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Systemic Equations and Stability Conditions for Precision Oxygen Analyzer (1st part)

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Abstract

In this work we describe the precision oxygen analyzer using adequate mathematic model and we establish the systemic equations for the response (a physical measure such as the electrical current intensity) produced by external condition modifications (the oxygen concentration in our case). The analyzer function can be studied through resolving these equations for known initial conditions. We are pointed out the linear dependence of the oxygen concentration by the electrical current intensity.

Key words: oxygen analyzer, magnetic field, concentration, magnetic force, temperature.

Introduction

The modeling of the oxygen analyzer (1) can be realized using the model of a system of two spherical diamagnetic (empty spheres filled with nitrogen, for example), very smalls ones with the help of a torsion wire. The entire system is sunk in an environment with paramagnetic properties (the oxygen) and in an unequal magnetic field. The forces - see relation (1) - which appear, due to the intensity's gradient of a magnetic field extremely unequal, are balanced by the electromagnetic forces made with the help of some electric currents which are passing trough a coil set towards the spherical shape and towards the torsion forces from the wire (see fig. 1).

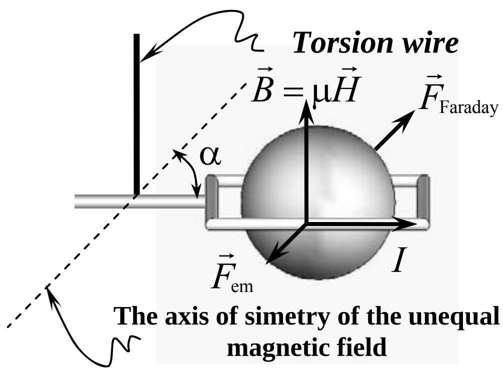

Fig. 1. The experimental modeling 3D of a torsion pendulum

A shape of small dimension, sunk in an environment with different magnetic susceptibility is subdued the action of a force given by the relation:

$$\vec{F}_{\text{Faraday}} = \left(\chi_1 - \chi_2\right) \text{v}\, H \,\text{grad}\, H \,, \tag{1}$$

where v represents the volume of the shape, χ_1 the magnetic susceptibility of the shapes, χ_2 the magnetic susceptibility of the environment and H the intensity of the magnetic field (1).

In the case of our modeling are used two very small identical spheres, like in the fig. 1, which, introduced in the inhomogeneous magnetic field, will interact with that depending on the difference of susceptibility (which characterizes the shape and also the environment).

The moment of Faraday force can be balanced by the moment of the torsion forces from the wire:

$$|\vec{F}_{\text{torsiune}}| = k(\alpha - \alpha_0) \tag{2}$$

where k represents a constant of torsion and $\Delta\alpha = \alpha - \alpha_0$ the wire's angle of torsion. The angle α_0 represents the initial position of the wire towards the chosen initial system of reference

In order to reduce the torsion wire in the balance position we must use an external force, the electromagnetically force, for example. In this way we can find a direct correlation between a measurable measure and the magnetic susceptibility.

In the case of electromagnetically force towards the element of length of conductor operates a force:

$$\mathrm{d}\vec{F}_{\text{em}} = I\left(\mathrm{d}\vec{l} \times \vec{B}\right). \tag{3}$$

The entire force can be evaluated through an integral:

$$\vec{F}_{\text{em}} = \int_{\Gamma} \mathrm{d}\vec{F}_{\text{em}} = \int_{\Gamma} I\left(\mathrm{d}\vec{l} \times \vec{B}\right). \tag{4}$$

The Obtaining of Systemic Equations

In order to reduce the complexity of the relations we suppose that in an environment exits only two types of gazes: nitrogen and oxygen. In the case of oxygen the magnetic susceptibility is more different than the one of nitrogen reason for which in a mixture of oxygen with nitrogen, both concentrate, relative will be strongly dependent on the susceptibility of the mixture, in a primary approximation a cool average.

$$\chi_2 = c_{\text{O}_2}\chi_{\text{O}_2} + c_{\text{N}_2}\chi_{\text{N}_2} \,. \tag{5}$$

Rewriting the relation (5) only depending on the oxygen's concentration, we obtain:

$$\chi_2 = c_{\text{O}_2}\chi_{\text{O}_2} + (1 - c_{\text{O}_2})\chi_{\text{N}_2} \,. \tag{6}$$

We replace the relation above in (1):

$$\vec{F}_{\text{Faraday}} = \left(\chi_1 - c_{\text{O}_2}\chi_{\text{O}_2} - (1 - c_{\text{O}_2})\chi_{\text{N}_2}\right) \text{v}\, H \,\text{grad}\, H \tag{7}$$

and we evaluate the intensity of the H field and the gradient in the point which is the centre of a volume sphere v (fig. 1) which coincide with the centre of a shaped rectangular frame.

As it is seen from the relation above, the Faraday force also depends on the angle between the axis of the disposition and the symmetry's axis of a magnetic distribution field.

We are going to shape the magnetic field with the help of the relation (2):

$$H(x,y) = 12000\cos\left(\frac{y[\text{mm}]}{5}\right)\left[\frac{\text{A}}{\text{m}}\right] \tag{8}$$

So that depending of the torsion's angle we have:

$$H(\alpha) = H\left(\left(d + L/2\right)\cos(\alpha), \left(d + L/2\right)\sin(\alpha)\right). \tag{9}$$

We are assuming that the disposition is brought back in the balanced position (initially settled in the absence of oxygen) so that the electromagnetically force s rebalancing the Faraday force through their moments:

$$2\left(d + L/2\right)F_{\text{Faraday}}\cos(\alpha) = 2\left(d + L/2\right)F_{\text{em}}. \tag{10}$$

It is multiplied with 2 because we have two spheres and $\cos(\alpha)$ appears because of the force's projection on a direction which is perpendicular on the axis of a system.

By simplifying we obtain the equality of the medium forces which take an extra action regarding the rebalance of the system:

$$F_{\text{Faraday}}(\alpha)\cos(\alpha) = F_{\text{em}}(\alpha). \tag{11}$$

Solving the equation it is obtained the concentration depending on the current's rebalance like a linear dependence.

The relations (4), (7), (9), (11) form a ensemble of systemic equations trough which we can shape the dependence equation of the concentration's intensity of the electric rebalanced current.

The Solving of the Systemic Equations

By forming a system of four equations:

$$\begin{cases} \vec{F}_{\text{em}} = \int_\Gamma \text{d}\,\vec{F}_{\text{em}} = \mu_0(\chi_2 + 1)\int_\Gamma I\left(\text{d}\vec{l} \times \vec{H}\right) \\ \vec{F}_{\text{Faraday}} = \left(\chi_1 - c_{\text{O}_2}\chi_{\text{O}_2} - (1 - c_{\text{O}_2})\chi_{\text{N}_2}\right)\text{v}\, H\,\text{grad}\, H \\ H(\alpha) = H\left(\left(d + L/2\right)\cos(\alpha), \left(d + L/2\right)\sin(\alpha)\right) \\ F_{\text{Faraday}}(\alpha)\cos(\alpha) = F_{\text{em}}(\alpha) \end{cases} \tag{12}$$

and solving them, leaving as changeable the intensity of electric current, it is obtained the concentration of oxygen the form of:

$$c_{\text{O}_2} = mI + n. \tag{13}$$

In the relation (13) the constants are:

$$n = \frac{-\chi_1 + \chi_{\text{N}_2}}{\chi_{\text{N}_2} - \chi_{\text{O}_2}}, \tag{14}$$

$$m = A\,\text{cosec}(2\alpha)\sin\left(\frac{l\cos(\alpha)}{2a}\right)\sin\left(\frac{L\sin(\alpha)}{2a}\right)\sec\left(\frac{(2d + L)\sin(\alpha)}{2a}\right), \tag{15}$$

$$A = -\frac{8a^2\mu_0(\chi_1 + 1)}{H_0\text{v}\left(\chi_{\text{N}_2} - \chi_{\text{O}_2}\right)}. \tag{16}$$

By the relation (13) we obtain a dependence between a no electrical measure the concentration of oxygen) and an electrical measure (the intensity of the electric current). Using the above mentioned values and the other constants as it follows:

(the sphere's beams) it is obtained a direct relation

$$c_{O_2}(I) = 7.8\,I[\text{mA}] + 0.2[\%]\,. \quad (17)$$

The graphic representation of the above expression shows the linear dependence and for a maximum (100%) concentration it would not excel $12-13\,\text{mA}$. In the following chart are calculated the values for the concentration of oxygen depending on the values of the electric current until it's obtained the maximum values of concentration.

Table 1.

Nr. Crt.	I(mA)	C(%)	Nr. Crt.	I(mA)	C(%)
1	0,00	0,20	21	6,56	51,38
2	0,33	2,76	22	6,89	53,94
3	0,66	5,32	23	7,22	56,50
4	0,98	7,88	24	7,55	59,06
5	1,31	10,44	25	7,87	61,62
6	1,64	12,99	26	8,20	64,17
7	1,97	15,55	27	8,53	66,73
8	2,30	18,11	28	8,86	69,29
9	2,62	20,67	29	9,19	71,85
10	2,95	23,23	30	9,51	74,41
11	3,28	25,79	31	9,84	76,97
12	3,61	28,35	32	10,17	79,53
13	3,94	30,91	33	10,50	82,09
14	4,26	33,47	34	10,83	84,65
15	4,59	36,03	35	11,15	87,21
16	4,92	38,58	36	11,48	89,76
17	5,25	41,14	37	11,81	92,32
18	5,58	43,70	38	12,14	94,88
19	5,91	46,26	39	12,47	97,44
20	6,23	48,82	40	12,79	100,00

References

1. M u n d a y, C. W. *A Precision oxygen analyzer for chemical plants*, Automatic Measurement of Quality in Process Plants (1958)
2. F â c ă, M., B o r s o s, Z., D i n u, O. *The Physical Model of the precision oxygen analyzer*, to appear in Bulletin of Petroleum Gas University of Ploiesti, (2008)

Condiții de stabilitate pentru analizorul de precizie pentru măsurarea concentrației de oxigen

Rezumat

Lucrarea de fata isi propune stabilirea ecuațiilor sistemice ale analizorului de precizie pentru masurarea concentiatiei de oxigen, plecind de la modelul sau matematic. Rezultatele sunt prezentate de grafice 2D si 3D. S-a urmarit stabilitatea sistemului in conditii initiale cunoscute si stabilite.