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Mathematical Support of the Sine-Delta PWM Control Strategy for Three-Phase Inverter

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Abstract

This paper deals with modeling of sine-delta PWM control strategy for three-phase inverter. The paper presents the mathematical model represented the intersection of sine and delta function, for the commutation program of the power switching devices of the three-phase bridge; this is a nonlinear equation which has not an exact solution represented by a mathematical function.
For generating sine-delta PWM control strategy by software it is proposed an approximated solution. The simulation of the mathematical support and the results were carried out using specific computational tools, such as Matlab toolbox, then the results can be loaded in a look-up table witch could be implemented on hardware inverter. The results of the simulation are very close to the theoretical ones; they confirm the validity of the proposed mathematical support.

Key words: *three-phase inverter, sine-delta PWM control strategy, Matlab toolbox.*

Introduction

The increase in performance of the power semiconductor devices (fast thyristors, Gate Turn Off - GTO thyristors, power transistors, Insulated Gate Bipolar Transistor - IGBT etc), and of the numerical integrated circuits (microprocessors, microcontrollers, specialized processors etc) opens the opportunity to implement sophisticated strategies for the command of inverters, for the synthesis of the sinusoidal shape of the voltage or of the output current [1, 4].

The optimal synthesis consists in decreasing or nullifying the low frequency harmonics in order to obtain waveforms close to sinusoidal ones. The command system of the inverter generates the commutation program for the power devices such that the widths of the output pulses of the inverter are modulated after a sinusoidal function (called sine-PWM control strategy, i.e. sine-Pulse Width Modulation). The inverter adjusts not only the frequency but also the amplitude for the fundamental harmonic of the voltage. The sine-PWM control strategy is based on one of the following methods: the comparison of a sinusoidal signal with a high frequency triangular signal (called sine-delta PWM control strategy), or the sampling of the angular position of the spatial phasor for the three-phase voltages, or the equality between the sampled area from a reference sinusoidal signal and the area of the pulse of the inverter output signal. The modulation of the pulse width is applied at frequencies $f < 50$ Hz and has a double role: the variation of amplitude of the fundamental correlated with its frequency and the nullifying of low frequency harmonics [3, 5].

In this paper, one elaborates two mathematical models of the sine-delta PWM control strategy for three-phase inverter based on which the commutation program of thyristors is determined. By numerical simulation one determines the angles and moments of commutation, modulated widths of commands impulses for conduction and blocking of thyristors.

Mathematical model of sine-delta PWM natural modulation

The principle electric scheme of the voltage three-phase inverter is presented in figure 1. The inverter is composed of the three-phase bridge with thyristors $T_1,...,T_6$ and the three-phase bridge with diodes $D_1,\ldots,D_6$ (called recovery diodes) in ant parallel connection. The bridge with the diode functions in the case of a charge (an electrical receptor) with an inductive character. This assures the current continuity through the charge during the time the reactive energy accumulated in the inductance of the charge is restored (unloads) to the direct current source and to the capacitor of the intermediate circuit.

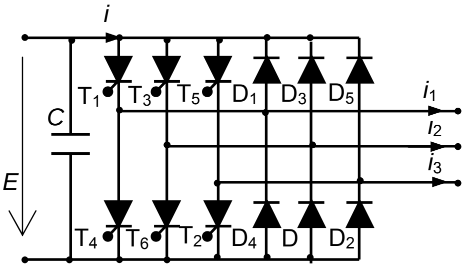

Fig. 1. The electric scheme of voltage three-phase

At frequency 50 Hz, the inverter functions in non-modulated regime. The interval of time when a thyristor is in the conduction state is *T*/2 (T =20 ms is the period of the output voltage), meaning that in each moment of time three thyristors are in simultaneous conduction state (with one thyristor on each phase); the thyristors of the same phase are successively conduction. The time interval when a group of three thyristors is in the conduction state is equal to *T*/6 and it one names tact; a period has six tacts. The output voltages of the inverter are positive and negative rectangular pulses shape of amplitude *E* (where *E* is the direct voltage in the intermediate circuit) for the line voltages and of amplitudes *E*/3 and 2*E*/3 for the phase voltages; the amplitude of voltage fundamental is equal $\left(2\sqrt{3}/\pi\right)E$ for the line voltage and $\left(2\ /\pi\right)E$ for the phase voltage.

The inverter functions in modulated regime at frequencies less than 50 Hz, in order to reduce the weight of low frequency harmonics (moreover nullifying or decreasing the 5^{th}, 7^{th}, 11^{th} harmonics). The principle of the sine-PWM control strategy consists in the conductions of the power devices during intervals of time with the durations modulated by a sinusoidal function [1, 2, 5]. The output voltage consists in a succession of pulses of amplitude *E* for the line voltages and of amplitudes *E*/3 and 2*E*/3 for the phase voltages, with widths modulated after a sinusoidal function. Consequently, the amplitude of the fundamental of the output voltages decreases as the width of the pulses becomes narrower, i.e. the durations of conduction of the thyristors are reduced. Even if the voltage *E* of the intermediate circuit is constant, the inverter adjusts not only the frequency, but also the effective value of the voltage fundamental [1, 5].

The analogue technique of sine-delta PWM modulation (called natural modulation) consist in the comparison of a sinusoidal three-phase signal of command with a delta signal of reference (having the shape of a triangle symmetrical with respect to the time axis) with the frequency equal to a multiple integer of the frequency of the given command signal. Depending on the result of the comparison the command impulses for conduction and blocking of thyristors (or of other power devices) are generated. During the time interval when the instant value of a sinusoidal mono-phase signal is higher than the value of the delta signal, the command impulse of conduction of the thyristor of a phase (with the anode of this thyristor connected to the positive pole of the direct voltage source) is generated. The other thyristor of the same phase (the one with the cathode connected to the negative pole of the direct voltage source) receives the command impulse of conduction during the time interval when the value of the delta signal is higher than the value of a sinusoidal mono-phase signal [1, 5].

The adjustment of the amplitude of the fundamental of the output voltage in correlation with its frequency is realized by the variation of the amplitude and of the frequency of the sinusoidal command signal, by keeping the same delta reference signal. By lowering the frequency and the amplitude of the sinusoidal command signal, the frequency and the amplitude of the fundamental of the output voltage are correlatively decreasing (for example amplitude/frequency = constant) and also the weight of the low-frequency harmonics is reduced (the harmonics 5^{th} and 7^{th} harmonics which negatively influence the functioning of the asynchronous motor that is connected to the inverter- are diminished). The sinusoidal command signal has the same frequency as the fundamental of the output voltage [1, 5].

In the case of the natural modulation, the commutation moments and the intervals of conduction and blocking of each thyristor can be determined from the equality condition between each sinusoidal signal and the delta signal (i.e. from the intersection points of the signals waves) [1].

The equation of the three-phase sinusoidal command signal is:

$$u_1 = A\sin\omega\, t\,,\quad u_2 = A\sin(\omega\, t - 2\pi/3)\,,\quad u_3 = A\sin(\omega\, t - 4\pi/3)\,, \tag{1}$$

where: A –is adjustable amplitude; f - is adjustable frequency ; T- is adjustable period ; ω - is adjustable angular frequency.
The equation of the delta reference signal is:

- for the line with positive slope:

$$u_0 = 4A_0/T_0(t - k\,T_0) = 4A_0 f_0 t - 4k\,A_0\,, \tag{2}$$

- for the line with negative slope:

$$u_0 = -4A_0/T_0(t - (2k-1)\,T_0/2) = -4A_0 f_0 t + 2A_0(2k-1)\,, \tag{3}$$

where: A_0 - is amplitude; T_0- is period; f_0 – is frequency; $p = f_0/f = T/T_0$ $p=f_0/f$ - is the ratio of the frequencies of the delta and of the sinusoidal signals (equal to the number of command impulses of a thyristor during one period T, or equal to the number of positive pulses of the line output voltage during one period T, or equal to the number of positive, negative and null pulses of the phase voltage during one tact [1, 5]).

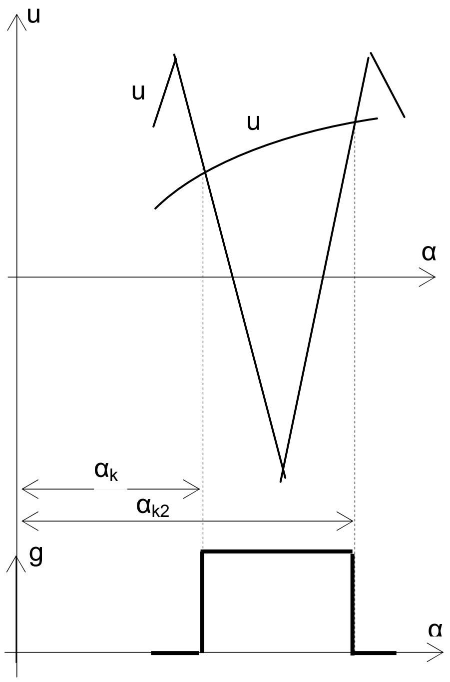

Fig.2. Principle of sine-delta PWM natural modulation.

Let us now introduce the following notations: $R = A/A_0 \le 1\,,\ \ \alpha = \omega\, t,\ \ f_0\, t = p\alpha/(2\pi)\,,$
R - denotes the modulation index (R=1 for f=50 Hz).

The mathematical model of the sine-delta PWM natural modulation for the first phase is represented by the equation:

$$u_1 = u_0\,,\ \text{meaning}$$

$$R\sin\alpha_{k1}^{(1)} = -2p\,\alpha_{k1}^{(1)}/\pi + 2(2k-1)\,, \tag{4}$$

where $\alpha_{k1}^{(1)}$ is the angle at which the command impulse with the number k starts;

$$R\sin\alpha_{k2}^{(1)} = 2p\,\alpha_{k2}^{(1)}/\pi - 4k\,, \tag{5}$$

where $\alpha_{k2}^{(1)}$ is the angle at which the command impulse with the number k ends;
or in just one equation:

$$R\sin\alpha_{kj}^{(1)} = (-1)^j 2p\,\alpha_{kj}^{(1)}/\pi + (-1)^{j+1}4k + 2(j-2)\,, \tag{6}$$

with j=1, 2.

The way to obtain the command impulse for the conduction of the thyristor T_1 is presented in figure 2.

For 2nd and 3rd phases the equations are:

$$R\sin(\alpha_{kj}^{(2)} - 2\pi/3) = (-1)^j 2p\,\alpha_{kj}^{(2)}/\pi + (-1)^{j+1} 4k + 2(j-2)\,, \tag{7}$$

$$R\sin(\alpha_{kj}^{(3)} - 4\pi/3) = (-1)^j 2p\,\alpha_{kj}^{(3)}/\pi + (-1)^{j+1} 4k + 2(j-2)\,. \tag{8}$$

The mathematical model of the sine-delta PWM natural modulation for the three phases is represented by the transcendental equation [4, 5]:

$$R\sin(\alpha_{kj}^{(i)} - 2(i-1)\pi/3) = (-1)^j 2p\,\alpha_{kj}^{(i)}/\pi + (-1)^{j+1} 4k + 2(j-2)\,, \tag{9}$$

where: i =1, 2, 3 – the phase number; k =1, 2,…, p–the number of command impulses ; j = 1, 2 – the start and end index of the command impulse ; $p = f_0/f$ - the ratio of the frequencies of signals.

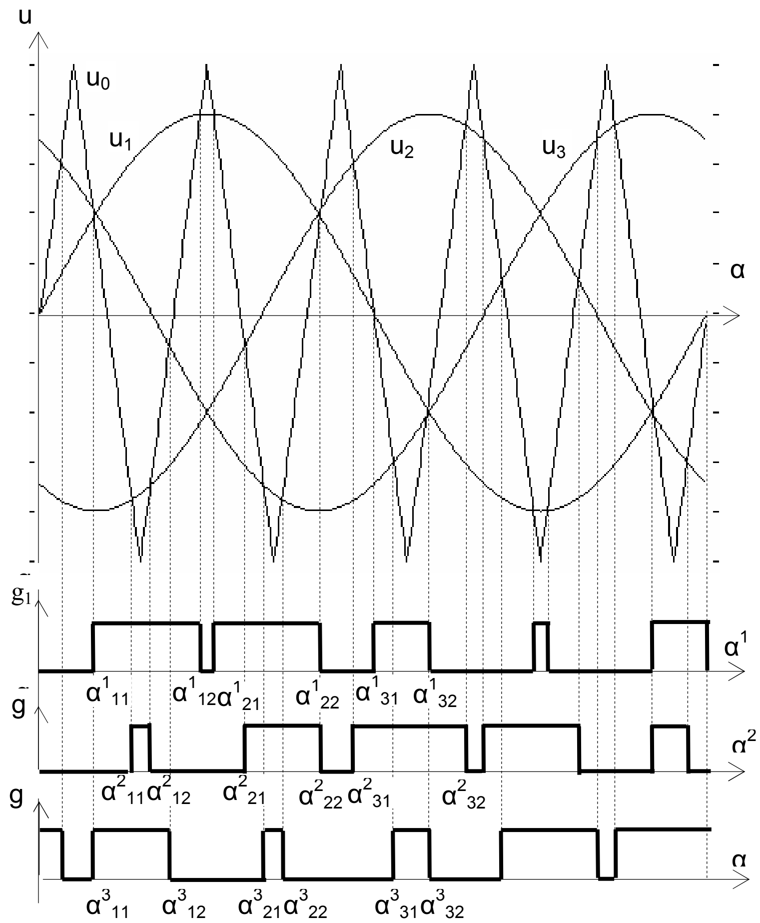

Fig. 3. The impulses of sine-delta PWM natural modulation

The angles of the command impulses are not defined by a mathematical function as the solution of the transcendental equation; they can be calculated numerically via non-linear numerical analysis methods that are implemented in application programs of a programming environment (for example: toolbox Matlab [2]).

Figure 3 presents the modulated command impulses g_1, g_3, g_5 for conduction of the thyristors T_1, T_3, T_5 in the case of the sine-delta PWM natural modulation, obtained by the comparison of the sinusoidal three-phase signal with delta signal of 5 times higher frequency; it can be seen the width modulation of the command impulses, i.e. the duration modulation of the conduction of thyristors. During the blocking of the thyristors T_1, T_3, T_5 the thyristors T_4, T_6, T_2 are conducting. Note that the impulses are neither equidistant, nor uniformly distributed in a period 2π [1].

The widths of the command impulses of the three phases can be determined from the transcendental equation:

$$\Delta\alpha_k^{(i)} = \alpha_{k2}^{(i)} - \alpha_{k1}^{(i)} = \pi/(2p)\left[R\sin(\alpha_{k1}^{(i)} - 2(i-1)\pi/3) + R\sin(\alpha_{k2}^{(i)} - 2(i-1)\pi/3) + 2\right], \quad (10)$$

where: i =1, 2, 3 – is the phase number; k =1, 2,…, p – is the number of command impulses.

The durations of the command impulses can be determined from the transcendental equation:

$$\Delta t_k^{(i)} = t_{k2}^{(i)} - t_{k1}^{(i)} = T_0/4\left[R\sin(\alpha_{k1}^{(i)} - 2(i-1)\pi/3) + R\sin(\alpha_{k2}^{(i)} - 2(i-1)\pi/3) + 2\right], \quad (11)$$

where: i =1, 2, 3 – is the phase number; k =1, 2,…, p – is the number of command impulses.

In conclusion, in the case of the natural modulation the angles and the widths of the command impulses can not be calculated directly in order to be implemented in the software of the command system with microprocessor (or with microcontroller) of the three-phase inverter, because the computing capacity of the microprocessor for the real time generation of the sine-delta PWM is exceeded.

Mathematical support of sine-delta PWM sampling modulation

Consider a mathematical support (mathematical model) defined by a simple mathematical function to be used for the calculation of the angles and of the widths of the command impulses and which could approximate as best as possible the angles and of the widths of the natural modulation [4].

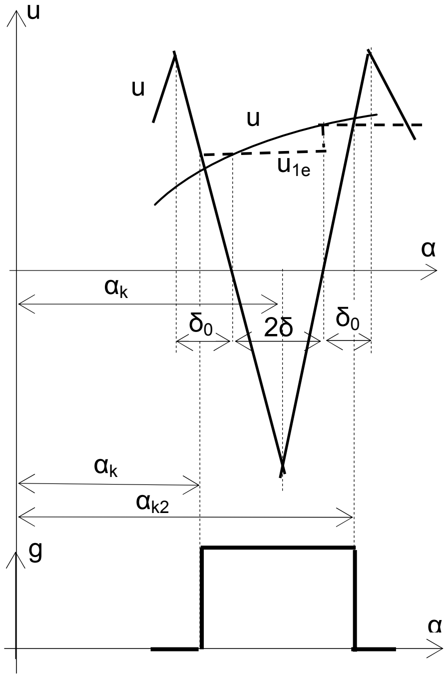

Fig.4. Principle of sine-delta PWM sampling modulation

Figure 4 presents the method to obtain the width modulated command impulse for the conduction of the thyristor T1. The sinusoidal command signal u_1 is sampled at the zeros of the delta reference signal and the curve u_{1e} is obtained, showing the command signal resulted by sampling. The comparison between the signal u_{1e} and the delta u_0 defines the intersection points of the waves, i.e. the angles of the command impulses. This is the principle of the sine-delta PWM sampling modulation, where the angles of the command impulses are defined by a mathematical function and can be directly calculated [1,4].

Let us now introduce the following notations: $\alpha_k = (4k-1)\delta_0$, $\delta_0 = \omega T_0/4 = \pi/(2p)$, k =1, 2,…, p – is the number of command impulses.

The mathematical model of the sine-delta PWM sampling modulation for the first phase is represented by the equation:

$$u_{1e} = u_0, \quad (12)$$

where u_{1e} is the sinusoidal signal sampled at the moments $(\alpha_k \pm \delta_0)$, meaning

$$u_{1e} = u_1(\alpha_k - \delta_0) = R\sin(\alpha_k - \delta_0) = -2p\,\alpha_{k1}^{(1)}/\pi + 2(2k-1), \quad (13)$$

where $\alpha_{k1}^{(1)}$ is the angle at which the command impulse with the number *k* starts;

$$u_{1e} = u_1(\alpha_k + \delta_0) = R\sin(\alpha_k + \delta_0) = 2p\,\alpha_{k2}^{(1)}/\pi - 4k\ . \quad (14)$$

where $\alpha_{k2}^{(1)}$ is the angle at which the command impulse with the number *k* ends.
The angles of the command impulses are given by the functions:

$$\alpha_{k1}^{(1)} = \alpha_k - \delta_0\left[1 + R\sin(\alpha_k - \delta_0)\right], \quad (15)$$

$$\alpha_{k2}^{(1)} = \alpha_k + \delta_0\left[1 + R\sin(\alpha_k + \delta_0)\right], \quad (16)$$

or in just one equation:

$$\alpha_{kj}^{(1)} = \alpha_k + (-1)^j\,\delta_0\left[1 + R\sin(\alpha_k + (-1)^j\,\delta_0)\right], \text{ with } j = 1, 2. \quad (17)$$

For 2nd and 3rd phases the functions are:

$$\alpha_{kj}^{(2)} = \alpha_k + (-1)^j\,\delta_0\left[1 + R\sin(\alpha_k + (-1)^j\,\delta_0 - 2\pi/3)\right], \quad (18)$$

$$\alpha_{kj}^{(3)} = \alpha_k + (-1)^j\,\delta_0\left[1 + R\sin(\alpha_k + (-1)^j\,\delta_0 - 4\pi/3)\right]. \quad (19)$$

The mathematical model of the sine-delta PWM sampling modulation for the three phases is defined by the function:

$$\alpha_{kj}^{(i)} = \alpha_k + (-1)^j\,\delta_0\left[1 + R\sin(\alpha_k + (-1)^j\,\delta_0 - 2(i-1)\pi/3)\right], \quad (20)$$

where: i =1, 2, 3 – is the phase number; k =1, 2,…, p – is the number of command impulses ; j = 1, 2 – is the start and end index of the command impulse ; $p = f_0/f$ the ratio of the frequencies of the delta and of the sinusoidal signals.
The widths and the durations of the command impulses can be determined by the functions:

$$\Delta\alpha_k^{(i)} = \delta_0\left[2 + R\sin(\alpha_k - \delta_0 - 2(i-1)\pi/3) + R\sin(\alpha_k + \delta_0 - 2(i-1)\pi/3)\right]. \quad (21)$$

$$\Delta t_k^{(i)} = T_0/4\left[2 + R\sin(\alpha_k - \delta_0 - 2(i-1)\pi/3) + R\sin(\alpha_k + \delta_0 - 2(i-1)\pi/3)\right]. \quad (22)$$

The angles of the command impulses are predetermined in terms of position and of width and can be directly generated in real time by the command system of the microprocessor of the three-phase inverter. Note that the command impulses are equidistant and uniformly distributed in a period 2π [1].

Numerical results of the simulation

The program package Matlab [2] has been used for the comparative analysis of the numerical results of the simulation of the mathematical models of the natural modulation and of the sampling modulation. The simulation program can be divided into 3 sections.

- Input of data

Input data are: R - modulation index; p - the ratio of the frequencies of the delta signal and of the sinusoidal signal; E - direct voltage of the intermediate circuit; f - frequency of the output voltage. For the numerical results there have been chosen: R =0,8, p =9, E = 100 V, f = 50 Hz.

- Calculation of the angles of the command impulses

For solving the transcendental equation of the natural modulation there has been used the 'fzero' function (for computation of the zeros of a nonlinear function). For the calculation of the angles of the command impulses of the sampling modulation the function (20) has been used. There are calculated the percentage errors between the values of the angles of the command impulses obtained by simulation in the case of the two modulation methods:

$$\varepsilon_{kj}^{(1)} = 100\left|(\alpha_{kj_{nat}}^{(1)} - \alpha_{kj_{smp}}^{(1)}) / \alpha_{kj_{nat}}^{(1)}\right| [\%]. \qquad (22)$$

where: $\alpha_{kj_{nat}}^{(1)}$ – the angles of the command impulses of the 1st phase for the natural modulation; $\alpha_{kj_{smp}}^{(1)}$ - the angles of the command impulses of the 1st phase for the sampling modulation.

The numerical results of the simulation are presented in table 1.

From the comparative analysis of the numerical results it can be seen that the errors are lower than 1,82 %, which proves that the mathematical support of the sampling modulation is feasible in the sine-delta PWM command strategy of the three-phase inverter.

Table 1. The numerical results of simulation

Natural modulation				**Sampling modulation**				**Error**			
Command angle		Impulse	Impulse	Command angle		Impulse	Impulse	$\varepsilon_{k1}^{(1)}$		$\varepsilon_{k2}^{(1)}$	
$\alpha_{k1}^{(1)}$	$\alpha_{k2}^{(1)}$	width	duration	$\alpha_{k1}^{(1)}$	$\alpha_{k2}^{(1)}$	width	duration				
[rad]	[rad]	[rad]	[ms]	[rad]	[rad]	[rad]	[ms]	[%]	[^{0}]	[%]	[^{0}]
0.3069	0.7981	0.4912	1.5636	0.3013	0.7879	0.4866	1.5488	1.8167	1.2815	0.3194	0.5860
0.9349	1.5358	0.6009	1.9128	0.9263	1.5338	0.6075	1.9337	0.9186	0.1326	0.4920	0.1166
1.6058	2.2067	0.6009	1.9128	1.6078	2.2153	0.6075	1.9337	0.1268	0.3892	0.1166	0.4920
2.3435	2.8347	0.4912	1.5636	2.3537	2.8403	0.4866	1.5488	0.4364	0.1967	0.5860	0.3194
3.1416	3.4485	0.3069	0.9768	3.1416	3.4429	0.3013	0.9591	0.0000	0.1617	.00000	0.3194
3.9397	4.0765	0.1368	0.4353	3.9295	4.0679	0.1384	0.4405	0.2596	0.2107	0.5860	0.4920
4.6774	4.7474	0.0700	0.2228	4.6754	4.7494	0.0741	0.2357	0.0435	0.0429	0.1166	0.1166
5.3483	5.4851	0.1368	0.4353	5.3569	5.4953	0.1384	0.4405	0.1606	0.1865	0.4920	0.5860
5.9763	6.2832	0.3069	0.9768	5.9819	6.2832	0.3013	0.9591	0.0933	.00000	0.3194	.00000

° Graphical representation of the command impulses

Figure 5 presents the command impulses of the conduction of thyristors T_1, T_3, T_5 for the sine-delta PWM sampling modulation.

Conclusions

This paper presents two mathematical models of the sine-delta PWM control strategy of the three-phase inverter, based on these models there are determined the modulated angles and widths of the command impulses of the thyristors.

In the case of the natural modulation the mathematical model is a transcendental equation which can not be implemented in the software of the real-time command system of the inverter.

In the case of the sampling modulation the mathematical model is a simple mathematical function with which the angles of the command impulses can be directly calculated. Because this model generates impulses with a good approximation compared to the model of the natural modulation, it can be implemented in the software of the command system with microprocessor of the three-phase inverter.

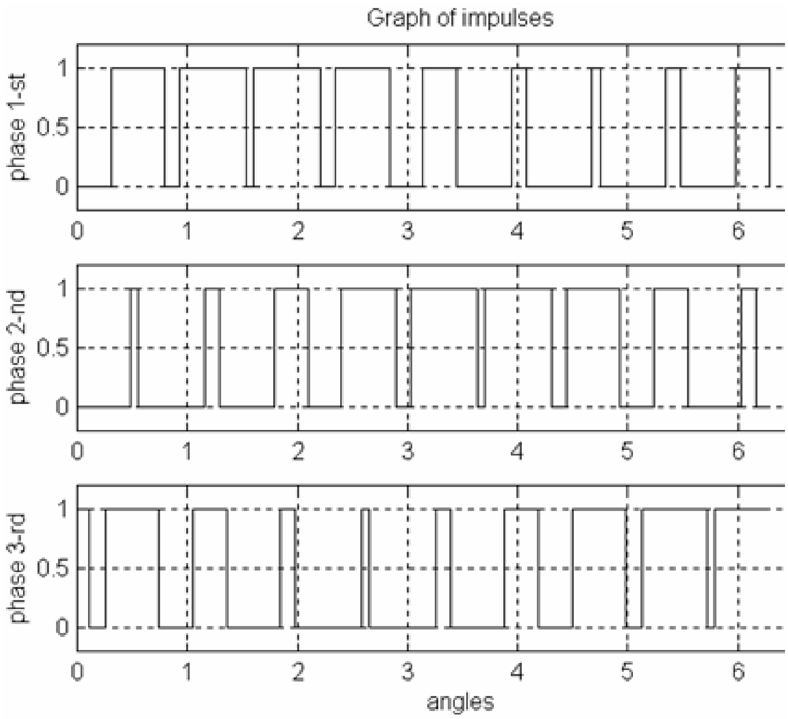

Fig. 5. The waveform of command impulses obtained by simulation

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Suportul matematic al strategiei de comanda PWM sine-delta a invertorului trifazat

Rezumat

Această lucrare se ocupa cu modelarea strategiei de comanda PWM sine-delta a invertorului trifazat. Lucrarea prezintă modelul matematic, reprezentând intersecția funcțiilor sinus si delta, al programului de comutație a dispozitivelor comutatoare de putere ale punții trifazate; acesta este o ecuație neliniară care nu are o soluție exactă reprezentata printr-o funcție matematică.
Pentru generarea strategiei de comanda PWM sine-delta prin software este propus un suport matematic pentru o soluție aproximata. Simularea suportului matematic si rezultatele au fost realizate utilizând un instrument de calcul specific, cum ar fi pachetul de programe Matlab, apoi rezultatele pot fi memorate in tabele de căutare care ar putea fi implementate pe structura hardware a invertorului. Rezultatele simulării sunt foarte apropiate de rezultatele teoretice; ele confirma validitatea suportului matematic propus.