BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LXII No. 4B/2010	7 - 12	Seria Tehnică

Graphics of the Involute Profile for Gear Drawing

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Abstract

In this paper, the authors present a program written in AutoLISP, with the aim of drawing the involute profile. All the steps of a gear representation are presented. It is well known that the involute curve profile for gears has a wide use and, usually, its representation in graphics is approximated by different curves. Rarely, the involute profile is drawn by calculated points. After establishing the necessary formulae in order to obtain the mathematical expression of the involute, the authors present the computer program named „Involute", using AutoLISP, which calculates many points` coordinates connected with a polyline.

Key words: *involute curve, gear, AutoLISP.*

Theoretical Considerations

Gears` profile use cyclic curves for teeth profile, as: epicycloid, hypocycloid, orthocycloid and involute. The involute is a curve with a wide use, being a cyclic curve traced by a point belonging to a straight line which is rolling on a fixed circle called base circle.

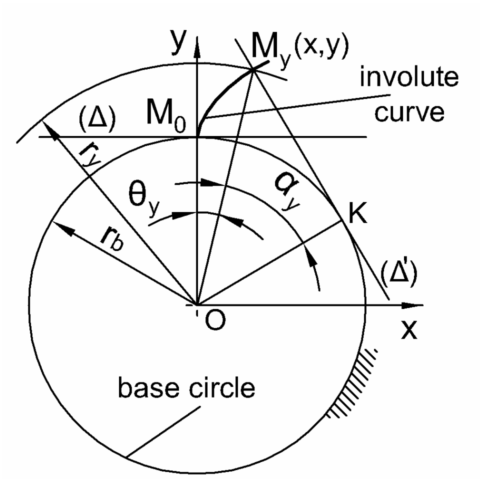


Fig. 1. Engendering the involute curve

First, there are established the formulae of the involute curve, using figure 1. In this figure, the following terms are used:

M_0- start point for generating the curve;
M_y- current point of the involute curve;

K – curvature centre of the involute;
(Δ)- straight line rolling on the base circle;
r_b- radius of the base circle;
α_y- current pressure angle.
After moving (Δ) to (Δ’) without sliding, we can write :

$$\widehat{M_0 K} = \overline{M_y K} \quad , \tag{1}$$

$$\widehat{M_0 K} = (\theta_y + \alpha_y) r_b ; \; \overline{M_y K} = r_b \cdot tg\alpha . \tag{2}$$

Replacing (2) in (1), one obtains (3):

$$\theta_y = tg\alpha_y - \alpha_y ; r_y = r_b / \cos\alpha_y , \tag{3}$$

where $\alpha_y[rad] = \alpha_y^0 \cdot \pi / 180^0$.

The terms from (3) are considered in the polar coordinate system. These formulae can be re-written in the Cartesian coordinate system, as it follows:

$$x_{My} = \frac{r_b}{\cos\alpha_y} \cdot \sin\theta_y ; \quad y_{My} = \frac{r_b}{\cos\alpha_y} \cdot \cos\theta_y ; (\theta_y = tg\alpha_y - \alpha_y) \; . \tag{4}$$

Drawing the Gears` Profile

Formulae (4) allow us to create the program named “*Involute*”, using AutoLISP. The program presented in figure 2 is written in Notepad with the extension *.lsp*. The program calculates a great number of points` coordinates which are connected with a polyline.

To use this program, the following steps must be completed:
-starting AutoCAD;
-setting up the highest precision for length and angle from *Format* → *Units* ;
-loading the program in AutoCAD as: *Tools* → *Load Application* → (*Look in* Desktop e.g , *File name* Involute.lsp) → *Load* → *Close*.

After loading the application, in *Command* line type ***Involute.***

The dialog which follows is:
Radius of the base circle: **56.381557**
Angular increment(degree): **0.01**
Number of points: **3000**

If one chooses a lower precision for the angular increment, then a longer curve can be obtained (e.g. *Angular increment*: **0.1**; *Number of points*: **500**).

On the screen an involute curve can be seen, drawn as it is presented in figure 3.a. The drawn gear has the next geometric parameters: number of teeth: $z = 20$; metric module: $m = 5$ mm ; base circle radius: $r_b = 56.381557$ mm ; outside circle radius (addendum): $r_a = 65$ mm ; root circle radius (dedendum): $r_f = 53.750$ mm ; pitch circle radius: $r = 60$ mm .

The steps for drawing gear’s profile are:
- drawing the base circle, r_b (see figure 3.b);
- rotating the involute curve with the angle $\Psi_b = 4.6039535^0$ (see figure 3.c);
- commanding MIRROR of the involute by a vertical line (see figure 3.d);

- drawing the outside circle r_a, the root circle r_f, the pitch circle r (see figure 3.e);
- drawing the fillet radius, ρ (see figure 3.f.);
- trimming the circle, in order to obtain only the tooth (see figure 3.g.);
- with command ARRAY, drawing the whole gear's teeth profile (see figure 3.h.).

The rotation angle on the base circle, $\Psi_b = 4.6039535^0$, was determined with:

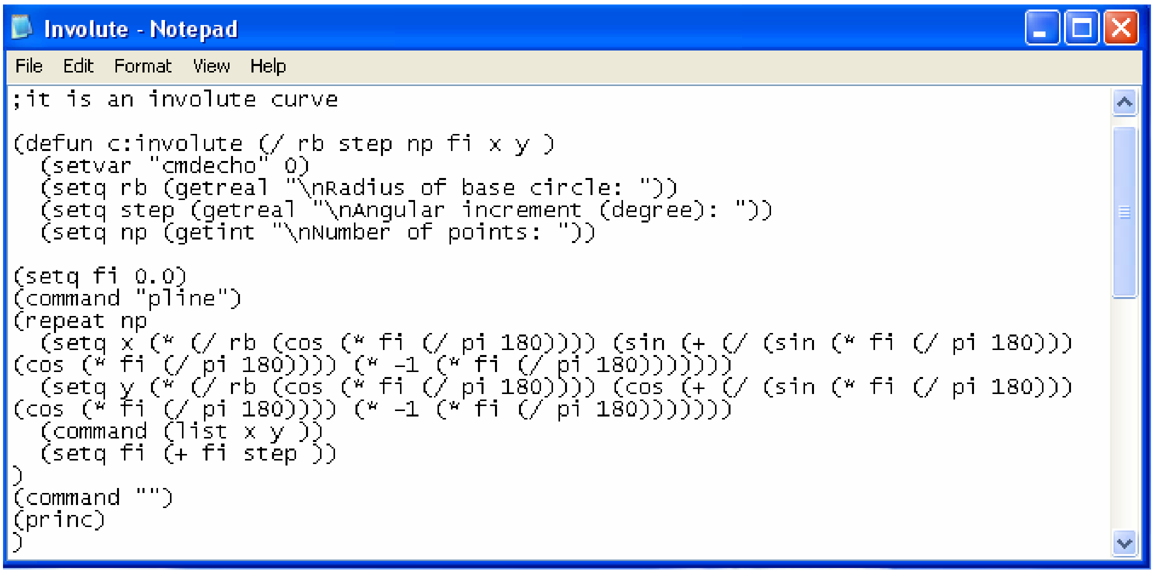
$$\Psi_b = \frac{s_b/2}{r_b} = \frac{9.0609838/2}{56.381557} = 0.0803541\,\text{rad} = 4.6039535^0 \quad . \tag{5}$$

where s_b is the tooth thickness on the base circle.

$$s_b = s\frac{\cos\alpha}{\cos\alpha_b} - 2r_b(\text{inv}\alpha_b - \text{inv}\alpha) = 7.8539816\cdot\frac{\cos 20^0}{\cos 0^0} - 2\cdot(0 - 0.01490438) = 9.0609838\,. \tag{6}$$

where s- tooth thickness on a pitch circle ($\text{inv}\alpha = \text{tg}\alpha - \alpha = \text{tg}20^0 - 20^0\cdot\pi/180^0 = 0.01490438$)

$$s = (0.5\cdot\pi + 2\cdot x\cdot\text{tg}\alpha)\cdot m = (0.5\cdot\pi + 2\cdot 0\cdot\text{tg}20^0) = 7.8539816\,. \tag{7}$$




```
;it is an involute curve

(defun c:involute (/ rb step np fi x y )
  (setvar "cmdecho" 0)
  (setq rb (getreal "\nRadius of base circle: "))
  (setq step (getreal "\nAngular increment (degree): "))
  (setq np (getint "\nNumber of points: "))

(setq fi 0.0)
(command "pline")
(repeat np
  (setq x (* (/ rb (cos (* fi (/ pi 180)))) (sin (+ (/ (sin (* fi (/ pi 180)))
(cos (* fi (/ pi 180)))) (* -1 (* fi (/ pi 180)))))))
  (setq y (* (/ rb (cos (* fi (/ pi 180)))) (cos (+ (/ (sin (* fi (/ pi 180)))
(cos (* fi (/ pi 180)))) (* -1 (* fi (/ pi 180)))))))
  (command (list x y ))
  (setq fi (+ fi step ))
)
(command "")
(princ)
)
```

Fig. 2. The program for drawing the involute curve

As a complex application of the involute profile drawing, in the following there are presented the cases of gear's construction when the correction coefficient, *x*, has different values (positive, negative or zero). In figure 4, a gear including a toothed wheel and a rack (in the three cases) is drawn.

Table 1

x	r [mm]	r_b [mm]	r_a [mm]	r_f [mm]	s_b [mm]	Ψ_b [^{0}]
-0.5	180	169.14467	185	162.500	16.38244	2.77468
0	180	169.14467	190	167.500	19.80265	3.35395
0.5	180	169.14467	195	172.500	23.22285	3.93323

The considered wheel has the following characteristics: $m = 10$ mm, $z = 36$ teeth. In the table 1, the necessary elements for drawing the gear including a toothed wheel and a generating rack are calculated.

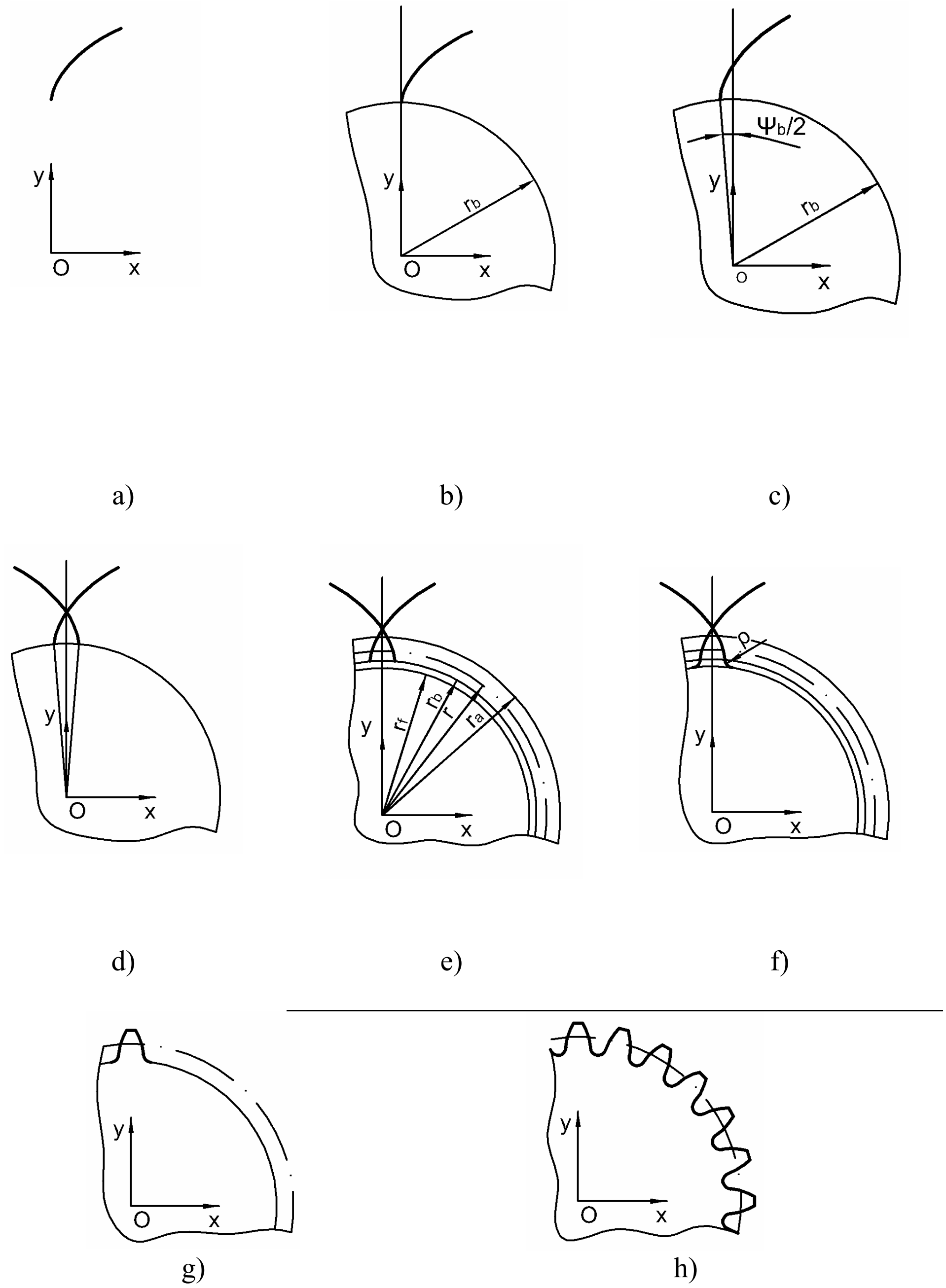


Fig. 3. The steps for gear's teeth engendering:
a) involute curve; b) drawing the base circle; c) curve rotation;
d) mirror feature; e) drawing all circles; f) drawing the fillet radius;
g) tooth's construction; h) gear's construction.

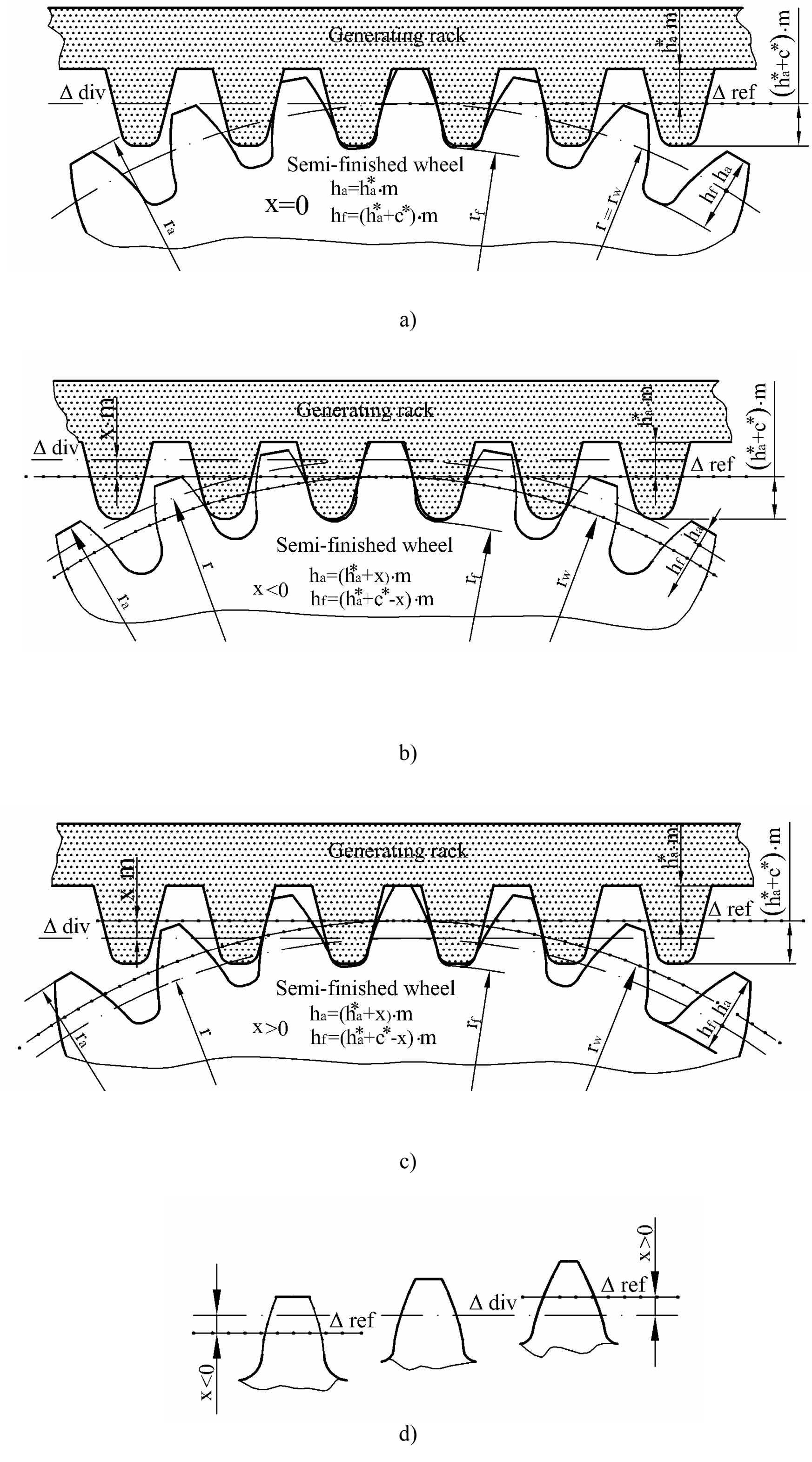


Fig. 4. Aspects regarding the gear`s engendering with profile correction".

Conclusions

The program for engendering the involute curve using a computer program and the examples offered by authors for its construction in this paper offer the necessary information for anyone who wants to draw toothed wheels or gears, being a practical instrument for industrial designers, teaching staff or students who wish to deepen their knowledge of machine parts.

References

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Grafica profilului evolventic utilizat în reprezentarea roților dințate

Rezumat

In această lucrare, autorii prezintă un program realizat în AutoLISP, cu scopul de a desena profilul evolventei. Toți paşii necesari reprezentării corecte a unei roți dințate sunt prezentați în continuare. Se cunoaşte că profilul evolentei este larg răspândit în construcția roților dințate şi că, de obicei, aceasta este reprezentată aproximativ, de cele mai multe ori, prin diverse curbe. Rareori, profilul evolventei este desenat prin puncte cu coordonate calculate. După stabilirea formulelor necesare pentru obținerea expresiilor matematice ale evolventei, autorii prezintă programul numit "Involute", utilizând AutoLISP, cu ajutorul căruia se calculează un număr suficient de puncte, cu coordonatele legate de o polilinie, astfel încât profilul rezultat al evolventei să fie unul foarte aproape de cel stabilit matematic.