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Some Aspects about Modelling of Asynchronous Motors Used for Driving of Depth-pumping Installations

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Abstract

The asynchronous motors, which have the rotor in short-circuit is used ofently in electrical drives because of the construction, which is relative simply and because of its price, which is smaller than other electrical motors. Mathematical model for this kind of motor is generally presented in form of Cauchy equations "in currents" or "in magnetic fluxes". In this paper is analyzed detailing of these equations having in view the effect of current rejection and the effect of magnetic saturation (longitudinal, respectively transversal). In the same time there are analyzed which are the effects of this detailing on programming effort and on computer working time.

Key words: *electric drives, asynchronous motor , mathematical model*

Introduction

Using of asynchronous motors, which have the rotor in short-circuit for electromechanical driving of installation have some advantages like relative simple construction, they are cheaper than other types of motors, more reliable and, in addition, they can be easier realized in antiexplosive construction. In addition, the asynchronous motors with tall bars, which will be taken into consideration in this paper, have the advantage of a starting torque that is relatively high. For example, the series of asynchronous motors ASI having the rotation of synchronism by 1500rot/min in a power range between 30 … 90 kW have the starting torque at a level of 90…92,5% from maximum value of the torque. The same range of values can be observed at the rotation of synchronism by 750rot/min. This kind of motors can be used in case of "heavy starts" that need a large starting torque in conditions of a direct start of the motor.

The asynchronous motors with tall bars can be used for these driving, having in view the advantage of a high starting torque. For example, the series of asynchronous motors ASI having the rotation of synchronism by 1500rot/min in a power range between 30 … 90 kW has the starting torque at a level of 90…92,5% from maximum value of torque.

Primary Mathematical Model of Asynchronous Motor

Mathematical model of asynchronous motor, in the phasor-compact variant, is the classical one, which is presented in the coordinates system (d,q):

$$\begin{aligned} \underline{U_1} &= R_1\underline{I_1} + \frac{d\underline{\Psi_1}}{dt} + j\omega\underline{\Psi_1}; \\ 0 &= R_2\underline{I_2} + \frac{d\underline{\Psi_2}}{dt} + js\omega\underline{\Psi_2}. \end{aligned} \tag{1}$$

The symbols are the usual: indexes 1,2 are referring to the stator/rotor of the motor. It was taken into consideration the fact that at the generalized electrical machine (having number of poles, p=1) we have:

$$\Omega_k - \Omega_2 = s\Omega_1 = s\omega. \tag{2}$$

Relations between magnetic fluxes and currents, in phasor-compact form, will be the following:

$$\underline{\Psi_1} = L_1\underline{I_1} + L_m\underline{I_2} \quad ; \quad \underline{\Psi_2} = L_2\underline{I_2} + L_m\underline{I_1} \ , \tag{3}$$

In these relations the notation of inductivities is the usual one.

Equations system (1) must be detailed in the form of Cauchy equations. Having in view relations (3), this detailing can be realized in form of equations “in currents” or “in fluxes”. Choosing the variant depends, in generally, by preference of the user but it can depend by some further developing of the equations, which can become convenient in a specific variant. For example, the Cauchy equations system for asynchronous motor, in terms of “currents” can be noted in the following form:

$$\begin{aligned} \frac{di_{1d}}{dt} &= a1*U_d - b1*i_{1d} + (c1 - d1*s)*i_{1q} + e1*i_{2d} + (1-s)*f1*i_{2q}; \\ \frac{di_{1q}}{dt} &= a1*U_q + (d1*s - c1)*i_{1d} - b1*i_{1q} - (1-s)*f1*i_{2d} + e1*i_{2q}; \\ \frac{di_{2d}}{dt} &= -a2*U_d + b2*i_{1d} - (1-s)*c2*i_{1q} - d2*i_{2d} + (e2*s - f2)*i_{2q}; \\ \frac{di_{2q}}{dt} &= -a2*U_q + (1-s)*c2*i_{1d} + b2*i_{1q} + (f2 - e2*s)*i_{2d} - d2*i_{2q}. \end{aligned} \tag{4}$$

The components of statorical/rotorical currents and the tension U are given by axis (d,q); the coefficients a1,...,f2 are functions by motor parameters, having the expressions:

$$\begin{aligned} &a1 = ax*X_2 \ ; \ a2 = ax*X_m \ ; \ b1 = a1*R_1 \ ; \ b2 = a2*R_1 \ ; \ c1 = a1*X_1 \ ; \ c2 = a2*X_1 \ ; \\ &d1 = a2*X_m \ ; \ d2 = e1*X_1/X_m \ ; \ e1 = a2*R_2 \ ; \ e2 = a1*X_1 \ ; \ f1 = a1*X_m \ ; \\ &f2 = a2*X_m, \text{ respective } ax = 2\pi f_1/(X_1*X_2 - X_m^2) \, . \end{aligned} \tag{5}$$

Equations system (4) has 5 unknowns: $i_{1d}, i_{1q}, i_{2d}, i_{2q}, s$. The fifth unknown, which is referring to the motor slip (s), assumes establishing of the fifth equation usually based on torques equation from driving system.

Detailing of Cauchy Equations

In principal, detailing of Cauchy equations can refer to the following aspects:
- Rejection of the current, especially because is about asynchronous motors with tall bars;
- Effect of longitudinal magnetic saturation, which is important in case of drives in that rotation is modified cyclically in large limits;
- Effect of transversal saturation, which is important when the load is modified cyclically in large limits;

The effect of current rejection affects parameters R_2, X_2 in their components from the cage, which are disposed in rotorical core. This can be noted in the following form:

$$R_2 = R_{rv} * k_r + R_{rc}, \text{ respective } X_2 = X_{rv} * k_x + X_{rc}, \qquad (6)$$

in which the corrective coefficients are given by the following relations:

$$k_r = \varepsilon \frac{sh(2\varepsilon) + \sin(2\varepsilon)}{ch(2\varepsilon) - \cos(2\varepsilon)}, \text{ respective } k_x = \frac{1.5}{\varepsilon} \frac{sh(2\varepsilon) - \sin(2\varepsilon)}{ch(2\varepsilon) - \cos(2\varepsilon)}, \qquad (7)$$

where $\varepsilon = h^{'} \sqrt{s}$ is a non-dimensional measure, named reduced high of cage bar. The measures R_{rv}, X_{rv}, are the components of the total value of resistance and rotorical reactance, which are affected by effects of current rejection. These components are calculated or are given by motor catalog.
The effect of longitudinal magnetic saturation affects magnetizing reactance X_m. This can be underline by relation (A. Bitoleanu):

$$\varphi_m = \Updownarrow \begin{cases} \Rightarrow k_s i_m, for: I_m \le 0.27 * I_{mn} \\ \Rightarrow \dfrac{a}{i_m}\left(1 + be^{-\alpha i_m}\right) - c i_m^{-\beta}, for: I_m > 0.27 * I_{mn} \end{cases} \qquad (8)$$

in which it was be noted : $\varphi_m = \Phi_m / \Phi_{mn}; i_m = I_m / I_{mn}; I_m = \left(\left(i_{1d} + i_{1q}\right)^2 + \left(i_{2d} + i_{2q}\right)^2\right)^{1/2}$,
respectively : $k = 1.28$; a = -1.185 ; b = -0.97 ; c = -2.108 ; $\alpha = 2.71; \beta = 0.185$. (9)

Relation of magnetizing flux from (8), superior variant, can be processed in the following manner:

$$\varphi_m = \frac{\Phi_m}{\Phi_{mn}} = k_s \frac{I_m}{I_{mn}} \times \frac{\Phi_{mn}}{I_m} \Rightarrow \frac{\Phi_m}{I_m} = k_s \frac{\Phi_{mn}}{I_{mn}} \Rightarrow L_m = k_s L_{mn} \Rightarrow X_m = k_s X_{mn}, pt.I_m \le 0.27 I_{mn}$$

Similarily it can be processed the „down" variant from (8) for φ_m, so that in the end it can be noted:

$$X_m = f_i X_{mn}, \text{ in which } f_i = \Updownarrow \begin{cases} \Rightarrow k_s, for: I_m \le 0.27 * I_{mn}; \\ \Rightarrow \dfrac{a}{i_m^2}\left(1 + be^{-\alpha i_m}\right) - c i_m^{-(1+\beta)}, for: I_m > 0.27 * I_{mn}. \end{cases} \qquad (10)$$

It is obviously that in this case is more convenient the expression „in current" for equations system from (4) but, in the processing of equations, which take into consideration the effect of longitudinal magnetic saturation, appears „the group" $d\left(X_m \underline{I_m}\right)/dt$ that must be treated so:

$$\frac{d\left(X_m \underline{I_m}\right)}{dt} = \frac{d\left(X_m \underline{I_m}\right)}{d\underline{I_m}} * \frac{d\underline{I_m}}{dt} = \frac{d\left(X_m I_m\right)}{dI_m} * \frac{d\underline{I_m}}{dt}, \qquad (11)$$

having in view that X_m is a real parameter and the phasors $\left(X_m \underline{I_m}\right)$, $\underline{I_m}$ are co-linear.

Regarding transversal magnetic saturation, it affects the statoric and rotoric reactance X_1, X_2. In this way rotoric reactance is affected by current rejection, respectively by transversal magnetic saturation. From point of view of rotoric reactance it can appear two problems:
- the way in which it can be taken into consideration the effect of transversal magnetic saturation (aspect that is hold good in case of statoric reactance too)
- if the two effects are cumulative or multiplicatif.

At a nominal current it have not transversal magnetic saturation or, more exactly, there is a saturation at which are calculated in fact, the reactance using their values from machine catalog, which have not be « adjusted ». At higher currents (especially in case of direct starts of asynchronous motors), the transversal magnetic saturation is taken into consideration by a proper increasing of the neck of statoric/rotoric notch.

For example, if the specific geometrical permeance for a type of statoric notch can be written by relation :

$$\lambda_{c1} = \frac{h_1 - h_4}{3b_c} k_\beta + \left[\frac{h_2}{b_2} + \frac{2h_3}{b_c + 2a_s} + \frac{h_0}{a_s} \right] k_\beta^{'} + \frac{h_4}{4b_c} , \tag{12}$$

in which we have some notch parameters and a_s is dimension of notch isthmus. This parameter (at the rotor with tall bar, usually, this isthmus is symbolized by b_0), at saturation must be written in the following form :

$$a_s^{'} = a_s + \frac{b_{d0}}{\mu^{'}} , \tag{13}$$

In this relation b_{d0} is the width of statoric (rotoric) tooth and $\mu^{'}$ is the relative magnetic permeability defined by relation:

$$\mu^{'} = \frac{B_{dk}}{\mu_0 (H_{dk} + \frac{I}{I_n} A_n)}, \tag{14}$$

where :

--- $A_n = A$, is current blanket used to design the car for I_n ;

--- I / I_n, is relative value of the load current;

--- B_{dk}, H_{dk} , represents the critical value of magnetic induction, in [T], respectively critical value of intensity of the magnetic field, in [A/m] ; for electrotechnic table used in electrical machines construction we have: $B_{dk} = 2{,}04$ [T] and $H_{dk} = 300.10^2$ [A/m], that is constant values. If we take into consideration the relations (13) and (14), it can be written

$$a_s^{'} = a_s + \frac{b_{d0}\mu_0}{B_{dk}} (H_{dk} + \frac{I}{I_n} A_n) , \tag{15}$$

that is, at saturation $a_s^{'}$ is a lineary function by the load current.

The relation (12) for λ_{c1}, at saturation, must be written as following:

$$\lambda_{c1s} = \frac{h_1 - h_4}{3b_c} k_\beta + \left[\frac{h_2}{b_2} + \frac{2h_3}{b_c + 2a_s^{'}} + \frac{h_0}{a_s^{'}} \right] k_\beta^{'} + \frac{h_4}{4b_c} , \tag{16}$$

It means that the terms 2 and 3 from paranthesis of relation (16) will be modified in inverse proportion with the relative value of the load current : at the start, the two terms will have minimal values, respectively λ_{c1s} (symbol « s », means « saturated ») will have minimal value, and in case of $I = I_n, \lambda_{c1s}$ must be considered equal with λ_{c1}.

The designer of asynchronous machine calculates this size (like similary others) only at the start of the motor, which means for $I / I_n = \lambda_I$ because he is interested only in determination as exactly is possible of starting parameters of asynchronous motor. He is not preoccupied by cases in which $I_n < I < I_p$ but extension of this idea is obviously possible because for some machines λ_I can have different values 5,5,…,7).

From point of view of rotoric dispersion, taken into consideration the most used rotoric notches with tall bars, specific geometrical permeances can be noted in the following form :

$$\sum \lambda_2 = \lambda_{c2}^{'} + \lambda_{c2}^{''} + \lambda_{d2} + \lambda_{f2} \ , \tag{17}$$

in which we have:

--- $\lambda_{c2}^{'}$, is notch component of λ_{c2} which is affected by current rejection;

--- $\lambda_{c2}^{''}$, is notch component of λ_{c2} which is affected by magnetic saturation;

--- λ_{d2}, the differential component, is totally affected by magnetic saturation but non-affected by current rejection ;

--- λ_{f2}, the frontal component, which is not modified by the two effects.

So, in principle, it can be written

$$X_{\sigma 2}^{'} = X_{\sigma 2r}^{'} + X_{\sigma 2s}^{'} + X_{\sigma 2n}^{'} , \tag{18}$$

in which it were made the following notations:

--- $X_{\sigma 2r}^{'}$, is component of $X_{\sigma 2}^{'}$ which is affected by current rejection (The symbol «‘» represents reporting of the dimension to the stator);

--- $X_{\sigma 2s}^{'}$, is component of $X_{\sigma 2}^{'}$ which is affected by magnetic saturation;

--- $X_{\sigma 2n}^{'}$, is component of $X_{\sigma 2}^{'}$ which is not modified by the two effects.

At the reactance $X_{\sigma 2}^{'}$ appears a complication in connection with $X_{\sigma 1}$: it is affected simultaneously by the both effects, because both effects are producing at the same sliding of asynchronous machine $s_n < s \leq 1$, but they are acting to distinct components of the reactance so, they can be considered simultaneously as cumulative effects.

But the components of reactance, anterior presented, are not usually given in the motors catalogues, even that the motors designer works with all these components. He is not interested by their underlying because he is preoccupied only by obtaining of starting parameters that are convenient for the motor. He have not the perspective of a driving user that must choose a driving motor and have to test its performances. In this situation it can be used only one solution: realizing of a powerful design instrument, using automated calculus tools for obtaining of a set of motors, convenient choused, helpful in testing of performances of a motor in a given driving.

Some Principle Aspects

Extension of detailing of Cauchy equations for an asynchronous driving motor must have allways a justification, because the form of equations becomes more complicated, programming effort grows significantly and computer-time follows an ascending curve. This aspect is important especially in case of some drives, when are necessary partial derivates of Cauchy equations, which have not at all a simple form.

In this idea it must appreciate the results obtained for some particular quantities of a given drive (forces, torques, angular dispalcements, etc.) at a specific „level” of detailing of Cauchy equations and must be analyzed the necessity of detailing on a superior level. It is possible that in detailing on a superior level, the obtained results be meaningless because of increasing of number of calculus, which make errors propagation be significant.

It can be noted that at some complex drives, using of equations without consideration the current rejection can lead to inadequate results (unstable functioning of the system).

Apparently there is not exactly criteria for appreciation of these aspects, which be used in a comfortable manner, especially because of a large diversification of mechanical part of drives, which leave their mark on the fifth equation of Cauchy system.

Conclusions

Asynchronous motor, especially that with tall bars, can be convenient used in the considered driving system. Mathematical model for this kind of motor, is presented, generally, in form of Cauchy equations "in currents" or "in magnetic fluxes". Detailing of these equations can be done having in view the effect of rejection of current and that of magnetic saturation (longitudinal, respectively transversal). Detailing of equations must be very well justified because this can lead at an important extension of equations form and of partial derivates (sometimes necessary). This fact involves an important programming effort and an increased computer time too. In case of considered driving system, the first step for detailing of Cauchy equations is referring at consideration of effect of rejection current.

References

1. Potlog, D.M., Mihăileanu, C.- *Acționări electrice industriale cu motoare asincrone,* Ed. Tehnică, Bucureşti, 1989.
2. Cioc, I., Nica, C.- *Proiectarea maşinilor electric*, Ed. Didactică şi Pedagogică, Bucureşti, 1994
3. Siro, B. - *La prise en consideration de l'effet de la saturation magnetic pour une machine asynchronous qui travaille en régime de moteur et de générateur*, SIELMEN, vol.II, Chişinău, september, 2003.
4. Siro, B. - *Une possibilité de verifier la choix correcte d'un moteur asynchrone por L'entraînement d'une installation de pompage de type canadien*, SIELMEN, vol. I, Chişinău, september, 2003.
5. Siro, B. - *Sur l'utilisation de la réactance de magnétisation en grandeurs variables dans les équation de la machine asynchrone*, Buletinul Institutului Politehnic din Iaşi, Tomul L(LIV), Fasc.5C, 2004.

Asupra unor aspecte privind modelarea motoarelor asincrone utilizate în acționarea ințalatiilor în pompaj de adâncime

Rezumat

Motorul asincron cu rotorul în scurtcircuit este des folosit în acționarile electrice datorită construcției relativ simple şi al prețului scăzut în raport cu celelalte motore electrice. Modelul matematic pentru un astfel de motor se prezintă, în general, sub forma ecuațiilor Cauchy „în curenți" sau „în fluxuri magnetice". În lucrare se analizeză detalierea acestor ecuații ținând cont de efectul refulării curentului şi cel al saturației magnetice (longitudinale, respectiv transversale) şi care sunt efectele acestei detalieri asupra efortului de programare şi al duratei de lucru al calculatorului.