2. Acarnley, P. Stepping motors: a guide to theory and practice. IEE, London, 2002.
- 3. Freitas, M.A., Andrade, D.A., Borges, T.T., Azevedo, H.R. *Driving the step motor with controlled phase currents.* Proceedings of the International Conference on Power Electronics Drives and Energy Systems for Industrial Growth, pp. 493-498, PEDES, Perth, Australia, 1998.
- 4. Mikhov M.R., Nakov P.S. A hybrid step motor drive system with high positioning accuracy. Proceedings of the International Conference on Electronic Devices and Systems, pp. 59-64, IMAPS, Brno, Czech Republic, 2005.

# Analiza unui sistem de poziționare bidimensională, pentru ameliorarea performanțelor de funcționare

## Rezumat

Articolul de față ia în discuție performanțele unui sistem de acționare hibridă asociat aplicațiilor de precizie. Este pusă în evidență influența sarcinii asupra traiectoriei de mișcare, fiind determinate profilele de accelerație/decelerație asociate. De asemenea, este analizată funcționarea efectivă a sistemului, în condiții reale de lucru. Rezultatele obținute pot fi folosite în etapa de proiectare a unor astfel de sisteme de poziționare precisă.