BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LXII No.4B/2010	125 - 132	Seria Tehnică

Analysis of Transitory Regime for Three-Phase Inverter Based on Sine-Delta PWM Control Strategy

Florinel Dinu, Gheorghe Cremenescu

Universitatea Petrol – Gaze din Ploieşti, Bd. Bucureşti 39, Ploieşti
e-mail: gcremenescu@upg-ploiesti.ro

Abstract

The paper presents the mathematical model of the three-phase configurated circuits between inverter and inductive-resistive charge in transitory regime. For analysis transitory regime one determines the differential equations of the circuits and for their numerical solving one applies the method of the transient regime with repeated initial conditions. The numerical simulation of the mathematical model and the results were carried out using Matlab toolbox. The obtained numerical results are presented in tables and waveforms of voltage and current are graphically represented.

Key words: *three-phase inverter, sine-delta PWM control strategy, transient regime with repeated initial conditions, Matlab toolbox.*

Introduction

In the adjustable electrical drives with alternative current motors, the inverter with PWM control strategy after a sinusoidal waveform provides functioning conditions for the motor close to a sinusoidal supply source. In modulated regime the voltage in rectangular waveform of a tact (own to inverter in non modulated regime) is divided in several pulses with the width modulated after a sinusoidal function (sine-PWM control strategy, i.e. sine-Pulse Width Modulation). The splitting of tact into a succession of pulses allows the adjustment of the effective value of the output voltage of the inverter, correlated with the frequency, by modifying the width of the pulses and the number pulses per tact [1, 4]. The inverter functioning after the sine-PWM control strategy assures the simultaneous control of frequency and amplitude of the fundamental of the applied voltage for the charge; the harmonic content of the output voltage is better (by nullifying of low frequency harmonics) than for the inverter in non-modulated regime. In this paper the PWM control strategy is based on method of the comparison of a sinusoidal three-phase signal with a delta signal of high frequency (named sine-delta PWM control strategy) [3].

The inverter yields non-sinusoidal currents and voltages, which determine a deformed regime in the motor and in the supply network. The 5^{th} and 7^{th} harmonics have negative effects on the functioning of the ensemble inverter – motor through: the increase of currents in the chain winding of the motor, the increase of the power loss, the apparition of oscillating couples and the worse of commutation phenomena in the power semiconductor devices [1, 4, 5].

In this paper, on the configurated inverter-charge circuits in the functioning, one analyses the transitory regime using the method for the transitory regime with repeated initial conditions. By numerical simulation of the mathematical model of the circuits the waveforms and the spectral analysis of the electric currents are determined.

Mathematical Model of Transitory Regime

The principle electric scheme of the voltage three-phase inverter with weak inductive charge in star connection is given in figure 1. The inverter is composed of the three-phase bridge with thyristors T_1,...,T_6 and the three-phase bridge with diodes D_1,…,D_6 (called recovery diodes) in ant parallel connection. The bridge with the diode functions in the case of a charge (an electrical receptor) with an inductive character. This assures the current continuity through the charge during the time the reactive energy accumulated in the inductance of the charge is restored (unloads) to the direct current source and to the capacitor of the intermediate circuit.

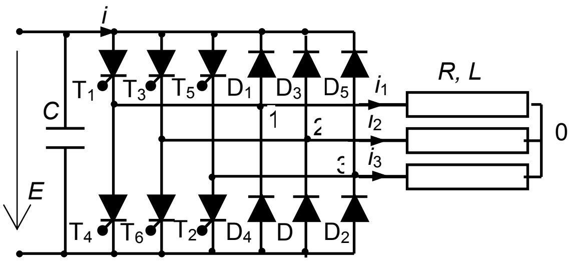

Fig. 1. The scheme of three-phase inverter with charge in star connection

At frequency 50 Hz, the inverter functions in non-modulated regime with three thyristors in simultaneous conduction state (with one thyristor on each phase); the thyristors of the same phase are successively conduction. The time interval when a group of three thyristors is in the conduction state is equal to $T/6$ and it one names tact (sixth part of a period); a period has six tacts. When passing from pulse to another, the thyristors on the same phase (but in different groups) commutate (the thyristor with an odd index together with the thyristor with an even index, and vice versa). The output voltages consist in a succession of equidistant pulses of the same width equal to $T/6$ (non-modulated pulses) of amplitude E (where E is the direct voltage in the intermediate circuit) for the line voltages and of amplitudes $E/3$ and $2E/3$ for the phase voltages.

At frequencies less than 50 Hz the inverter functions in modulated regime with commanded thyristors in conformity with sine-delta PWM control strategy, in order to reduce the weight of low frequency harmonics (moreover nullifying or decreasing the 5^{th}, 7^{th}, 11^{th} harmonics). The output voltages consist in a succession of rectangular pulses of amplitude E for the line voltages and of amplitudes $E/3$ and $2E/3$ for the phase voltages, with widths modulated after a sinusoidal function. Let us denote by $p = f_0/f$, the ratio between the frequency of the reference delta signal and the frequency of the three-phase command signal. The output line voltage has p positive pulses and p negative pulses in a period T, and the phase voltage has p negative, positive and null pulses within one tact [3]. The non null pulses of phase voltage correspond to the conduction state of the semiconductor devices (thyristors, or thyristors and diodes) contained in the two groups of bridges (devices with odd and even indexes), and the null pulses correspond to the conduction state of the semiconductor components belonging to a single group of bridges (devices with odd or even indexes). Even if the voltage E of the intermediate circuit is constant, the inverter adjusts not only the frequency, but also the effective value of the voltage fundamental.

In the case of a resistive-inductive charge, when thyristors commute for passing from a pulse to another, one of the diodes from the phase of the blocking thyristor (that one directly polarized) begins conducting; the reactive energy stored in the inductance of the charge discharges through it, thus maintaining the same sense of the current through the phase of the charge. The discharge current closes between phases, either through the condenser C, or through the source of direct current (if this is possible). Because the reactive energy can come back from the charge to the direct current source, the diodes D_1…D_6 are called recovery diodes The discharge current is decreasing and zeroes in a non null pulse then changes its sense through the respective phase and one closes through the thyristor ordered to conduct, and increases in time [4]. One considers the currents being positive when they have the senses displayed in figure 1.

Depending on the conduction state of the semiconductor devices of the bridges, the electrical circuits of type inverter-charge that are configured in the pulses of a tact can be as follows:

- circuit with 2 thyristors from one group and with 1 thyristor from the other group in conduction state; it produces non-null pulses for the phase voltages (e. g. fig. 2,*a*);
- circuit with 2 thyristors and 1 diode from the same group (with odd or with even indexes) in conduction state; it produces null pulses for the phase voltages (e. g. fig 2,*b*);
- circuit with 1 thyristor and 1 diode from the same group and with 1 thyristor from the other group in conduction state; it produces non-null pulses for the phase voltages (e. g. fig. 2,*c*);
- circuit with 1 thyristor and 2 diodes from the same group in conduction state; it produces null pulses for the phase voltages (e. g. fig 2,*d*);
- circuit with 1 thyristor and 1 diode from the same group and with 1 diode from the other group in conduction state; it produces non-null pulses for the phase voltages (e. g. fig. 2,*e*).
- Figure 2 shows examples of electrical circuits that are configured in the pulses of phase voltage during tact 1.

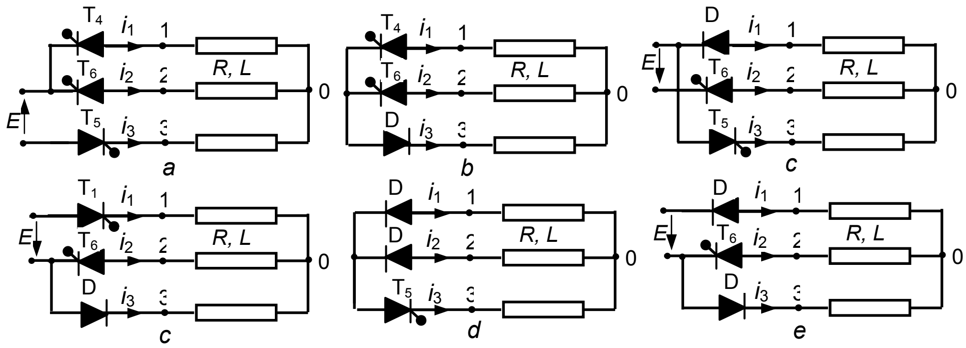

Fig. 2. Configurated inverter-charge circuits during the pulses of 1st tact

The inverter-charge circuits work in non-sinusoidal transitory regime [4, 5]; for the calculus of currents absorbed by the charge, the mathematical model for the configured circuit during a k^{th} pulse of phase voltage is deduced. The mathematical model of the transitory regime during one pulse is given by the following system of differential equations with unknown non-null initial conditions:

$$
\begin{cases}
\begin{cases}
L\dfrac{\mathrm{d}i_1(t)}{\mathrm{d}t}+Ri_1(t)=u_{10}(t)\\
i_1(t_k)=I_1(k),\ u_{10}(t_k)=U_{10}(k),\ i_1(t_{k+1})=I_1(k+1)
\end{cases}\\
\begin{cases}
L\dfrac{\mathrm{d}i_2(t)}{\mathrm{d}t}+Ri_2(t)=u_{20}(t)\\
i_2(t_k)=I_2(k),\ u_{20}(t_k)=U_{20}(k),\ i_2(t_{k+1})=I_2(k+1
\end{cases}\\
\begin{cases}
L\dfrac{\mathrm{d}i_3(t)}{\mathrm{d}t}+Ri_3(t)=u_{30}(t)\\
i_3(t_k)=I_3(k),\ u_{30}(t_k)=U_{30}(k),\ i_3(t_{k+1})=I_3(k+1)
\end{cases}
\end{cases}
\qquad (1)
$$

where: $t_{k,}$ t_{k+1} represent the start time and end time (limits of time) of the k^{th} pulse.

From equations (1) the phase currents are inferred:

$$\begin{cases} i_1(t) = \left(I_1(k) - U_{10}(k)/R\right) \mathrm{e}^{-(t_{k+1}-t_k)/\tau} + U_{10}(k)/R \\ i_2(t) = \left(I_2(k) - U_{20}(k)/R\right) \mathrm{e}^{-(t_{k+1}-t_k)/\tau} + U_{20}(k)/R \\ i_3(t) = \left(I_3(k) - U_{30}(k)/R\right) \mathrm{e}^{-(t_{k+1}-t_k)/\tau} + U_{30}(k)/R \end{cases} \tag{2}$$

where $\tau = L/R$ is the time constant of the charge.

At the end of each pulse, the currents have the following values:

$$\begin{aligned} i_1(t_{k+1}) &= I_1(k+1) = a(k)\, I_1(k) + b_1(k) \\ i_2(t_{k+1}) &= I_2(k+1) = a(k)\, I_2(k) + b_2(k) \\ i_3(t_{k+1}) &= I_3(k+1) = a(k)\, I_3(k) + b_3(k) \end{aligned} \tag{3}$$

where : $k = 1, 2, \ldots, 6p$ - represents the pulse number during a period; $a(k) = \mathrm{e}^{-(t_{k+1}-t_k)/\tau}$, $b_1(k) = (1-a(k))U_{10}(k)/R$, $b_2(k) = (1-a(k))U_{20}(k)/R$, $b_3(k) = (1-a(k))U_{30}(k)/R$.

The method of the transitory regime with repeated initial conditions imposes (from the periodicity condition and from the continuity of currents), that the solutions to have the final values after 1 half-period (after three tacts) equal to the initial ones (from the beginning of the period) with changed sign or to have the final values after 1 period equal to the initial ones [1, 4]:

$$\begin{cases} I_1(3p+1) = -I_1(1) \\ I_2(3p+1) = -I_2(1) \\ I_3(3p+1) = -I_3(1) \end{cases}, \text{ or } \begin{cases} I_1(6p+1) = I_1(1) \\ I_2(6p+1) = I_2(1) \\ I_3(6p+1) = I_3(1) \end{cases}. \tag{4}$$

These conditions imply three systems of algebraic equations having unknowns the initial values of currents in each pulse during first half-period (the unknown initial conditions):

$$\begin{cases} I_1(k+1) - a(k)\, I_1(k) = b_1(k) \\ I_1(3p+1) = -I_1(1) \end{cases}, \tag{5}$$

$$\begin{cases} I_2(k+1) - a(k)\, I_2(k) = b_2(k) \\ I_2(3p+1) = -I_2(1) \end{cases}, \tag{6}$$

$$\begin{cases} I_3(k+1) - a(k)\, I_3(k) = b_3(k) \\ I_3(3p+1) = -I_3(1) \end{cases}, \tag{7}$$

where: $k = 1, 2, \ldots, 3p$ - represents the pulse number during a half-period.

By solving the systems of equations (5), (6), (7) the initial values of currents for each pulse during first half-period are obtained. Taking into account the fact that the values of the phase voltages and of the phase currents from the pulses of the first tact do repeat also during the next 5 tacts according to the recurrence relations from table 1, only the system of equations (5) can to be numerically solved.

The changing in sign of two consecutive values of the phase current shows in which pulse it zeroes (for example $I_1(k) < 0$, $I_1(k+1) > 0$, where k is the number of the pulse during which the current i_1 nullifies). During a k^{th} pulse the phase current i_1 has an exponential variation between the values $I_1(k)$ and $I_1(k+1)$.

The waveforms of the phase voltages and of the phase currents during pulses of second half-period are determined using the recurrent relations from table 1 [1, 4]. The voltages and the currents are doubly indexed: the superior index corresponds to the first tact and the inferior index corresponds to the phase.

Table 1. The recurrent relations

Size \ Tact	1	2	3	4	5	6
u_{10}, i_1	$u_{10}^{(1)}$, $i_1^{(1)}$	- $u_{20}^{(1)}$, - $i_2^{(1)}$	$u_{30}^{(1)}$, $i_3^{(1)}$	- $u_{10}^{(1)}$, - $i_1^{(1)}$	$u_{20}^{(1)}$, $i_2^{(1)}$	- $u_{30}^{(1)}$, - $i_3^{(1)}$
u_{20}, i_2	$u_{20}^{(1)}$, $i_2^{(1)}$	- $u_{30}^{(1)}$, - $i_3^{(1)}$	$u_{10}^{(1)}$, $i_1^{(1)}$	- $u_{20}^{(1)}$, - $i_2^{(1)}$	$u_{30}^{(1)}$, $i_3^{(1)}$	- $u_{10}^{(1)}$, - $i_1^{(1)}$
u_{30}, i_3	$u_{30}^{(1)}$, $i_3^{(1)}$	- $u_{10}^{(1)}$, - $i_1^{(1)}$	$u_{20}^{(1)}$, $i_2^{(1)}$	- $u_{30}^{(1)}$, - $i_3^{(1)}$	$u_{10}^{(1)}$, $i_1^{(1)}$	- $u_{20}^{(1)}$, - $i_2^{(1)}$

The current i in the intermediate circuit only exists during the non null pulses and has an exponential variation. As long as the current i is positive, the source gives energy to the charge, meaning that the energy given by the source to the charge is greater than the energy recovered from charge; when the current i is negative, the source receives energy from the charge, meaning that the energy given by the source to the charge is smaller than the energy recovered from charge.

Spectral Analysis of the Phase Current

For the spectral analysis, one uses the Fourier series expansion of the current $i_1(t)$:

$$i_1(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\,\omega t + b_n \sin n\,\omega t), \tag{8}$$

where the coefficients are given by the formulas:

$$a_0 = 1/T\int_0^T i_1(t)\,dt = 1/(2\pi)\int_0^{2\pi} i_1(\alpha)d\alpha,\ \alpha = \omega t, \tag{9}$$

$$a_n = 2/T\int_0^T i_1(t)\cos n\omega t\,dt = 1/(\pi)\int_0^{2\pi} i_1(\alpha)\cos n\alpha\,d\alpha, \tag{10}$$

$$b_n = 2/T\int_0^T i_1(t)\sin n\omega t\,dt = 1/(\pi)\int_0^{2\pi} i_1(\alpha)\sin n\alpha\,d\alpha, \tag{11}$$

The effective values of the harmonics and the distortion coefficients are computed with:

- the effective value of the n^{th} harmonic:

$$I_{efn} = \left(\frac{a_n^2 + b_n^2}{2}\right)^{1/2},\ n = 1,2,3,...\infty, \tag{12}$$

- the total effective value:

$$I_{ef.t} = \left(\frac{1}{T}\int_0^T i_1{}^2(t)\,\mathrm{d}t\right)^{1/2} = \left(a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)\right)^{1/2}, \tag{13}$$

- the total effective value of the high harmonics:

$$I_{ef.t.a} = \left(\frac{1}{2}\sum_{n=2,..}^{\infty}\left(a_n^2 + b_n^2\right)\right)^{1/2}, \tag{14}$$

- the distortion coefficients:

$$k_{d1} = \frac{I_{ef.t.a}}{I_{ef.1}},\quad k_{d2} = \frac{I_{ef.t.a}}{I_{ef.t}}. \tag{15}$$

Numerical Results of the Simulation

For the numerical simulation of the transitory regime of inverter-charge circuits one has used the Matlab toolbox [2], which has facilities for solving linear systems of algebraic equations, for the spectral analysis of the phase current, for the construction of time-current vectors, for the graphical representation of three-phase current and frequency spectrum of the current harmonics. The simulation program is structured into three sections:

- Input of data
  The input data are:
  - p - the ratio of the frequencies of the delta signal and of the sinusoidal signal;
  - $\alpha_{kj}^{(i)}$ - the angles of the command impulses for conduction of the thyristors, where: i =1, 2, 3 – is the phase number; k =1, 2,…, p – is the number of command impulses ; j = 1, 2 – is the start and end index of the command impulse ;
  - E - direct voltage of the intermediate circuit;
  - f - frequency of the output voltage;
  - R, L – resistance and inductance of charge.

  For the numerical results there have been chosen: p =9, $\alpha_{kj}^{(i)}$ with values from work [3], E = 100 V, f = 50 Hz, R = 10 Ω, L = 50 mH.

- Processing of data
  By data processing after relations (5 – 15) one obtain: the initial values of currents in each pulse during first half-period, number of the pulse during which the current nullifies, the effective values of the current harmonics and the distortion coefficients.

- Display of the numerical results
  The output data are:
  - the initial values of currents in each pulse;
  - the effective value of the current harmonics up to order 25;
  - the distortion coefficients k_{d1}, k_{d2};
  - the graph of the spectrum of the harmonics amplitude;
  - the graphical representation of the first phase voltage and of the three-phase current.

The numerical results of the simulation for chosen input data are given in tables 2 (the initial values of three-phase current) and table 3 (the numerical results of the spectral analysis).

Table 2. The numerical results of the simulation

Current	**No.pulse**	**Initial values of currents [A]**								
I_1	1 - 9	-1.5312	-1.6522	-1.5114	-1.5772	-1.0408	-0.9954	-0.5225	-0.2628	-0.2409
	10 – 18	0.2358	0.4496	0.4121	0.5177	1.1894	1.1376	1.7416	1.7991	1.6491
	19 - 27	1.7653	2.1036	1.9282	2.0994	2.2342	2.1368	2.2676	2.0652	1.8930
I_2	1 - 9	-0.2320	-0.4402	-0.4027	-0.5086	-1.1813	-1.1299	-1.7347	-1.7925	-1.6430
	10 – 18	-1.7597	-2.0983	-1.9234	-2.0947	-2.2301	-2.1329	-2.2640	-2.0618	-1.8899
	19 - 27	-1.5294	-1.6539	-1.5160	-1.5817	-1.0447	-0.9992	-0.5259	-0.2660	-0.2439
I_3	1 - 9	1.7632	2.0924	1.9141	2.0858	2.2221	2.1253	2.2573	2.0553	1.8839
	10 – 18	1.5238	1.6487	1.5113	1.5771	1.0406	0.9953	0.5224	0.2627	0.2408
	19 - 27	-0.2359	-0.4497	-0.4122	-0.5177	-1.1895	-1.1376	-1.7417	-1.7992	-1.6492

Table 3. The numerical results of the spectral analysis

Harmonic number	**The effective value of the current harmonic** [A]							**Tot.eff. val.** [A]	**Tot.eff. val.of harm.**[A]	**Distortion coefficients**	
n	$I_{ef\,n}$, $n = 1,2,3,...28$							$I_{ef.t}$	$I_{ef.t.a}$	k_{d1}	k_{d2}
1 – 7	1.5181	0.0014	0.0012	0.0011	0.0044	0.0010	0.0713	1.5223	0.1127	0.0742	0.0740
8 – 14	0.0010	0.0010	0.0010	0.0460	0.0010	0.0045	0.0010				
15 – 21	0.0010	0.0010	0.0426	0.0011	0.0360	0.0011	0.0011				
22 – 28	0.0011	0.0100	0.0011	0.0169	0.0011	0.0011	0.0011				

The graphical representations of the spectrum of the harmonics up to order 25 is presented in figure 3.

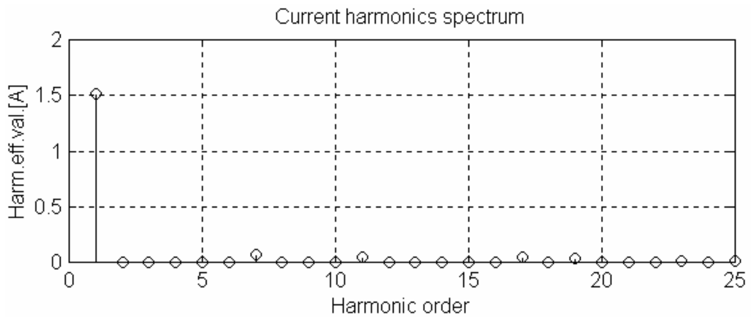

Fig. 3. The graph of current harmonics spectrum

In figure 4 the waveforms of first phase voltage and three-phase current are graphically represented; the variation in time of voltage has a shape of rectangular pulses and that of three-phase current is alternatively exponential.

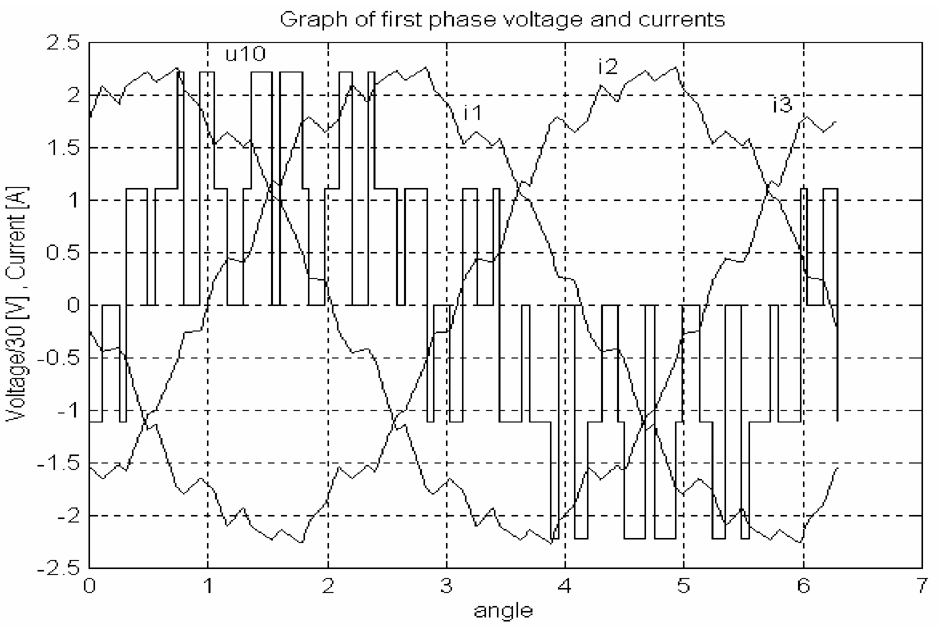

Fig. 4. The waveforms of first phase voltage and of three-phase current

Conclusions

In this paper, the inverter in sine-delta PWM regime which supplies a resistive- inductive charge in star connection is analysed. The inverter yields non-sinusoidal currents and voltages, which determine a deformed regime in the charge and in the supply network.

For the circuits configured in the pulses of the phase voltage during a semi period are determined their equations in transitory regime, then are computed the solutions of the phase currents. The method of the transient regime with repeated initial conditions proposed for the analysis of the inverter-charge circuits is easier to apply than the Fourier series expanding method. For the numerical solving of the systems of equations and for the graphical representation of voltage and currents, the Matlab toolbox is used. By numerical simulation of the mathematical model of the circuits the waveforms and the spectral analysis of the electric currents are determined for an entire period.

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Analiza regimului tranzitoriu al invertorului trifazat bazat pe strategia de comandă PWM sine-delta

Rezumat

Lucrarea prezintă modelul matematic al circuitelor trifazate configurate intre invertor şi sarcina inductiv-rezistivă in regim tranzitoriu. Pentru analiza regimului tranzitoriu se determină ecuațiile diferențiale ale circuitelor si pentru rezolvarea numerica a lor se aplica metoda regimului tranzitoriu cu condiții inițiale repetate. Simularea numerica a modelului matematic si rezultatele au fost realizate utilizând pachetul de programe Matlab. Rezultatele numerice obținute sunt prezentate in tabele si reprezentate grafic tensiunile si curenții.