| BULETINUL Universităţii Petrol – Gaze din Ploieşti | Vol. LXII No. 4B/2010 | 154 - 160 | Seria Tehnică |
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# The Bifurcation Diagram for Oxygen Detector's Chaotic Regime

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## Abstract

*In the case of small modifications in initial conditions if we have large modifications for a complex system evolution, the system behavior is chaotic. For the precision oxygen analyzer exists characteristic parameters for those the measurements are sensitive. In this paper, we analyze the existence of such parameters and the chaotic behavior (i.e. stability conditions) of the oxygen analyzer function of the characteristic constants modifications. To point out this behavior the bifurcation diagram, in dynamic regime, must be built.*

**Key words:** *oxygen analyzer, chaotic behavior, bifurcation diagram, Lyapunov exponent***.**

## Introduction

In order to make the precision measures it is important the studying of the system's stability depending on the oscillation of the external factors which can influence the measures. These factors can be of different types and the most important ones, in the case of the oxygen analyzer are: the distance of the centre of gravity of dump – bell from the axis of suspension, the magnetic field strength, the net magnetic susceptibility of the dumb – bell coil assembly, the volume of the spheres of the dumb – bell, the restoring torque of the suspension strip, the tension of the suspension spring, mechanical pressure on the dumb-bell caused by circulation of gases around, the combined effect of temperature etc.

In the following we are going to study the influence of the temperature and the intensity of the magnetic field towards the stability of the made measures.

## Elements of Physics of Chaos

The chaos is a disoriented system and uncontrollable (the sensitive dependence for the initial conditions) which means a little change in the status of a chaotic system that can rapidly lead to big changes in the status of the system, measured at a later moment, even if the status of the system is completely dependent on the initial conditions.

The current definition of the chaos implies at least four measures, from which two most important are the Lyapunov exponent and topological entropies. An other way to explore the chaotic behavior for a dynamical system can be made using the bifurcation diagram. By these measures we can define the determined chaos as being that status of a determined system which is characterized by the local instability (the Lyapunov exponent to be positive) and global mixing (the typological entropies to be positive). Here we have made a difference between the determined chaos and the real chaos (or the general chaos), the first one is completely determined trough the continue un linear differential equations, only their solutions are characterized, in certain conditions, by the complexity and disorder which, in other way, they appear chaotic.

Let’s consider a particle which is moving on a given trajectory by the vector of position $\vec{r} = \vec{r}(t)$ towards a chosen referential. If the system of the differential equations which describe the movement is nonlinear, then there are conditions for that particle to have a chaotic trajectory (determined chaos) and not a simple stable trajectory. We can quantitatively measure the „chaoticity” of a system? The answer is yes and the number which characterizes the chaotic character is the Lyapunov exponent. The introduction of this measure can be made by the studying of the system’s sensitivity with small modifications of the initial conditions. If as a response to small changes of the initial conditions results big effects than we say that the system is unstable to the initial conditions. If, on the other hand, there are no differences in the measures of the changes in the initial conditions the system tends to the same trajectory (attractor) or a set of trajectories (strange attractor), then we say that the system is stable. For the simplicity, we consider that the system may be characterized by a single co-ordinate $q = q(t)$ (generalized co-ordinate). At the initial moment, the particle’s position is characterized by $q_0 = q(t_0)$ which implies a trajectory $q = q(t, q_o)$. Let’s suppose that we are changing a little the initial conditions with a value $q = q(t, q_o)$, then we are expecting a changing of the trajectory so that at moment the difference between the two positions is:

$$\Delta q = q(t, q_0 + \Delta q_0) - q(t, q_0)\,. \tag{1}$$

The sensitivity towards the initial conditions, taking into consideration the distance is exponential in time, can be quantified in the expression:

$$|\Delta q| \cong \mathrm{e}^{\lambda t} |\Delta q_0| \tag{2}$$

where $\lambda$ represents the Lyapunov exponent. From the above expression we can define this exponent by the relation:

$$\lambda = \lim_{\substack{t\to\infty \\ \Delta q_0 \to 0}} \frac{1}{t} \frac{|\Delta q|}{|\Delta q_0|} \tag{3}$$

Depending on the values of $\lambda$ we distinguish the following three cases:

a) $\lambda < 0$, if the Lyapunov exponent is negative, then all the trajectories converges to a stable status (a fix point or a fixed eye socket); that value characterizes the neoconservative or dissipative systems which tend to get to the attractor’s system, while they are losing energy; the maximum value of the stability is succeeded when $\lambda < -\infty$ and these status are described by the super stable statuses.

b) $\lambda = 0$, this value characterizes the conservative systems. Two trajectories in the space’s stage will remain at the same distance in time (neither they get closer nor they get further). We say that the system is characterized by the Lyapunov stability.

c) $\lambda > 0$ in these conditions, the movement becomes unstable and chaotic. Two points, no matter how close they are, they will get distant – in fact in the space of stages a point will have the possibility to get in any neighboring point. Even if the system is deterministic, we cannot observe the regularities or order regarding the structure of the trajectories.

Still, we have to take into consideration that a Lyapunov positive exponent is not sufficient for a real chaos, which must also happen is the coming-back of the particle in a delimitated space for the superposed trajectories to produce, what we call, chaos. In this way, in the case of an $\lambda$ positive exponent may happen that the trajectories to get so distant that they leave the interest space and lead to the teasing of the system, there won't be observed the local instabilities. His "mixing" status of successive coming-backs of the particles in the local spaces characterized by instabilities, may be characterized by a different number, the typological entropy H.

## The Study of the Parameter's Dependence by Temperature

We are going to study two measures which depend on the temperature: the volume and the magnetic susceptibility. In the volume's case we are going to use the relation and in the susceptibility's case we are going to use the magnetic relation. Using the solutions of the systemic equation given in the paper (2) we can find the dependent relations of temperature:

$$n = \frac{-\chi_1 + \chi_{N_2}}{\chi_{N_2} - \chi_{O_2}}. \tag{4}$$

$$m = A\operatorname{cosec}(2\alpha)\sin\left(\frac{l\cos(\alpha)}{2a}\right)\sin\left(\frac{L\sin(\alpha)}{2a}\right)\sec\left(\frac{(2d+L)\sin(\alpha)}{2a}\right) \tag{5}$$

$$A = -\frac{8a^2\mu_0(\chi_1+1)}{H_0 v\left(\chi_{N_2} - \chi_{O_2}\right)} \tag{6}$$

Presuming that in these situations we have the dependence:

$$\chi_{O_2}\left(T\right) = \frac{C_{O_2}}{T[K]} = \frac{C_{O_2}}{t[°C]+273{,}15} \tag{7}$$

and

$$v = v_0\left(1+\alpha t[°C]\right) \tag{8}$$

With these variations the A constant becomes

$$A(t) = -\frac{8a^2\mu_0(\chi_1+1)}{H_0 v_0\left(1+\alpha(t-t_0)\right)\left(\chi_{N_2} - \chi_{O_2}\dfrac{t_0+T_0}{t+T_0}\right)} \tag{9}$$

Where we had noted with $T_0 = 273{,}15$, and the constant $C_{O_2}$ it had been determined from the interpolating curve proposed for the susceptibility's dependence of temperature.
The temperature's dependence of the concentration is represented in fig.1.

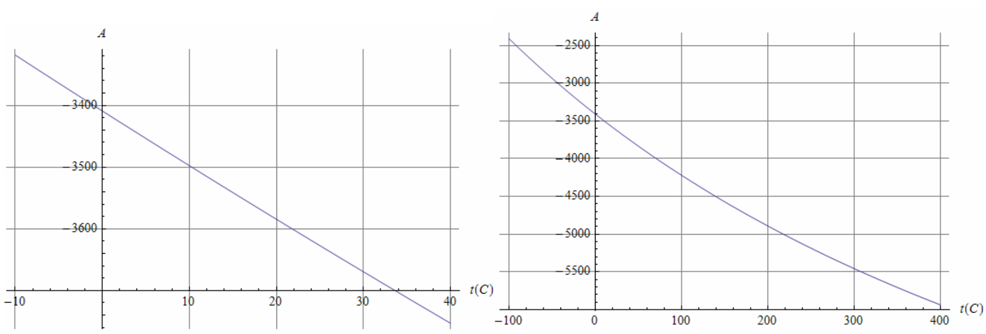



**Fig. 1.** The dependence of the A constant of temperature on the interval $-10 \div 40$ °C and $-100 \div 400$ °C

It can be observed that this dependence is not nonlinear, if the temperature interval will arise. The procedural variation of the constant on the temperature interval $-10 \div 40$ °C has also an approximate linear dependence, taking as example the room's temperature $t_0 = 20$ °C (fig. 2).

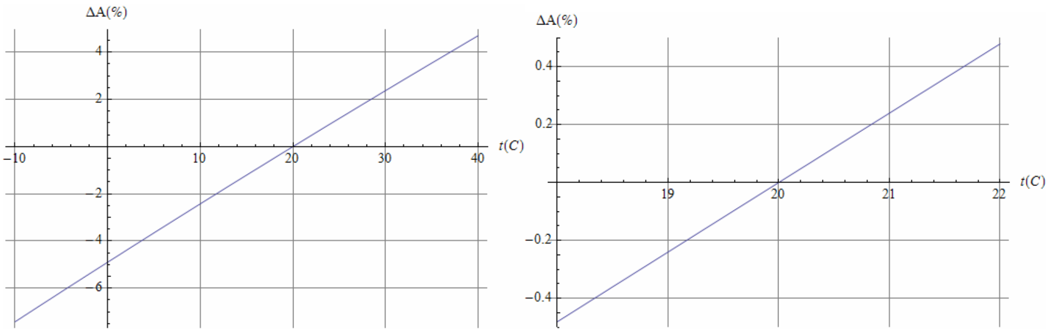



**Fig. 2.** The procedural variation of A constant on the interval $-10 \div 40$ °C and $18 \div 22$ °C

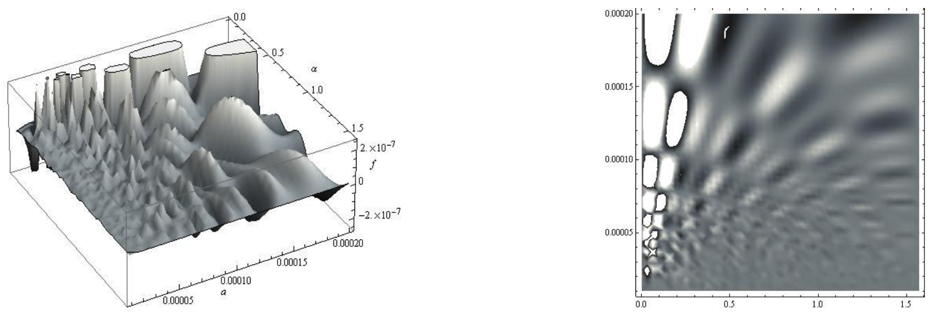
In the case in which the temperature's interval is reduced on the interval $18 \div 22$ °C, meaning variations of grades order, it can be observed also the constant variations which imply modifications in the calculated values for a concentration which depend on the intensity of the electric current. Yet, there cannot be observed the instability elements (the variations on small intervals of temperature being practically linear), from where we can draw the conclusion that the oxygen's precision analyzer is practically unstable (even if it introduces measurements errors, it hasn't got a chaotic regime) on the variations of temperature which influences the measure by the implicate variables in our model.

## The Shaping of Passing from the Stable Regime to the Chaotic Regime

The chaotic regime is obtained for certain values of the constants which characterizes the system trough the $F_{em}(\alpha)$ force.

$$F_{em}(\alpha) = \frac{C}{\sin(\alpha)} \sin\left(\frac{l\cos(\alpha)}{2a}\right) \sin\left(\frac{L\sin(\alpha)}{2a}\right) \sin\left(\frac{(2d+L)\sin(\alpha)}{2a}\right), \qquad (10)$$

where $C = 4aH_0\mu_0(\chi+1)I$ ( for the measure signification which intervene.



*a)* *b)*

**Fig. 3.** The passing from a stationary regime to a chaotic regime depending on the values of parameter *a*.

Taking into consideration the slow dependence of external factors of the variables which ivolved, we are going to study the dependence depending only on the distribution of the inhomogeneous magnetic field of a parameter. The passing from a stationary regime to a chaotic regime is gradual made, as it is shown in figure 4*a* and *b*, where the values of parameter $a$ and $\alpha$ indicated in the superior part of the figures. As it is shown in the above figures, simultaneously with the reduction of the parameter $a$, the fluctuations become more frequent at a rotation of some degrees, sufficient for the system to enter in a chaotic regime. For this reason, the values of a parameter must be chosen in such way that the some degrees variations to remain practically constant.

## The Bifurcation Diagram for Dynamic Regime

In case of oscillation of the oxygen analyzer under small oscillations (induced by small perturbations) the evolution of system, characterized by the torsion angle $\theta$, is described by the following equation of motion in approximation of small oscillations

$$I\ddot{\theta} = -C\dot{\theta} - k\sin(\theta) + M(\theta) \text{ or } I\ddot{\theta} = -C\dot{\theta} - k\theta + M(\theta) \tag{11}$$

In order to construct de bifurcation diagram the equation (11) must be reconsidered using an iteration method using the following notations: $y = \theta$ and $z = \dot{\theta} = \left(\dot{\theta}(t+h) - \dot{\theta}(t)\right)/h \cong y_{k+1} - y_k$.

The second derivative of $\theta$ can be expressed as $\ddot{\theta} = \dot{z} \cong z_{k+1} - z_k$. Using this approximations, eq. (11) can be expressed as a system of recurrence equations, in nonlinear case:

$$\begin{cases} I(z_{k+1} - z_k) = -C\,z - k\sin(y) + M(y) \\ y_{k+1} - y_k = z \end{cases} \tag{12}$$

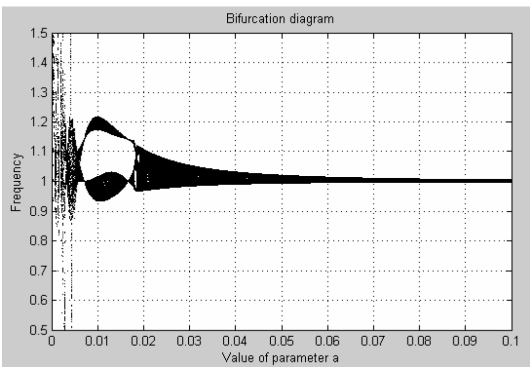
Using numerical values [2] equation (12) become

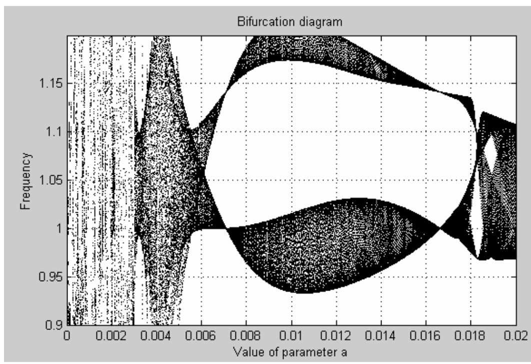
$$\begin{cases} z = \dfrac{a\,2.44174\cdot 10^{-6}}{\sin(y)10^{-7}} f(y,a) - 6\cdot 10^{-12}\sin(y) + z \\ y = z + y \end{cases} \tag{13}$$

where

$$f(y,a) = \sin\left(0.02\frac{\cos(y)}{a}\right)\sin\left(0.02\frac{\sin(y)}{a}\right)\sin\left(0.012\frac{\sin(y)}{a}\right). \tag{14}$$

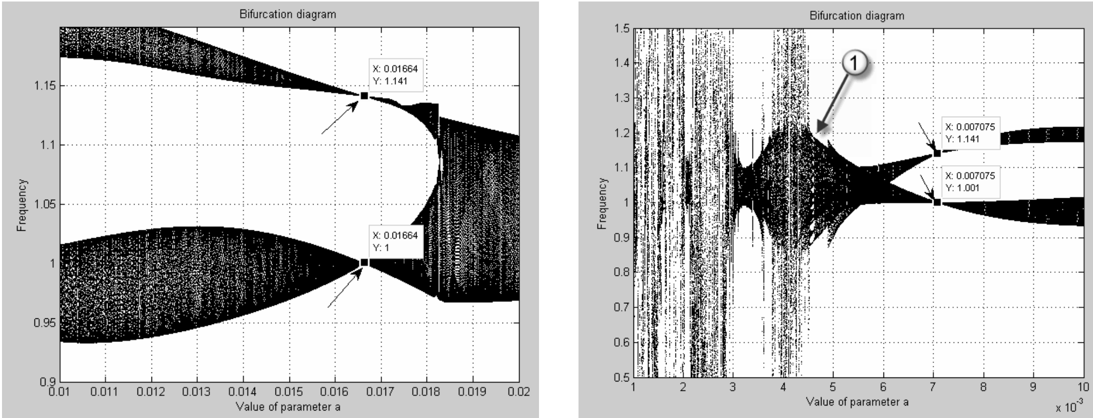
The solutions for eq. (13) ar represented on fig. 4, 5 and 6. The modification of parameter $a$ implies a non-regular bifurcation diagram fig. 4*a)* where the first major bifurcation appear at $a = 0.01664$ fig. 4*b)* and fig. *5a)*. The mixing mechanism imply a second bifurcation at $a = 0.007075$, see fig. 5*b)*.




*a)* *b)*

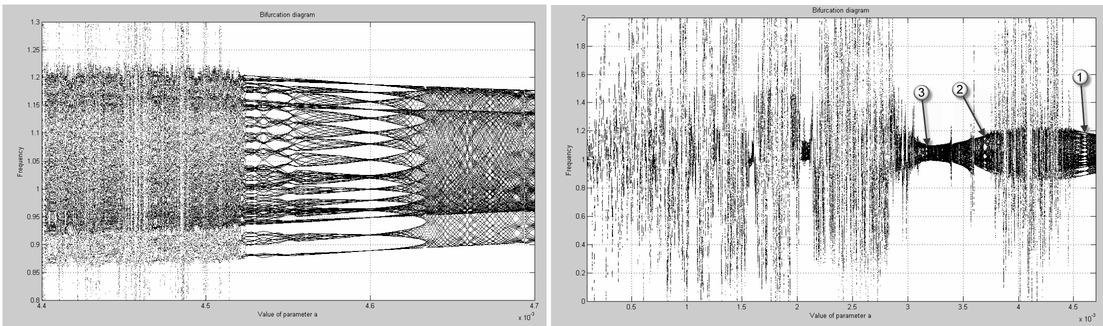
**Figure 4.** The bifurcation diagram *a)* expanded zoom level, *b)* zoom in level at first bifurcations




*a)* *b)*

**Fig. 5.** *a)* the first major bifurcation *b)* the second bifurcation an mixing characteristic ①

On fig. 5*b)* also can be observed a particular characteristic of general bifurcation diagrams, the collapse of chaotic regime into an approximate ordered motion (see zone ①) that is transient, i.e. with decrease of parameter *a* the system go into other chaotic regime of function. The transient zone ①, can be observed on clearly on fig. 6*a)*, with mixing multiple bifurcations.




*a)* *b)*

**Fig. 6.** *a)* the mixing characteristic of multiple bifurcation, ① from fig. 5*b)* an other zoom level, *b)* the chaotic regime of oxygen detector for small values of *a*, and other multiple bifurcation zones ①, ② and ③

On fig. 6*b)* are evidenced the chaotic zones for small values of $a$ $(a < 10^{-3})$. The critical chaotic zones are also evidenced on fig. 3.

## Conclusions

It has be high lightened the measures' dependence of certain external factors (the temperature and the intensity of the magnetic field) in two cases. In the first case, (the dependence depending on the temperature) the variations are approximate linear and in the second case (the dependence depending on the distribution of the magnetic field's intensity) they are non-linear and, at the emphasized variations, leading even to uncontrolled variations depending on the deviation's angle.

The bifurcation diagram is a more accurate method to separate the chaotic regimes from the stable (or semi – stable) regimes for nonlinear dynamic systems. In present investigation of the oxygen detector dynamics can be established a new kind of bifurcation diagrams the can named as "*multifurcation diagram*". This name is motivated because on diagrams appear mixing multilevel bifurcations, that may be represent a particular characteristic for oscillators in nonlinear magnetic fields.

It is obvious that the phenomenon must not happen in order not to increase the measurements imprecision but there are situations (see the stochastic resonance [3-5]) where the noise produced by a chaotic phenomenon may be used in order to increase the signal. In this way, there can be evaluated the situations in which, even if the device enters in a chaotic regime, the measures can be made by extraction of an ordered signal from the chaotic signal with the help of the stochastic resonance.

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# Diagrama de bifurcație pentru regimul haotic al unui analizor de oxigen

## Rezumat

*În cazul unor mici modificări ale condiților inițiale, dacă avem modificări mari în evoluția unui sistem complex, spunem că dinamica sistemului este haotică. Pentru analizorul de oxigen de precizie există parametri caracteristici pentru care măsurătorile efectuate sunt sensibile. În această lucrare, vom analiza un astfel de parametru și corelația sa cu comportamentul haotic (i.e. condițiile de stabilitate) prin modificarea constantelor caracteristice. Pentru a sublinia acest comportament vom construi diagrama bifurcație în regim dinamic.*