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The Steady-State Modeling and Simulation of a Heat Exchanger

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Abstract

To study the performances of the temperature control systems, the authors have elaborated a mathematical model of the heat exchanger. The process is represented by a multivariable system. The authors have developed a numerical model of steady-state heat transfer in a shell and tube heat exchanger. It has been developed a computer program for solving the model equations. Using the comparison between the experimental data and the numerical results, the authors have validated the proposed model.

Key words: *multivariable system, heat exchanger, mathematical modeling, numerical method.*

Introduction

The temperature control system, which uses the heat exchanger, represents an uncomfortable problem. Though the temperature control structure is very simple, the performances of the temperature control are not good [6]. The study of these performances of the temperature control system is possible if the mathematical model of the exchanger is known.

In this context, the mathematical modeling of the heat transfer process represents an important problem, as for both design and operation stage of the chemical and petrochemical plants. At this moment, globally are available program systems for chemical processes simulation, including heat exchangers [4, 5]. These simulation media treats global function of the heat exchanger, with focusing on sizing the heat transfer area and less on analysis of functioning designed exchangers.

For solving the temperature control problem, the authors have investigated the possibility of the modeling and simulation of the heat operation exchanger already designed, to verify the control systems that have a heat exchanger in the process structure [3].

Heat exchangers structure

In figure 1 is presented a shell and tube heat exchanger having fluxes in contra flow. The heat exchangers are multivariable systems, characterized by four inlet and two outlet variables.

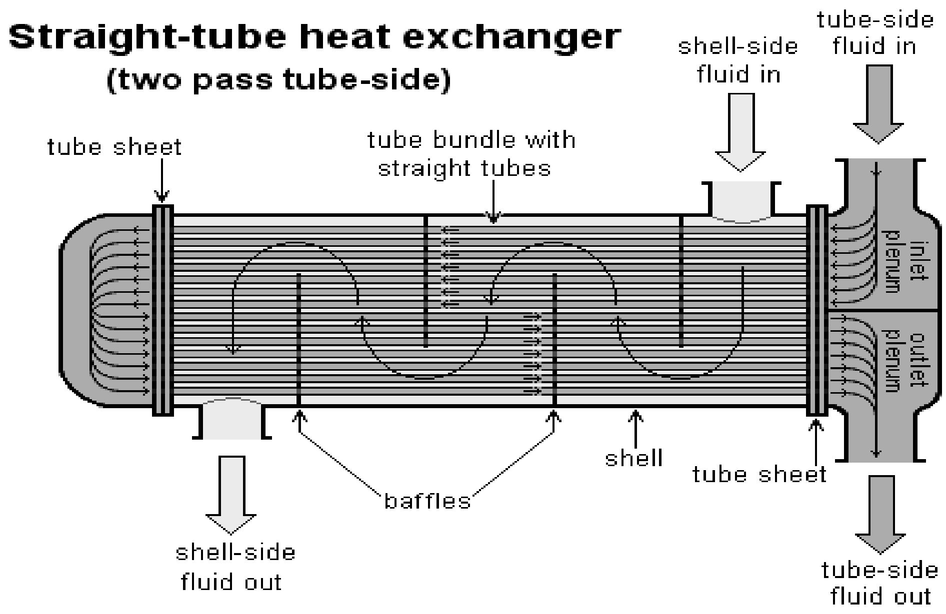

Fig. 1. Heat exchanger with bundle of tubes in shell.

The inlet variables are: temperature and flow rate of warm fluid entering the exchanger, t_{h1} and G_{hot}, and temperature and flow rate of cold fluid entering the exchanger, t_{c1} *and* G_{cold}. A bloc diagram of the heat exchanger is presented in figure 2. The outlet variables are: temperatures of hot and cold fluid at the heat exchanger output, t_{h2} and t_{c2}.

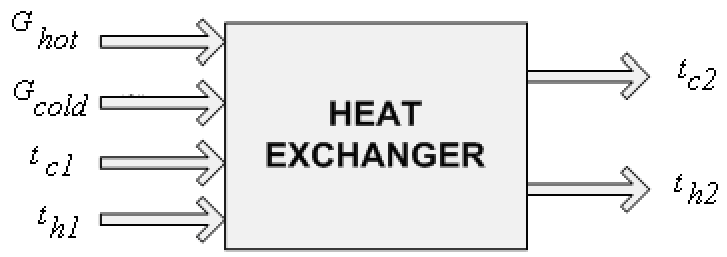

Fig. 2. Block scheme of the heat exchanger seen as multivariable system.

The treatment of tubular beam heat exchanger as multivariable system requires modeling and simulation in steady state and dynamic state of process. So far, the authors have developed and tested a mathematical model for steady state. Mathematical modeling includes the following steps:

a) mathematical modeling of heat transfer inside tubes;
b) mathematical modeling of heat transfer in the shell;
c) mathematical modeling global of heat transfer in the exchanger;
d) mathematical model of the exchanger.

Mathematical modeling of heat transfer inside tubes

Mathematical modeling of heat transfer inside tubes is represented by convection coefficient inside tubes [1].

$$\alpha_i = \frac{\lambda_r}{d_i} \cdot Re^{0,8} \cdot Pr^{1/3} . \tag{1}$$

Helpful relations are presented in table 1.

Table 1. Helpful relations used for mathematic modeling inside tubes [1]

Signification	Calculation formula	Relation
Reynolds criteria	$Re = \frac{d_i \cdot \omega_r \cdot \rho_r}{\mu_r}$	(2)
Cold fluid velocity in tubes	$\omega_r = \frac{m_r \cdot 4 \cdot N_p}{\rho_r \cdot \pi \cdot d_i^2 \cdot n_t \cdot 3600}$	(3)
Prandtl criteria	$Pr = \frac{c_p \cdot \mu_r}{\lambda_r}$	(4)

Mathematic modeling of heat transfer in the shell

Mathematical modeling of heat transfer in the shell is materialized though convection coefficient outside de tubes, the calculation formula used being

$$\alpha_e = \frac{\lambda_c}{d_e} \cdot 0,285 \cdot C_1 \cdot C_2 \cdot C_3 \cdot Re^{0,629} \cdot Pr^{1/3} . \tag{5}$$

The computation relations needed in relation (5) are presented in table 2.

Global mathematic modeling of heat transfer in the exchanger

The global model of heat transfer in the heat exchanger is based on the overall coefficient of heat transfer for heat exchanger without deposits determination

$$k_e = \frac{1}{\frac{1}{\alpha_i} \cdot \frac{d_e}{d_i} + \frac{1}{\alpha_e}} , \tag{19}$$

where α_i and α_e represents the convection heat transfer coefficient inside and outside of the tubes. Relations used for calculating the correction convection coefficients are presented in table 3.

Mathematic model of the heat exchanger

Mathematical model of heat exchanger is obtained by heat balance equations associated with the two material flows, the hot flow and the cold flow [1]

$$\begin{cases} G_{hot}\, c_{p,hot}(t_{h1} - t_{h2}) = G_{cold}\, c_{p,cold}(t_{c2} - t_{c1}) \\ G_{hot}\, c_{p,hot}(t_{h1} - t_{h2}) = k_e\, A\, \dfrac{(t_{h1} - t_{c2}) - (t_{h2} - t_{c1})}{ln \dfrac{t_{h1} - t_{c2}}{t_{h2} - t_{c1}}} \end{cases} . \tag{24}$$

Table 2. Helpful relations used for mathematic modeling in the heat exchanger shell [1]

Signification	Calculation formula	Relation
Correction factor C_1	$C_1 = z + 0{,}524 \cdot (1-z)^{0{,}32} \cdot \left(\frac{S}{A_{lf}}\right)^{0{,}03}$	(6)
Flow area	$S = x \cdot \left[D_i - D_f + \frac{D_f - d_e}{s} \cdot (s - d_e)\right]$	(7)
Free area of the window	$A_{lf} = A_f - n_f \cdot \frac{3{,}14 \cdot d_e^2}{4}$	(8)
Total area of the window	$A_f = 0{,}11182 \cdot D_i^2$	(9)
Number of tubes in window	$n_f = n_t \cdot \frac{1-z}{2}$	(10)
Correction factor C_2	$C_2 = f\left(\frac{a_{sm} + a_{to}}{S}; \frac{a_{sm}}{a_{sm} + a_{to}}\right)$	(11)
Flow area between tubes and holes	$a_{to} = 0{,}3927 \cdot (d_o^2 - d_e^2)(1+z) \cdot n_t$	(12)
Flow are between baffles and shell	$a_{sm} = \frac{3{,}14}{4} \cdot (D_i^2 - D_s^2) \cdot \frac{360 - \varphi}{360}$	(13)
Correction factor C_3	$C_3 = \exp\left\{-1{,}25 \cdot \frac{(D_i - D_f) \cdot x}{S} \cdot \left[1 - \left(\frac{2 \cdot N_{sl}}{N_{if}}\right)^{1/3}\right]\right\}$	(14)
String number of tubes placed between edges of windows	$N_{if} = \frac{D_i \cdot \left(\frac{2 \cdot h}{D_i} - 1\right)}{s}$	(15)
Reynolds criteria	$Re = \frac{d_e \cdot \omega_c \cdot \rho_c}{\mu_c}$	(16)
Hot fluid velocity in the shell	$\omega_c = \frac{m_c}{\rho_c \cdot S \cdot 3600}$	(17)
Prandtl criteria	$Pr = \frac{c_p \cdot \mu}{\lambda}$	(18)

From mathematical point of view, the system (24) is a system of two nonlinear equations with two unknowns

$$\begin{cases} f_1(t_{h2}, t_{c2}) = 0 \\ f_2(t_{h2}, t_{c2}) = 0 \end{cases}, \tag{25}$$

where these unknowns are hot fluid temperature t_{h2} and cold fluid temperature t_{c2}, both temperatures being associated with the outlet from heat exchanger.

Table 3. Helpful relations used for mathematic modeling of global heat transfer [1]

Signification	Calculation formula	Relation
Corrected convection coefficient inside tubes	$\alpha_{if} = \alpha_i \cdot \left(\frac{\mu_p}{\mu}\right)^{0,14}$	(20)
Corrected convection coefficient outside tubes	$\alpha_{ef} = \alpha_e \cdot \left(\frac{\mu_p}{\mu}\right)^{0,14}$	(21)
The temperature difference outside the tubes	$\Delta t_e = \frac{k_e \cdot (t_c - t_r)}{\alpha_{e_f}}$	(22)
Wall temperature	$t_p = t_c - \Delta t_e$	(23)

Solving of the mathematical model

The system of nonlinear equations can be solved using Newton-Raphson algorithm. The system Jacobian contains the following expressions for partial derivatives:

$$\frac{\partial f_1}{\partial t_{h2}} = -G_{hot}\, c_{p,hot} ; \qquad (26)$$

$$\frac{\partial f_1}{\partial t_{c2}} = -G_{cold}\, c_{p,cold} ; \qquad (27)$$

$$\frac{\partial f_2}{\partial t_{h2}} = -G_{hot}\, c_{p,hot} - k_e\, A \frac{-ln\frac{t_{h1}-t_{c2}}{t_{h2}-t_{c1}} - (t_{h1}-t_{h2}+t_{c1}-t_{c2})\,\frac{1}{t_{h2}-t_{c1}}}{\left(ln\frac{t_{h1}-t_{c2}}{t_{h2}-t_{c1}}\right)^2} ; \qquad (28)$$

$$\frac{\partial f_2}{\partial t_{c2}} = -k_e\, A \frac{ln\frac{t_{h1}-t_{c2}}{t_{h2}-t_{c1}} - (t_{h1}-t_{h2}+t_{c1}-t_{c2})\,\frac{1}{t_{h1}-t_{c2}}}{\left(ln\frac{t_{h1}-t_{r2}}{t_{h2}-t_{r1}}\right)^2} . \qquad (29)$$

The authors have developed a computer program that solves the mathematical model of a heat exchanger, characterized by the following constructive-functional elements: the constructive data of the exchanger, physical and operating data of the hot fluid, physical and operating data of the cold fluid [1]. By using the computer program, there have been calculated the output variables of the multivariable system, respectively the output temperatures of the hot and cold fluid of the exchanger. The solution of the nonlinear system (24) has been obtained using the Newton-Raphson algorithm in 7 iterations. For the following input data: the input temperature of hot fluid $t_{h1} = 180°C$, the flow rate of hot fluid $G_{hot} = 1.63\times10^5 kg/h$, the input temperature of $t_{c1} = 103°C$, the flow rate of entering cold fluid $G_{cold} = 5\times10^5 kg/h$, the solution of the mathematical model of the heat exchanger is the following: the hot fluid temperature $t_{h2} = 137.04$ and the cold fluid temperature $t_{c2} = 119.27$ °C. These values obtained by numerical solving of the model equations are similar with the published data [1]. The small errors between the calculated values of the output temperatures and the published values have validated the mathematical model of the heat exchanger and the simulation program developed by the authors. A complete simulation of the multivariable heat exchanger will be made in a next paper.

Conclusions

The authors have developed an original mathematical model for the steady-state heat exchanger process. The model will by used to design a temperature control system with well performances. The obtained multivariable model is a nonlinear algebraic equation system. The computer program developed by the authors has permitted the calculation the output variables of the heat exchanger. The numerical results have validated the model.

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Modelarea şi simularea în regim staţionar a unui schimbător de căldură

Rezumat

În vederea studierii performanţelor sistemelor automate de reglare a temperaturii, autorii au elaborat un model matematic al unui schimbător de căldură. Procesul este un sistem multivariabil cu patru variabile de intrare şi două de ieşire. A fost modelat procesul în regim staţionar şi a fost elaborat un program de simulare a procesului, pe baza modelului matematic propus. Modelul matematic elaborat a fost validat prin compararea rezultatelor numerice obţinute cu date experimentale din literatura de specialitate.