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Modelling Dynamic Systems Operating with Almost Periodic Functions

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Abstract

A method of boring research dynamics is almost periodic functions using classical sense and almost periodic probability to study the dynamics of petroleum facilities, taking into account a large number as random factors.

Key words: *dynamics, random phenomena, model, petroleum facilities*

Introduction

Dynamics of machinery and plant oil-fields equipment operations for extraction requires substantiation equivalent mathematical models, generally comprising a number of discrete masses (concentrated), joined by elastic links or elements with distributed parameters.

Given the complexity are a number of simplifying assumptions, considering that the masses are concentrated rigid bodies, elastic connecting elements have mass, and the influences of nature are not considered random.

Critical to solving the corresponding dynamic problem, its systems work is equivalent to building mathematical models and simplifying assumptions election.

Problem Formulation

Be the equation of motion written in handling drum shaft in the general case:

$$J_r \cdot \frac{\mathrm{d}\omega}{\mathrm{d}t} = M_m - M_r, \tag{1}$$

where:
J_r is the reduced mass moment of inertia;
ω - angular velocity roads pump;
M_m - engine torque, $M_m = M_m(\omega)$;
M_r - reduced when handling drum resistant tree, $M_r = M_r(v) = M_r(k\omega)$.

Relation (1) is the classic expression of equivalent mathematical model of a working system, in this case: the operating system. Taking into account expressions (4.1.6.6 and 4.1.6.56, [1]), epression engine when taking into account random phenomena, is:

$$M_m \equiv M_m(\omega, a_k, t) = M_m(\omega) - \sum_{k=1}^{m}\sum_{i=1}^{n} p_{ik}(a_k)\cdot q_{ik}(t); \tag{2}$$

and of the resistance, taking into account the relations (4.1.6.7 and 4.1.6.55, [1]), is:

$$M_r \equiv M_r(v, b_h, t) = M_r(\omega) + \sum_{h=1}^{l}\sum_{j=1}^{q} r_{jh}(b_h)\cdot s_{jh}(t), \tag{3}$$

where:
$M_r(\omega)$ are reduced moments; $Rjh(bh)$ resistant forces
rjh (bh) - random reduced to the actuator of the operating system

$$J_r \cdot \frac{d\omega}{dt} = M_m(\omega) - \sum_{k=1}^{m}\sum_{i=1}^{\infty} p_{ik}(a_k)\cdot q_{ik}(t) - M_r(\omega) - \sum_{h=1}^{l}\sum_{j=1}^{\infty} r_{jh}(b_h)\cdot s_{jh}(t); \tag{4}$$

Equivalent mathematical model representing the operating system generally unconventional for the dynamic study, where $M_m(\omega)$, M_r (ω) is deterministic components, and:

$$\sum_k\sum_i M_{ik};\ \sum_h\sum_j F_{jh} \tag{4'}$$

is random perturbations model

M_{ik} and F_{jh} as and (4), obtained by generations and reduce random moments drum handling resistant tree. Solving the mathematical model (4) is not accessible at this point. From studies, if functions F_{jh}, M_{ik} and certain conditions, solving the model becomes available, can be obtained the general form of the model solution.

This phase starts to form premises simulation steps using digital programs of specific applications in mechanical drives, the electromechanical field in general and oil in particular.

Definitions:

$$P_\varepsilon(t) = \sum_{i=1}^{n} c_{ik}\varphi\cdot_{ik}(t) = \sum_{i=1}^{n} c_{ik}\cdot e^{(\sqrt{-1}\cdot\lambda_{ik}\cdot t)}, \qquad i = \overline{1,n}; \tag{5}$$

$$P_{\varepsilon'}(t) = \sum_{j=1}^{q} c'_{jh}\,\varphi'_{jh}(t) = \sum_{j=1}^{q} c'_{jh}\cdot e^{(\sqrt{-1}\cdot\lambda'_{jh}\cdot t)}, \qquad j = \overline{1,q}; \tag{6}$$

M_{ik} and F_{jh} say are random functions almost periodic in the classical sense ***FAPC***. These families of functions, the customizations on the actual situation will be function that will create the mathematical model study.

The relationship (4) becomes:

$$J_r \cdot \frac{d\omega}{dt} = M_m(\omega) - \sum_{k=1}^{m}\sum_{i=1}^{n} c_{ik}\cdot\varphi_{ik}(t) - M_r(\omega) - \sum_{h=1}^{l}\sum_{j=1}^{q} c'_{jh}\cdot\ \varphi'_{jh}(t). \tag{7}$$

If conditions (5) and (6) are not met by approximating the trigonometric polynomials (7) is recommended to use random trigonometric polynomials, of the form:

$$P_{\varepsilon}(a_k,t)=\sum_{i=1}^{n}c_{ik}(a_k)\cdot\ \varphi_{ik}(t)=\sum_{i=1}^{n}c_{ik}(a_k)\cdot e^{(\sqrt{-1}\cdot\lambda_{ik}\cdot t)} \tag{8}$$

$$P_{\varepsilon'}(b_h,t)=\sum_{j=1}^{q}c'_{jh}(b_h)\cdot\ \varphi'_{jh}(t)=\sum_{j=1}^{q}c'_{jh}(b_h)\cdot e^{(\sqrt{-1}\cdot\lambda'_{jh}\cdot t)} \tag{9}$$

If polynomials (8) and (9) satisfy the conditions (5) and (6), we say that functions are functions F_{jh}, M_{ik} and almost periodic random probability ***FAPP***.

$$J_r\cdot\frac{d\omega}{dt}=M_m(\omega)-\sum_{k=1}^{m}\sum_{i=1}^{n}c_{ik}(a_k)\cdot\ \varphi_{ik}(t)-M_r(\omega)-\sum_{h=1}^{l}\sum_{j=1}^{q}c'_{jh}(b_h)\cdot\ \varphi'_{jh}(t). \tag{10}$$

Consider a linear variation in the difference deterministic components:

$$M_m(\omega)-M_r(\omega)=\mu\cdot\omega, \tag{11}$$

If $M_m(\omega)$ = A – B·ω, and $M_r(\omega)$ = A'- B'·ω, according to (15) and substituting (11) in (9) or (10), justified by the analysis of possible cases where the DC electric drive we obtain:

$$J_r\cdot\frac{d\omega}{dt}=\mu\cdot\omega-\sum_{k=1}^{m}\sum_{i=1}^{n}c_{ik}\cdot\varphi_{ik}(t)-\sum_{h=1}^{l}\sum_{j=1}^{q}c'_{jh}\cdot\ \varphi'_{jh}(t) \tag{12}$$

case of almost periodicity in the classical sense of random phenomena, namely

$$J_r\cdot\frac{d\omega}{dt}=\mu\cdot\omega-\sum_{k=1}^{m}\sum_{i=1}^{n}c_{ik}(a_k)\cdot\varphi_{ik}(t)-\sum_{h=1}^{l}\sum_{j=1}^{q}c'_{jh}(b_h)\cdot\varphi'_{jh}(t) \tag{13}$$

case of almost periodicity in the probability of phenomena. Relations (4) and (13) the general form of generalized mathematical model for system dynamics study of flexibility to both forms of almost periodicity of random phenomena. Mathematical model solution (12), is:

$$\omega(t)=-\frac{1}{J_r}\cdot\sum_{k=1}^{m}\sum_{i=1}^{n}\left[\int_t^{\infty}e^{\frac{\mu(t-u)}{J_r}}\cdot c_{ik}\cdot\varphi_{ik}(u)du\right]-\frac{1}{J_r}\cdot\sum_{h=1}^{l}\sum_{j=1}^{q}\left[\int_t^{\infty}e^{\frac{\mu(t-u)}{J_r}}\cdot c'_{jh}\cdot\varphi'_{jh}(u)du\right], \tag{14}$$

For μ > 0 (14) and μ < 0 in (15)

$$\omega(t)=\frac{1}{J_r}\cdot\sum_{k=1}^{m}\sum_{i=1}^{n}\left[\int_{-\infty}^{t}e^{\frac{\mu(t-u)}{J_r}}\cdot c_{ik}\cdot\varphi_{ik}(u)du\right]+\frac{1}{J_r}\cdot\sum_{h=1}^{l}\sum_{j=1}^{q}\left[\int_{-\infty}^{t}e^{\frac{\mu(t-u)}{J_r}}\cdot c'_{jh}\cdot\ \varphi'_{jh}(u)du\right], \tag{15}$$

Model solution (15) is similar to relations (12), (13)

$$\omega(a_k, b_h, t) =$$

$$= -\frac{1}{J_r} \cdot \sum_{k=1}^{m} \sum_{i=1}^{n} \left[\int_t^{\infty} e^{\frac{\mu(t-u)}{J_r}} \cdot c_{ik}(a_k) \cdot \varphi_{ik}(u) \mathrm{d}u \right] - \frac{1}{J_r} \cdot \sum_{h=1}^{l} \sum_{j=1}^{q} \left[\int_t^{\infty} e^{\frac{\mu(t-u)}{J_r}} \cdot c'_{jh}(b_h) \cdot \varphi'_{jh}(u) \mathrm{d}u \right],$$

for $\mu > 0$, (16)

and:

$$\omega(a_k, b_h, t) =$$

$$= \frac{1}{J_r} \cdot \sum_{k=1}^{m} \sum_{i=1}^{n} \left[\int_{-\infty}^{t} e^{\frac{\mu(t-u)}{J_r}} \cdot c_{ik}(a_k) \cdot \varphi_{ik}(u) \mathrm{d}u \right] + \frac{1}{J_r} \cdot \sum_{h=1}^{l} \sum_{j=1}^{q} \left[\int_{-\infty}^{t} e^{\frac{\mu(t-u)}{J_r}} \cdot c'_{jh}(b_h) \cdot \varphi'_{jh}(u) \mathrm{d}u \right]$$

for $\mu < 0$. (17)

Analysis of Possible Cases

If $\mu > 0$, k = 1 and $h = 1$, which means that the influence is considered a single random variable from engine to the actuator.

a1) Where almost periodicity in the classical sense.

$$\omega(t) = \frac{e^{-\mu \cdot t}}{J_r} \cdot \sum_{i=1}^{n} \left[c_{i1} \cdot \frac{1}{\left(\frac{\mu}{J_r}\right)^2 + (i\omega_0)^2} \cdot \left(-\frac{\mu}{J_r} \cdot \sin(i\omega_0 t) - i\omega_0 \cdot \cos(i\omega_0 t) \right) \right] +$$

$$+ \frac{e^{-\mu \cdot t}}{J_r} \cdot \sum_{j=1}^{q} \left\{ c'_{j1} \cdot \left\{ \frac{\sin(j\omega_0 t)}{\left(\frac{\mu}{J_r}\right)^2 + (j\omega_0)^2} \cdot \left[j\omega_0 \cdot \sin(\phi_0) - \frac{\mu}{J_r} \cdot \cos(\varphi_0) \right] - \right.\right.$$

$$\left.\left. - \frac{\cos(j\omega_0 t)}{\left(\frac{\mu}{J_r}\right)^2 + (j\omega_0)^2} \cdot \left[j\omega_0 \cdot \cos(\phi_0) - \frac{\mu}{J_r} \cdot \sin(\varphi_0) \right] \right\} \right\} + \omega_0;$$

(18,a)

a2) If the probability of almost periodicity

$$\omega(t)=\frac{e^{-\mu\cdot t}}{J_r}\cdot\sum_{i=1}^{n}\left[c_{i1}(a_1)\cdot\frac{1}{\left(\frac{\mu}{J_r}\right)^2+(i\omega_0)^2}\cdot\left(-\frac{\mu}{J_r}\cdot\sin(i\omega_0 t)-i\omega_0\cdot\cos(i\omega_0 t)\right)\right]+$$

$$+\frac{e^{-\mu\cdot t_1}}{J_r}\cdot\sum_{j=1}^{q}\left\{c'_{j1}(b_1)\cdot\left\{\frac{\sin(j\omega_0 t)}{\left(\frac{\mu}{J_r}\right)^2+(j\omega_0)^2}\cdot\left[j\omega_0\cdot\sin(\phi_0)-\frac{\mu}{J_r}\cdot\cos(\varphi_0)\right]-\right.\right.$$

$$\left.\left.-\frac{\cos(j\omega_0 t)}{\left(\frac{\mu}{J_r}\right)^2+(j\omega_0)^2}\cdot\left[j\omega_0\cdot\cos(\phi_0)-\frac{\mu}{J_r}\cdot\sin(\varphi_0)\right]\right\}\right\}+\omega_0; \tag{18,b}$$

b) Where A and B according to and considerations point to remain valid. The general solution of the model (14) becomes:

$$\omega(t)=\frac{2A}{B}-\frac{\Omega(t)}{B}-\left(\frac{A}{B}-\omega_0\right)\cdot e^{-\frac{B}{J_r}\cdot t} \tag{19}$$

where:

$$\Omega(t)=-\frac{B}{J_r}\cdot\sum_{i=1}^{n}\left[c_{i1}\cdot\frac{e^{-\mu\cdot t}}{\left(-\frac{B}{J_r}\right)^2+(i\omega_0)^2}\cdot\left(\frac{B}{J_r}\cdot\sin(i\omega_0 t)-i\omega_0\cdot\cos(i\omega_0 t)\right)\right]-$$

$$-\frac{B\cdot e^{-\mu\cdot t}}{J_r}\cdot\sum_{j=1}^{q}\left\{c'_{j1}\cdot\left\{\frac{\sin(j\omega_0 t)}{\left(-\frac{B}{J_r}\right)^2+(j\omega_0)^2}\cdot\left[j\omega_0\cdot\sin(\varphi_0)+\frac{B}{J_r}\cdot\cos(\varphi_0)\right]-\right.\right.$$

$$\left.\left.-\frac{\cos(j\omega_0 t)}{\left(-\frac{B}{J_r}\right)^2+(j\omega_0)^2}\cdot\left[j\omega_0\cdot\cos(\varphi_0)+\frac{B}{J_r}\cdot\sin(\varphi_0)\right]\right\}\right\}+\omega_0.$$

Figures at the end paper are simulated using computer applications.

c) Cases studied previously, are cases of work-specific systems has consistently yield.
Using relations, the functionality of the system actuator considering

$$M_m(\omega) = A - B \cdot \omega \quad (20)$$

is given by expression (11) becomes:

$$M_m(\omega) - M_r(\omega) = (A - B \cdot \omega) \cdot \left(1 - \frac{k_F}{k \cdot x} \cdot \eta(x) \cdot \eta\right); \quad (21)$$

valid for systems with variable yield, where k_F is the conversion factor resistance

$$M_r = k_F M \quad (22)$$

It proposes the following notations:

$$A(x) = A \cdot \left(1 - \frac{k_F}{k \cdot x} \cdot \eta(x) \cdot \eta\right) \qquad B(x) = B \cdot \left(1 - \frac{k_F}{k \cdot x} \cdot \eta(x) \cdot \eta\right) \quad (23)$$

The general solution of the model (19), in this case, becomes:

$$\omega(t) = \frac{2A(x)}{B(x)} - \frac{\Omega_x(t)}{B(x)} - \left(\frac{A(x)}{B(x)} - \omega_0\right) \cdot e^{-\frac{B(x)}{J_r} \cdot t} \quad (24)$$

where :

$$\Omega_x(t) = -\frac{B(x)}{J_r} \cdot \sum_{i=1}^{n} \left\{ c_{i1} \cdot \int_t^{\infty} e^{-\frac{B(x)}{J_r} \cdot (t-u)} \cdot \sin(i\omega_0 u) du \right\} - $$
$$- \frac{B(x)}{J_r} \cdot \sum_{j=1}^{q} \left\{ c'_{j1} \cdot \int_t^{\infty} e^{-\frac{B(x)}{J_r} \cdot (t-u)} \cdot \sin(i\omega_0 u + \varphi_0) du \right\}; \quad (25)$$

case of almost periodicity in the classical sense.

Case of almost periodicity in probability under the conditions agreed, the general solution is (24) or (25), but coefficients c_{i1} and c'_{j1} will take values $c_{i1}(a_1)$ and $c'_{j1}(b_1)$.

Simulations for Installation of Pumps with Progressive Cavity Pump

Plant fluids from wells with pumping progressive cavity pumps, submersible Figure 1 has the following main components. Operating system (SM — ST — SCA) in the rotating pump impeller and support burden from the probe, surface mounted Figure 1. Pipes (P) that are screwed to the stator to be placed in column operation.

Pumping rods (PR)which transmit the rotation of the rotor drive system with progressive cavity pump, pipes are placed in the probe, with the rotor screw to the bottom of the ram pump road.

Progressive cavity pump (PCP), submersible known as other names that screw pump, eccentric screw pump, rotary pump thaw, or thaw pump rotating twister. The following figures are numerical applications of these results.

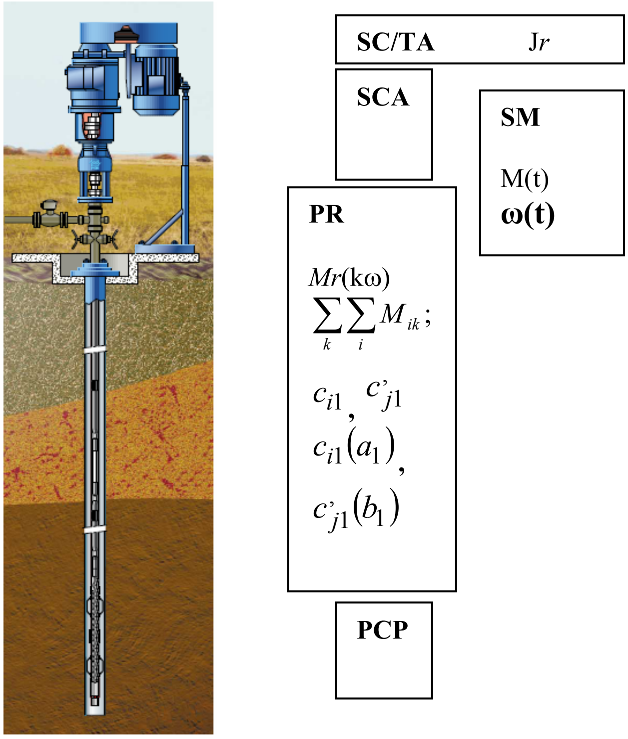

Fig. 1. Installation of pumps with progressive cavity pump.

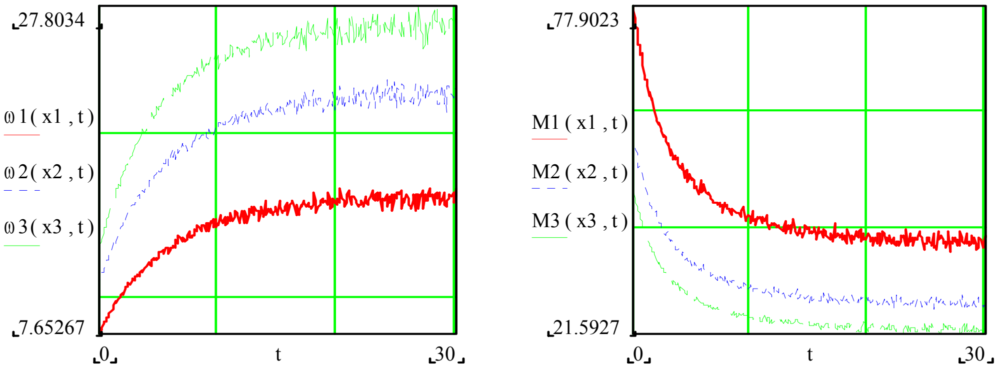

Fig. 2. Variation of angular velocity and momentum with variable yield case of almost periodicity in the classical sense

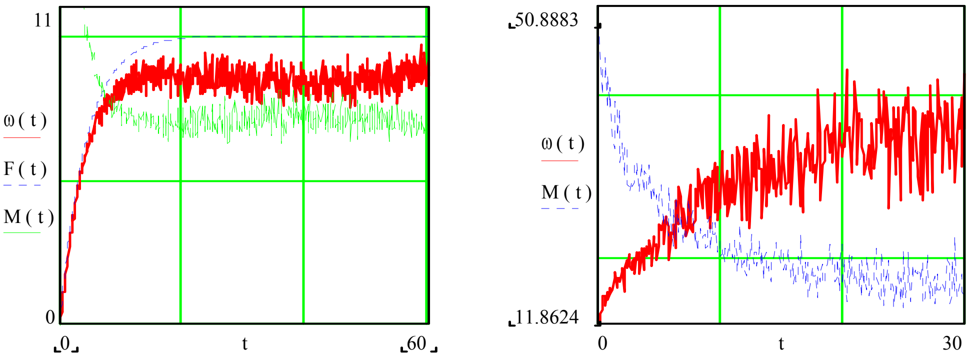

Fig. 3. Variation of angular speed and engine torque case of almost periodicity in the probability

Conclusions

Modern pumping is equipped with a modern system of engagement of the upper part of the head drive system which is used to carry out special functions for screw pump. A great importance in the dynamic study for flow oil in technological pumping system operation is given to the way in which the system structure is set. A first concern in this regard was to set the structure of the working system in order to design the dynamic simulation as a structural system.

By comparison with mathematical models established by already existing probability functions method proposed by various researchers, some original contributions have been made by including the mathematical equations of the structural model, the effect given by the resistance of the roads screw pump in rotation and pumping phenomena by hydrodynamic flow for phase solid – liquid – gas, pressure occurrence during bit pumping operation.

It has also been considered that energy losses of the road string result from viscous and dry amortization. More, a new element has been discovered. The load peak results from the system inertia, overcoming resistance and hydrodynamic pressure due to action effect.

Produced mechanical waves can become dangerous for both the drill string (area of threaded joints) and the surface guidance structure on which the waves have effect. Identifying these types of variations of the variables of force and speed respectively makes possible the study of the induced effects in the structure of top drive rolling.

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Modelarea dinamica a sistemelor de acționare cu ajutorul funcțiilor aproape periodice

Rezumat

In cadrul acestui articol este prezentat modelul matematic al ansamblului sistem compus din echipamentul de antrenare de la partea superioară a garniturii de pomare şi a ansamblul garnitură de pompare. Principalele elemente de noutate aduse modelului constau in considerarea perturbatiilor aproape periodice in sens clasic si in probabilitate, ambele situatii definite in lucrare, aparute in timpul funcționării in sonda. Rezultatele se materializează prin elaborarea unui program de calcul computerizat a carui rezultate grafice sunt prezentate in urma simularii funcționarii sistemului la o anumita adâncime.