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# **Optimal Fuzzy Analysis of Manufacturing Systems**

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# Abstract

This article presents two algorithms for optimal analysis of information provided by the supervision system of a production system. Uncertain knowledge of such task asks for specific reasoning and adaptive fuzzy logic modeling and analysis fuzzy methods. The proposed method interfaces the results given by the fuzzy supervision model with an algorithm that identify the real state of the monitored system. The degradation supervision system emits fuzzy fault signal. These ones represent inputs values in an information system which identifies the degradation of the system at current moment through recurrent identification cycles. These one have also a prediction component that determines a possible evolution of the state towards the critical minimum of the system loss function.

Key words: fuzzy maintenance, prediction, optimal identification, critical operation, critical reliability.

# Introduction

The fuzzy supervision provides refined information for actions of optimal analysis of system's performance indicators. The major difficulty consists in the identification of mathematical models appropriately to these indicators which, in a fuzzy approach, will also have to integrate the fuzzy variable's values. We concentrate our study on the optimal analysis of information given by fuzzy monitoring tools.

The supervision of the discrete events systems needs taking into consideration some specific reasoning and modeling methods put on our disposal by the artificial intelligence techniques. The fuzzy logic provides the environment to exploit the fuzzy information which seen from the qualitative point of view offers more refined results than the classic logic. From this point of view in the category of modeling instruments, the Fuzzy Petri network (RdPF) is an adequate instrument for studying the discrete events systems described by such fuzzy information. A very complete study of various approaches regarding the existent Fuzzy Petri networks (RdPF) was published by J. Cardoso and B. Pradin-Chezalviel in [3]. Various types of logic (classic, linear and fuzzy) [3, 4, 9, 7] used in system's description, generate two main categories of RdPF models: the first class of models is represented by the fuzzy expert systems. Another class of applications is modeled through RdPF models which express information's vagueness [2].

Using a temporal fuzzy research, RdPFS modeling instrument allows generalization of the analysis upon the production system from the point of view of fault's tracking down and diagnosis. RdPFS models the state of error's temporal evolution expressed trough a descriptive instrument: fault tree analysis (ADD). The propagation and temporal variation of errors are evaluated through temporal sections on network's marking levels. Through his interface, the instrument provides selective fuzzy information from the point of view of the defaults gravity of

the detection. This information represents the inputs in an algorithm of cyclic identification of system's state modeled through an objective function. We shall propose two numerical methods which locate the current state through the information given by the fuzzy supervision model. The objective function can be a performance indicator or the analytical expression of system's state. We chose the functions which express the system's reliability in operation as being exclusively dependent on the error variables situated on the error critical way [10] and provided by the RdPFS temporal model.

# Analytical Methods for Identifying the Critical States in Operation of the Production Systems

Considering a n-dimensional space which describes the objective function  $f: \mathbb{R}^n \to \mathbb{R}^n$ ,  $f(x_1, x_2, .., x_n)$ , called *critical state in operation* of a dynamic system. We shall propose an algorithm who analyzes the current state of the system in order to predict a possible future state of the system that can be assimilated with a failure called *critical state of the system*. From the set  $\{a_1, a_2, .., a_{n+m}\}$  of possible errors to appear in system's operation, we consider the set  $\{a_1, a_2, .., a_n\}$  which represents the set of crucial errors in system's maintenance action. These errors are those situated on the *critical error curve* in a Petri network specialized in modeling the maintenance function of a RdPFS production system [7, 8, 9]. The model is based upon the causal relation between these events expressed by ADD, respectively the apparition of errors and their propagation.

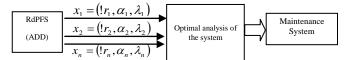


Fig. 1. Block schema for optimal analysis of manufacturing systems.

The model built with the RdPFS specialized network shows intermediary error states which can be critical as for the repetitive experiences of the human expert.

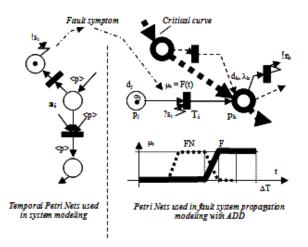
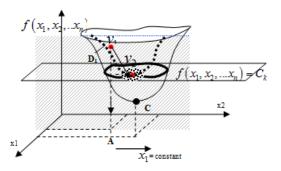


Fig. 2. Fuzzy Petri network specialised, based on ADD for manufacturing monitoring systems modelling.

We learn the fuzzy aspect of the modeling expressed through fuzzy values  $\{\alpha_1, \alpha_2, .., \alpha_n\}$  associated to the variables which define the critical errors, errors shown by the

model through  $\{!r_i\}_{i=\overline{1,n}}$  signals. The  $x_1, x_2, ..., x_n$  variables are in correspondence with the set of states that define the critical error way  $\{x_1 \equiv s_1, x_2 \equiv s_2, ..., x_n \equiv s_n\}$ . The  $\{s_1, s_2, ..., s_n\}$  states corresponding to the critical way lead the supervised system into states of critical weakness if the limit value associated to its error state is exceeded by the fuzzy value which describes the system's state in the RdPFS associated model:  $\alpha_k \ge \lambda_k$ .



**Fig. 3.** Analytical representation of loss function based on considered faults in RdPFS and ADD.

We define the *critical reliability state*, n-uplet  $\{x_1, x_2, ..., x_n, f(\bullet)\}$  corresponding to the minimum value of function f. The evolution of the system towards state  $C \in \{f(x_1, x_2, ..., x_n)\}$  imposes the movement of the point corresponding to system's current state on a level track descendant from current point **A** towards point **C**.

We define the system's state corresponding to point **C** state of critical reliability of the supervised system where  $C = \min\{f(x_1, x_2, .., x_n)\}$ . The information provided by the supervision system in the case of system's evolution towards the state of critical reliability, shows worsen states of the supervised system through the *additive apparition* of new errors. We consider the initial state  $y_1$ . In a temporal approach, it corresponds to the evolution error state generated by the apparition of a critical error  $d_1 = x_1$ , while the other errors (n-1) remain stationary. Accordingly to the optimal numeric techniques for searching a minimum value of a variable *n* function, these states can be determined through evaluation of the objective function  $f(x_1, x_2, .., x_n)$  for the condition  $(x_1, .., x_{i+1}, .., x_n) = const.$ . The problem is transforming into a classical problem of successive linear search, on direction of coordinate axes of  $\mathbb{R}^n$  systems, of relative minimum value of one variable function.

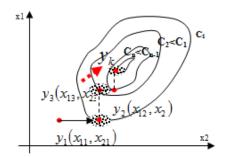


Fig. 4. Graphical representation of isolevel curves associated to critical status identified with faults monitoring system.

The correspondent state of the supervised system at the appearance of  $d_1$  error is represented by  $D_1$  curve obtained by sectioning  $f(\bullet)$  with cu plan $(x_2, x_3, ..., x_n) = const$ . If the  $d_1$  error is worse for system's state of reliability, then it will set the current state in a circled nucleus  $y_2 = \min\{f(x_1, (x_2, x_3, ..., x_n) = const)\}$ . The line  $\overline{d_{12}} = (y_1, y_2)$  is obtained by plan projection  $(x_2Ox_1)$  of  $(\overline{y_1y_2}) = \overline{Oy_2} - \overline{Oy_1}$  vector where  $y_2$  point is situated on the level curve  $f = C_1$ . The point  $y_2$  can vary on the domain  $(y_2 - \delta_2, y_2 + \delta_2)$ . The value  $\delta_i = \alpha_i$  is given by the temporal fuzzy supervision system designed to emit fuzzy signals for warning about the appearance of an error situated on the critical error way.

The appearance of the next error  $d_2$  leads the system into  $y_3$  state correspondent to  $y_3 = \min\{f(x_2, (x_1, x_3, ..., x_n) = const)\}$ . The current state of the system becomes a point from  $(y_3 - \delta_3, y_3 + \delta_3)$ . Through augmentation, the appearance of the last error  $d_k$  will lead the system into the critical error way:  $y_{k+1} = \min\{f(\bullet), (x_1, ..., x_{k-1}, x_{k+1}..., x_n) = const\}$ .

The system's current state becomes a point from the interval  $(y_k - \delta_k, y_k + \delta_k)$ where  $\delta_k$  corresponds to the fuzzy value attached to the signal announcing the appearance of  $d_k$ error. If the state of the supervised system gets worse through the appearance of all *n* errors, then it will be in a *state of reliability in degraded operation* which doesn't exactly mean that it is the critical one. The analysis instrument becomes efficient if we introduce the independent variable *t* into the set of variables of the objective function, correspondent to an orthogonal direction from space  $\mathbb{R}^n$ . The function which describes the *state of reliability in operation* gets the form of a specific model for the evolutionist systems:  $\Phi: \mathbb{R}^{n+1} \to \mathbb{R}^{n+1}, \Phi(f(x_1, x_2, ..., x_n), t)$ . It shows the dynamic aspect of system's supervision.

**Proposition 1:** if a system's error state is continuous, the system can develop towards the *state* of critical reliability in operation correspondent to the minimum value of the objective function  $\Phi(f(x_1, x_2, ..., x_n), t)$ . The presence of time variable on one of the coordinate axes shows the non stationary aspect of a system in a certain weakness state. The analysis has the aspect of some stationary cyclic evaluations on the *n* rectangular axes axe as well as a temporal one through evaluation of the objective function in each new moment  $\Phi(f(x_1, x_2, ..., x_n), t + k \cdot \Delta t)_{k \in \mathbb{Z}_k}$ .

**Proposition 1**: if  $||y_{n+1} - y_n|| < \varepsilon$  and  $\delta_m = \frac{\sum_{k=2}^{n+1} \delta_k}{n} > \left(\frac{\lambda_1 + \lambda_2 + ..\lambda_n}{n} + \varepsilon^2\right)$  the system is in a state of

critical reliability  $y_{n+1} = \min \{ \Phi(\bullet) \}$ , where

- $\Phi$  is the objective function which describes the reliability in operation of it, depending on the critical error states that the system has in time.
- (n+1) represents the last direction (last variable  $x_{n+1}$ ) in a cycle of identification/evaluation of system's state.

#### Algorithms for identifying the critical state of a production system

The method for dynamic identification of the state of reliability in operation appeals to system's state identification algorithms by searching the state in space  $R^{n+1}$  with variable steps on the directions of the coordinate axes. The dynamic aspect of the analysis is determined by the input

data, respectively the fuzzy signals  $\{!r_i\}_{i=\overline{1,n}}$  emitted by the RdPFS model. The analytically determination of critical state in operation supposes the of  $(x_{1i}, x_{2i}, ..., x_{ni}, t_i)$ n-uplet on the characteristic of function  $\Phi(f(x_1, x_2, ..., x_n), t)$ . If the series  $\{X_i\}_{i=1,n+1}, X_i = (x_{1i}, x_{2i}, ..., x_{ni}, t_i)$  leads to a convergent evolution of the function  $\Phi$  towards its minimum point, it is a minimized series for  $\Phi$  and system's state leads towards the critical state in operation of the system.

The analysis is based on monitored data at every  $\Delta t$  moment. In each  $t \rightarrow t + \Delta t$  moment the  $\{!r_i\}_{i=\overline{1,n}}$  signals are obtained. After it, there is a complete cycle for identification of the current state of the system in relation with evolution model of function  $\Phi$ . The algorithm is cyclic and indicates a movement on the characteristic  $\Phi(f(x_1, x_2, ..., x_n), t)$  by smaller and smaller steps/levels. From a moment  $t_n$  the algorithm can't distinguish anymore the difference between states. This moment is obtained in relation with the condition  $||y_{n+1} - y_n|| < \varepsilon$  and indicates a limit of the method.

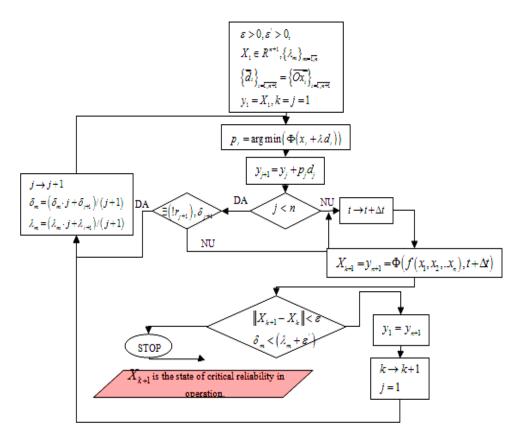


Fig. 5. Logical schema of critical states of manufacturing system identified through fuzzy output of the monitoring system

The method's limit can be verified through comparison between the theoretical conclusion of the numeric calculus algorithm which can indicate the system failure and its real state which can be sever but not yet fatal. The method's accuracy consists in having a higher number of analysis points for the supervised system. Processing the data is a time-consuming operation and the result can sometimes be wrong because of the stop condition (not so restrictive)  $||y_{n+1} - y_n|| < \varepsilon$ .

# Identifying algorithm of critical functioning state of a system using accelerated step method

The earlier method has proposed a cyclic research method on coordinated axes without guaranteeing the uniqueness of the solution. In this context, we have looking for a more complex algorithm capable to indicate with a small error, in case of  $\Phi(f(x_1, x_2, ..., x_n), t)$  convergence to the point C, the intersection point between the current point  $\Phi$  obtain with digital analysis and C point. The stop condition imposed by the previous algorithm does not reflect the real stop conditions:  $\|\nabla f(x)\| < \varepsilon$  ( $\nabla f$  represents the gradient of the *f* in the analyzed point). We propose a second algorithm, based on Hooke Jeeves algorithm [15] for numerical computation of minimal value of n variables function. This method is known as the accelerated step research method. It intercalated between two rounds of research on the coordination axes, a research on a particular direction determinate by minimal points results of the last two searches.

We apply this algorithm for the identification of optimal critical state in functioning of manufacturing system, described by the objective function  $\Phi(f(x_1, x_2, ..., x_n), t)$ . Between two identification cycles of failure system state, we propose an implementation of the identification algorithm presented in the previous paragraph but on a supplementary direction determined by the last two cyclical researches. From a practical point of view, this additional direction indicates more relevant the system degradation direction, precisely the closeness of current point C (figure 6).

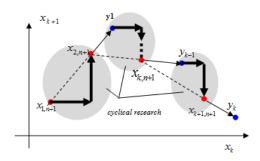


Fig. 6. Representation in state space of estimated failure state with conventional accelerated step research method

It is used the following notation  $x_{k,n+1} \in \Phi(f(x_1, x_2, .., x_n), t)$  for the identified point between the (n+1) minimum relative points (Figure 5). Between two states of the system, represented at every  $\Delta t$  moments, the modification of one or more variables - respectively the occurrence of new faults, determines on the characteristic  $\Phi(f(x_1, x_2, .., x_n), t)$  different points of identification. Every  $\{x_{k,n+1}\}_{k=\overline{1:n+1}}$  point is associated with an identification cycle on the (n+1) orthogonal axes on  $\mathbb{R}^{n+1}$ . Between two consecutive states is running an intermediary identification on the result direction:  $\overline{d_{k,k+1}} = \overline{Ox_{k+1,n+1}} - \overline{Ox_{k,n+1}}$ . The  $d_{k,k+1}$  direction represents the system evolution trend and is obtain through the superposition of general effects of intermediary states  $\{k, k+1\}$ . For  $\{y_k\}_{k=\overline{2:n+2}}$  have been considered crisp values and now have been associated with fuzzy values 1.

The stop condition of the algorithm became:  $\|x_{k,n+1} - x_{k+1,n+1}\| < \varepsilon$  and  $\sum_{k=2}^{n+1} \delta_k > \left(\frac{\lambda_1 + \lambda_2 + \dots + \lambda_n}{n} + \varepsilon^2\right)$ .

This second method is recommended; especially for rapid systems changing that evolution is fast towards failure moment. The algorithm is faster and needs a bigger number of acquisitions for the identification of a certain state.

#### Conclusions

In this paper, two classic numerical methods dedicated to indicator's modeling: a system's critical reliability in operation has been implemented. The two methods are based upon general methods of numeric search of multivariable function's extremes in test points. The presented algorithms are based upon fuzzy information provided by the model of supervision function implemented with the RdPFS dedicated tool. These ones can be applied for any other multivariable objective function for which is desired the supervision of the current state related to its extreme point which corresponds to a limit situation. The route generated by the representation of current state predicts the closeness or not of the system towards the critical state located in the extreme point. Both algorithms are based upon conventional methods usually used in multivariable minimum identification. The method based on accelerated step improves the research results because the critical fault estates are determined with a smaller step number. The difficulty of the method consists in mathematic model's identification of the objective function and fuzzy logic introduction of the variable independent of time and of fuzzy values associated with the variables of objective function.

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# Analiza optimală fuzzy a sistemelor de producție

#### Rezumat

Acest articol prezintă doi algoritmi de analiză optimală a informației furnizate de sistemul de supervizare a sistemelor de producție. Descrierea logică cu integrarea logicii fuzzy este adaptată modelării cunoștințelor care nu pot fi reprezentate prin structurile logicii convenționale. Metodele propuse interfațează informațiile furnizate de modelul de supervizare fuzzy, cu un algoritm de identificare a stării sistemului monitorizat. Sistemul de supervizare a defecțiunilor avertizează apariția defectelor. Acestea reprezintă mărimi de intrare într-un sistem informatic destinat identificării stării de degradare, bazat pe un mecanism recurent de analiză optimală. Algoritmii propuși au o componentă predictivă care poate estima evoluția posibilă a stării sistemului prin determinarea punctului de minim al funcției cost.