BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LXI No. 3/2009	167 - 172	Seria Tehnică

MP and LQ Control with Compensation of Dead Zone for a Real Time Experiment

Andreea Șoimu, Florin Stîngă, Dan Popescu

University of Craiova, 13 A.I. Cuza, 200585 Craiova, ROMÂNIA
e-mail: {andreea, florin, dpopescu}@automation.ucv.ro

Abstract

In this paper, a laboratory experiment flexible link has been used to investigate some aspects about the tracking performance in model predictive (MP) and linear quadratic (LQ) control. In LQ control an integral action is introduced in order to eliminate (to minimize) stationary error. Due to neglected nonlinearities and the dead zone of the real plant we need to add a variable structure part to our control scheme, in order to reduce the vibrations of the tip of the flexible link. Experimental results are included in order to illustrate the performances of the proposed control schemes.

Key words: *Quanser Flexible Link, MPC, LQR, DLQR, Optimal Control.*

Introduction

The Flexible Link module consists of a Quanser DC servomotor and a Flexgage module. We developed two control schemes, one with a model predictive controller and other with LQ based controller (LQR and DLQR). A Quadratic Regulator yield a state feedback that can achieves the stabilization as well as minimizing a performance index. A predictive control algorithm solves an on-line and optimal control problem subject to system dynamics and variable constraints.

Mathematical model of the system

The SRV02 rotary plant module serves as the base component for the Quanser rotary family of experiments.

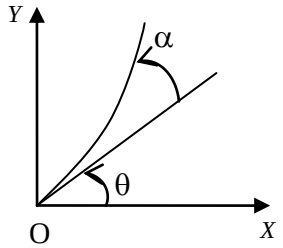


Fig. 1. Schematic representation of the Flexible Link.

We consider for the combined servomotor and the flexible link module the state variables $x(t) = [\theta \quad \alpha \quad \omega \quad \upsilon]^T$, where $\omega = \dot{\theta}$, $\upsilon = \dot{\alpha}$; θ is the motor shaft angle; α is the tip deflection angle; ω is the motor angle velocity; υ is the deflection angle velocity.

If J_{arm} is the link's moment of inertia, the torque due to the link acceleration is:

$$T_{J_{arm}} = J_{arm}\frac{d^2\gamma}{dt^2} = J_{arm}(\ddot{\theta}+\ddot{\alpha}) = J_{arm}(\dot{\omega}+\dot{\upsilon}) \ . \tag{1}$$

The link torque due to torsional spring stiffness is assumed to be proportional to the link's deflection

$$J_{arm}(\dot{\omega}+\dot{\upsilon}) + K_{stiff}\alpha = 0 \ . \tag{2}$$

The servomotor output torque is given by the following relation:

$$J_{eq}\dot{\omega} + B_{eq}\omega + J_{arm}(\dot{\omega}+\dot{\upsilon}) = T_L \ . \tag{3}$$

After several substitutions we obtain the following state-space model:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{4}$$

where,

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{K_{stiff}}{J_{eq}} & -\dfrac{\eta K_m^2 K_g^2 + B_{eq}R_a}{J_{eq}R_a} & 0 \\ 0 & -\dfrac{K_{stiff}(J_{eq}+J_{arm})}{J_{eq}J_{arm}} & \dfrac{\eta K_m^2 K_g^2 + B_{eq}R_a}{J_{eq}R_a} & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ \dfrac{\eta K_m K_g}{J_{eq}R_a} \\ -\dfrac{\eta K_m K_g}{J_{eq}R_a} \end{bmatrix}. \tag{5}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

where R_a is the armature resistance, K_m is the motor voltage constant, J_{arm} is the link moment of inertia, K_g is the high gear ratio, K_{stiff} is the equivalent spring constant, J_{eq} is the equivalent moment of inertia, B_{eq} is the equivalent viscous friction.

Model predictive control

The benefits of MP control compared to general controllers are its ability to handle constraints on the control and output signals. The model predictive control system uses model based predictions of the plant outputs to manipulate the plant inputs in such a way that deviations from set-points are minimized, subject to constraints on inputs and outputs. Consider the discrete-time system model

$$\begin{aligned} x(k+1) &= A_d x(k) + B_d u(k) \\ y(k) &= C_d x(k) \end{aligned} \tag{6}$$

where $x(k) \in R^{n_x}$ are the states, $u(k) \in R^{n_u}$ are the manipulated inputs and $y(k) \in R^{n_y}$ are the measured outputs. The cost function used in MPC is:

$$J(k) = \sum_{i=1}^{N_p} \left\| \hat{y}(k+i/k) - r(k+i) \right\|_{Q(i)}^2 + \sum_{i=1}^{N_u} \left\| \Delta u(k+i-1) \right\|_{R(i)}^2 \ . \tag{7}$$

subject to constraints specified on the inputs, outputs and input increments:

$$u_{\min} \le u(k) \le u_{\max}$$
$$y_{\min} \le y(k) \le y_{\max}$$
$$\Delta u_{\min} \le \Delta u(k) \le \Delta u_{\max}$$

where: $Q(i)$ - positive definite error weighting matrix; $R(i)$ - positive semi-definite control weighting matrix; $\hat{y}(k+i/k)$ - vector of predicted output signals; $r(k+i)$ - vector of future set-point; $\Delta u(k+i)$ - vector of future control actions; N_p - prediction horizon; N_u - control horizon.

The control law has regular form:

$$\begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N_u-1) \end{bmatrix} = K_{MPC} e(k) = \begin{bmatrix} K_0 \\ K_1 \\ \vdots \\ K_{N_u-1} \end{bmatrix} e(k)\,, \tag{8}$$

where $e(k)$ is the tracking error vector.

LQ based control

Considering the continuous-time model (4), the state-feedback law

$$u(t) = -(R^{-1}B^T P)x(t) = Kx(t) \tag{9}$$

minimizes the quadratic cost function

$$J(u) = \int_0^\infty (x^T Qx + u^T Ru)dt\,, \tag{10}$$

where P is the positive definite solution of the following algebric Riccati equation:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0\,. \tag{11}$$

Considering the discrete-time system (6), the DLQ control law which minimizes the following performance index:

$$J(u) = \sum_{i=1}^{\infty} [x_i^T Qx_i + u_i^T Ru_i] \tag{12}$$

is given by

$$u_k = -(R + B^T PB)^{-1} B^T PAx_k = -Kx_k\,, \tag{13}$$

where P is the positive definite solution of the following algebric Riccati equation:

$$P = A^T PA + Q - A^T PB(R + B^T PB)^{-1} B^T PA\ . \tag{14}$$

Experimental results

In MP controller case, we consider the following input data: control horizon $N_u = 4$; prediction horizon $N_P = 21$; sample time $T_s = 0.2$; input constraints $-10 \le u(k) \le 10$.
K_{MPC} from relation (8) is computed using the discretized model of the plant described by the continuous system (4), (5).

$$K_{MPC} = \begin{bmatrix} 3.9757 & -0.0001 & -4.1878 & 0.2957 \\ -0.2949 & -0.1897 & -3.9757 & 0.0001 \end{bmatrix}^T$$

Integral action is used to eliminate steady state errors when tracking constant signals. Integral action can be introduced in LQ tracking controller by considering the integral of the tracking error as an extra set of state variables.

$$\begin{aligned} w &= \int e\, dt = \int (r - y)\, dt = \int (r - Cx)\, dt \\ \frac{dw}{dt} &= r - Cx \end{aligned} \tag{15}$$

The augmented state space representation of the continuous-time system (4) is:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{w} \end{bmatrix} &= \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \\ y &= Cx \end{aligned} \tag{16}$$

The integral controller is (here we considered $C = [1\,0\,0\,0]$):

$$u = -[K \quad K_I] \begin{bmatrix} x \\ w \end{bmatrix}. \tag{17}$$

The state and control weighting matrices Q and R are chosen with the particular form:

$$Q = diag([0.01 \quad 0.1 \quad 0.01 \quad 0.1 \quad 1]); \; R = 1. \tag{18}$$

With this input data, K, K_I have following form:

$$K = [\,2.4101 \quad 0.1752 \quad 0.1115 \quad 0.0708] \quad K_I = [-0.6339].$$

In DLQ case we use the accumulated tracking error as part of the state:

$$w_{k+1} = w_k + e_k\,. \tag{19}$$

Augmented state space representation becomes:

$$\begin{aligned} \begin{bmatrix} x_{k+1} \\ w_{k+1} \end{bmatrix} &= \begin{bmatrix} A_d & 0 \\ -C_d & 1 \end{bmatrix} \begin{bmatrix} x_k \\ w_k \end{bmatrix} + \begin{bmatrix} B_d \\ 0 \end{bmatrix} u_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r_k \\ y_k &= C_d x_k \end{aligned} \tag{20}$$

where discrete matrices A_d, B_d, C_d are obtained by using a ZOH discretization of continuous system (4) with sample time $T_s = 0.2$. With Q and R, chosen with particular form (18), we obtain the following control law:

$$u_k = -[K \quad K_I] \begin{bmatrix} x_k \\ w_k \end{bmatrix} = -[2.1741 \quad -10.5184 \quad 0.2702 \quad -0.5081 \quad -2.2361] \begin{bmatrix} x_k \\ w_k \end{bmatrix}. \tag{21}$$

For all cases presented above in order to compensate the neglected nonlinearities and the dead zone of the real plant, we add to our standard control scheme a variable structure part. The structure implements the following algorithm:

$$if\ |u| \geq 0.2\ then\ u^* = u$$
$$else\ if\ |u| < 0.2\ \&\ |e| \geq 0.2\ then\ u^* = 0.2 * \mathrm{sgn}(u)$$
$$else\ u^* = 0$$

We implement the above algorithm, by using Stateflow chart presented in figure 2.

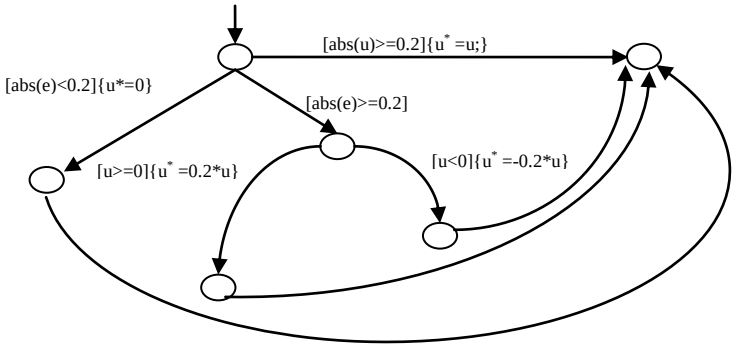
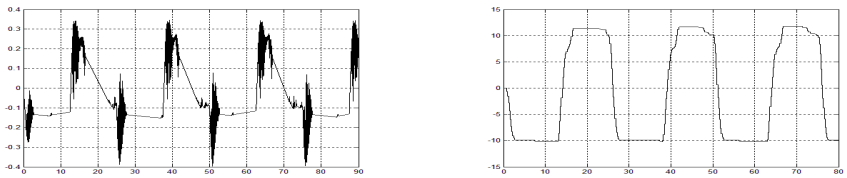


Fig. 2. Variable structure part of the controller.

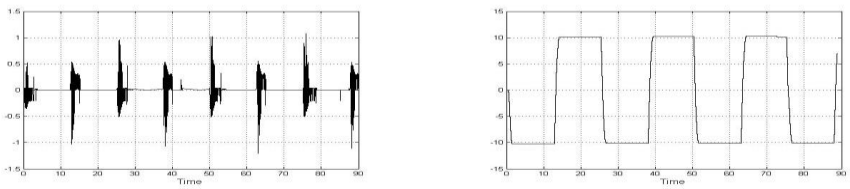
Figure 3 illustrates angular position of the shaft θ and applied command u obtained by using LQ method without variable structure. In this case, we observe an insensibility zone of the motor for the input control $u \in (-0.2, 0.2)$ and a stationary error for considered output.



a. Input Control **b**. Motor shaft angle

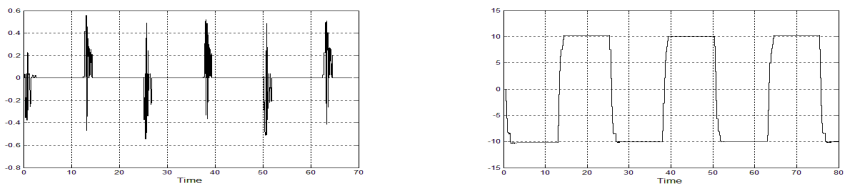
Fig. 3. LQ experimental response without variable structure part of the controller.

Figures 4 and 5 illustrate angular position of the shaft and applied command u^* by using different control schemes with variable structure part for compensation of the insensibility zone; accordingly, the considered experimental output is better: small rise time and small stationary error.



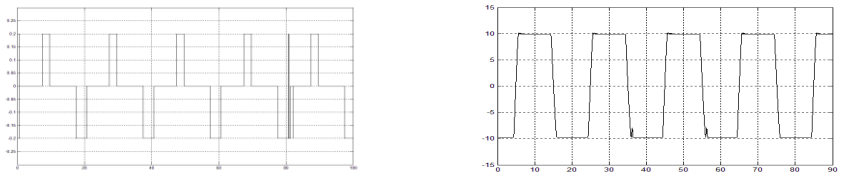
a. Input Control **b**. Motor shaft angle

Fig. 4. LQ experimental response with variable structure part of the controller.



a. Input Control **b**. Motor shaft angle

Fig. 5. DLQ experimental response with variable structure part of the controller.



a. Input Control **b**. Motor shaft angle

Fig. 6. MPC experimental response with variable structure part of the controller.

The obtained results using MP controller are illustrated in figure 6; it are similar with DLQ case, but in this case appear a small overshooting for motor shaft angle and a larger rise time.

Conclusions

In this paper we presented a Quanser real-time experiment: flexible link. Here, we proposed a MP control scheme and LQ based control scheme. In order to compensate the insensibility zone of the real experiment we improved our controllers by adding a variable structure part. In DLQ and LQ cases we obtained good responses for considered output. MP case is similar with DLQ case, however if the constraints are present we observe a larger rise time and presence of overshooting. These performance indicators increase if the constraints become harder. The presented experimental results illustrate the performances of the proposed control schemes when the controllers with variable structure are used, since the insensibility zone is compensated.

References

1. Lazăr, C. – *Predictive control of process with known model* (in Romanian). Ed. Matrix, Bucuresti, 1999.
2. Maciejowski, J. – *Predictive Control with Constraint*. Prentice Hall, 2002.
3. Popescu, D., Ionete, C., Sendrescu, D. – *Robust Control Methods Applied to a Flexible Beam Quanser Experiment*. Int. Carpathian Control Conference, Slovak Republic, 2007.
4. Stîngă, F., Roman, M., Şoimu, A., Bobasu, E. – *Optimal and MPC Control of the Quanser Flexible Link Experiment*. 7th WSEAS International Conference on Non-Linear Analysis, Non-Linear Systems and Chaos, Greece, 2008.
5. Stoleru, R. – *State-Space Feedback Control(Courses Notes)*. Virginia University, 2007.
6. * * * – Quanser Consulting Inc. *Flexible Link experiment*, 1998.

Controlul predictiv şi liniar pătratic cu compensarea zonei de insensibilitate pentru un experiment în timp real

Rezumat

În această lucrare este utilizat un experiment în timp real pentru a studia anumite aspecte legate de eroarea de urmărire în controlul predictiv şi liniar pătratic. În cazul controlului liniar pătratic o componentă integrală este introdusă pentru eliminarea (minimizarea) erorii staţionare de poziţie. Din cauza neliniarităţilor neglijate şi zonei de insensibilitate a instalaţiei, este necesară adăugarea la schema de control a unei părţi cu structură variabilă pentru reducerea vibraţiilor vârfului braţului. Rezultatele experimentale prezentate ilustrează performanţele schemelor de control.