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The Dependency Relations Between the Formation Factor and the Tortuosity

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Abstract

The formation resistivity factor, F, is determinated by porosity and tortuosity. Diferent modeles using for determinating formation factor are over simplifications for a complex sistem capillaries. In this paper is studied the relationship between formation factor and tortuosity using a very simple model, but with conclusion interesting.

Keywords: *formation's factor resistivity, resistivity of the formation, true rezistivity, water rezistivity, porosity, water saturation, tortuosity , capillary channel*

Introduction

It is known that from the current conduction point of view, the rock's mineral frame (the matrix) is non-conductive, its resistivity generally being bigger than 10^7 Ωm, the rock's resistivity being conditioned by the quantity and the resistivity of the conductive fluid contained in the rock's interstitial space, respectively the formation water. Based on the resistivity values obtained in the laboratory, on different rock samples charged with formation water, there was noticed the existence of a proportionality relation between the resistivity of the rock charged with formation water, ρ_{Ri} and the resistivity of the formation water charging the rock, ρ_{ai}, as the relation:

$$\rho_{Ri} = F\rho_{ai} \tag{1}$$

where F is a proportionality factor named *formation's resistivity factor* or *formation factor*.

The formation factor is thus defined as the relation between the resistivity of the rock charged with formation water and the resistivity of the formation water charging the rock:

$$F = \frac{\rho_{Ri}}{\rho_{ai}} \tag{2}$$

For a given porosity, the relation $\frac{\rho_{Ri}}{\rho_{ai}}$ stays about constant for all the values $\rho_{ai} \leq 1\Omega m$. Some experimentations showed that in case there are some more resistive formation waters, the value of F is reduced with the increase of the value ρ_{ai} as well as with the reduction of the dimensions of the rock grains, especially in the case of sands.

Dependency between the formation factor and the porosity

For the deduction of the dependency between the formation factor and porosity, it is considered a rock medium as a cube with the volume $V = 1m^3$, respectively with the side $1 = 1$ m (fig. 1).

We consider the pore space is in the form of some capillary channels charged with formation water crossing the cube from side "a" to side "b". The length of a capillary is $l_c = 1$, and the statistical average cross area of a capillary is A_c.

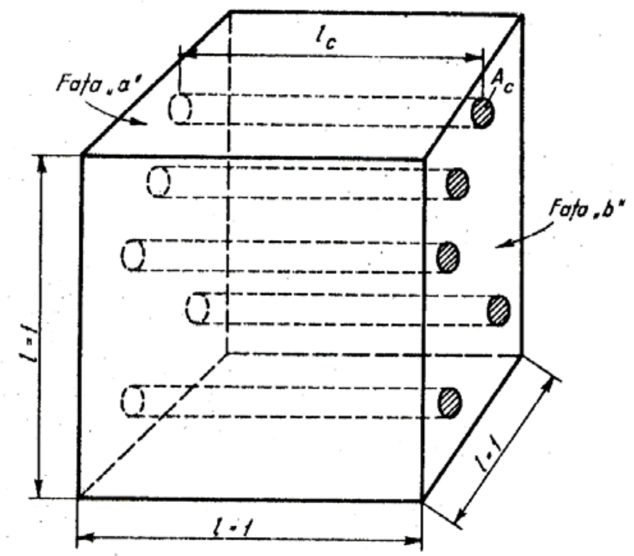

Fig.1. Model of pore space permeable rock saturated with formation water

The volume of the pore space, V_p containing n capillaries shall be:

$$V_p = nA_c l_c \tag{3}$$

The porosity Φ was defined as the relation between the volume of the pore space V_p and the total volume of the rock, V_t

$$\Phi = \frac{V_p}{V_t} = \frac{n \cdot A_c \cdot l_c}{l^3} = nA_c l_c = V_p \tag{4}$$

where l is the cube side equal to the unit.

The wasteful resistance between side "a" and side "b" can be written as:

$$R_0 = \rho_{Ri} \frac{1}{A} = \rho_{Ri} \tag{5}$$

where ρ_{Ri} is the resistivity of the rock charged with formation water and l and *A* represent the length, respectively the total cross area of the cube, both equal to the unit. Considering the fact that the wasteful resistance R_0 of the cube is given by the resistance of the mineral frame teed together with the resistance given by the formation water found in the pore space, the wasteful resistance can be expressed as:

$$\frac{1}{R_0} = \frac{1 - V_p}{R_{ma}} + \frac{V_p}{R_c} \tag{6}$$

where R_{ma} is the wasteful resistance of the rock matrix and R_c - the wasteful resistance of all the capillaries charged with formation water.

The matrix resistance R_{ma} is very high, we can consider it tends to infinite and the relation value:

$$\frac{1 - V_p}{R_{ma}} \to 0 \tag{7}$$

It results that relation (6) considering (7), can be written as:

$$\frac{1}{R_0} = \frac{V_p}{R_c} \quad \text{sau} \quad R_0 = \frac{R_c}{V_p} \tag{8}$$

As the model was considered (much simplified), the capillary channels are conducting wires the resistivity of which is given by the formation water resistivity teed together. The wasteful resistance of all the capillaries R_c can be expressed as:

$$\frac{1}{R_c} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n} \tag{9}$$

where R_1, R_2, , R_n are the capillaries' wasteful resistances.

As the capillaries length, area and the formation water resistance are the same, it results that:

$$R_1 = R_2 = R_3 = \ldots = R_n = R_{c1} \tag{10}$$

In this case, relation (9) can be written:

$$\frac{1}{R_c} = \frac{n}{R_{c1}} \quad \text{sau} \quad R_c = \frac{R_{c1}}{n} \tag{11}$$

The wasteful resistance of a capillary is:

$$R_{c1} = \rho_{ai} \frac{l_c}{A_c} \tag{12}$$

and the "n" capillaries number is equal to:

$$n = \frac{\text{volumul porilor}}{\text{volumul unui capilar}} = \frac{V_p}{V_c} = \frac{V_p}{A_c \cdot l_c} \tag{13}$$

By replacing in relation (11) the relations (12) and (13) and considering that $l_c = 1$ it results that:

$$R_c = \frac{\rho_{ai} \cdot \frac{l_c}{A_c}}{\frac{V_p}{A_c \cdot l_c}} = \rho_{ai} \frac{l_c^2}{V_p} = \rho_{ai} \frac{1}{V_p} \tag{14}$$

The total wasteful resistance given by the relation (8) shall be equal to:

$$R_0 = \rho_{ai} \frac{1}{V_p^2} \tag{15}$$

but for a unitary cube the pores volume $V_p = \Phi$ (rel.4), so that relation (5) is transcribed as:

$$R_0 = \rho_{ai} \frac{1}{\Phi^2} \tag{16}$$

Considering the equation (5), for ρ_{Ri} it results the expression:

$$\rho_{Ri} = \rho_{ai} \frac{1}{\Phi^2} \qquad \text{sau} \qquad \frac{\rho_{Ri}}{\rho_{ai}} = \frac{1}{\Phi^2} \tag{17}$$

Considering the relation defining the formation factor (2), it results *the dependency relation between the formation factor and the porosity for the idealized rock model:*

$$F = \frac{1}{\Phi^2} \tag{18}$$

The formation factor established according to relation (18) is independent of the resistivity of the electrolyte in the pore space, for a certain considered rock, with the porosity Φ, for any value, except from the above mentioned situations.

In the case of the rock composing the systems of beds, they have a totally different pore distribution as compared to that of the rectilinear capillaries, as in the case of the idealized rock model, but in the form of some pore spaces interconnected by thin channels, the rock presenting a certain *tortuosity.* Besides the porosity, the formation factor also depends on the rock structure and texture.

The general dependency relation between the formation factor and the porosity, experimentally obtained is given by *Archie's general relation*:

$$F = \frac{a}{\Phi^{\nu}} \tag{19}$$

where: a is a constant value empirically determined depending on the lithology and

ν – the cementation exponent or the structural index.

Practical dependency relations between the formation factor and the porosity

In the practice of the interpretation of the geophysical log it is generally accepted as a dependency relation, in consolidated formations, a variant of the general relation, known as *Archie's formula*:

$$F = \frac{1}{\Phi^{\nu}} \tag{20}$$

where ν has values according to table 1.

Table 1. The values of the cementation exponent ν of Archie's formula (rel. 20) [3].

Rock	**ν**
Low consolidated sands	1,3 – 1,6
Sandstones	1,8 – 2,0
Intergrain porosity, chalky limestone	2,0
Compact limestone, chalky limestone	2,2 – 2,5
Dolomite	2,2 – 2,8

For non-consolidated rocks (sands, loam sands and non-consolidated sandstones), it is accepted as general dependency the Humble's formula:

$$F = \frac{0{,}62}{\Phi^{2{,}15}} \tag{21}$$

If we renounce to the approximation l_c=1, the formation factor becomes:

$$F = \frac{\overline{l_c^2}}{\Phi^2} \tag{22}$$

where: $\overline{l_c} = \frac{l_c}{l}$, 1 = 1.

The minimum length of a capillary for the fictive rock considered is one and consequently $\overline{l_c} > 1$. By introducing relation (22) in the formula for the calculation of the saturation for clean formations we obtain:

$$S_a^n = \frac{F \cdot \rho_{ai}}{\rho_R} = \frac{\overline{l_c^2}}{\Phi^2} \cdot \frac{\rho_{ai}}{\rho_R} \tag{23}$$

From relation (23) it results that:

$$\overline{l_c^2} = \Phi^2 \cdot \frac{\rho_R}{\rho_{ai}} \cdot S_a^n \tag{24}$$

The saturation in water can have at most the value 1 or considering an irreductible saturation in water of 0,2 (a value currently met), there was further calculated l_c for porosities between 0,1 and 0,3 and values of the relation $\frac{\rho_R}{\rho_{ai}}$ between 1 and 5000, these values being written in table 2.

Table 2. Lenght of capilar determination function of porosity

$\frac{\rho_R}{\rho_{ai}}$	Φ =0.10		Φ =0.15		Φ =0.20		Φ =0.25		Φ =0.30	
	Sa = 1	Sa =0.2	Sa =1	Sa =0.2	Sa =1	Sa =0.2	Sa =1	Sa = 0.2	Sa = 1	Sa =0.2
	lc1	lc2	lc1	lc2	lc1	lc2	lc1	lc2	lc1	lc2
1	0.100	0.020	0.150	0.030	0.200	0.040	0.250	0.050	0.300	0.060
2	0.141	0.028	0.212	0.042	0.283	0.057	0.354	0.071	0.424	0.085
3	0.173	0.035	0.260	0.052	0.346	0.069	0.433	0.087	0.520	0.104
4	0.200	0.040	0.300	0.060	0.400	0.080	0.500	0.100	0.600	0.120
5	0.224	0.045	0.335	0.067	0.447	0.089	0.559	0.112	0.671	0.134
6	0.245	0.049	0.367	0.073	0.490	0.098	0.612	0.122	0.735	0.147
7	0.265	0.053	0.397	0.079	0.529	0.106	0.661	0.132	0.794	0.159
8	0.283	0.057	0.424	0.085	0.566	0.113	0.707	0.141	0.849	0.170
9	0.300	0.060	0.450	0.090	0.600	0.120	0.750	0.150	0.900	0.180
10	0.316	0.063	0.474	0.095	0.632	0.126	0.791	0.158	**0.949**	0.190
15	0.387	0.077	0.581	0.116	0.775	0.155	0.968	0.194	1.162	0.232
20	0.447	0.089	0.671	0.134	**0.894**	0.179	**1.118**	0.224	**1.342**	0.268
25	0.500	0.100	0.750	0.150	1.000	0.200	1.250	0.250	1.500	0.300
30	0.548	0.110	0.822	0.164	1.095	0.219	1.369	0.274	1.643	0.329
35	0.592	0.118	0.887	0.177	1.183	0.237	1.479	0.296	1.775	0.355
40	0.632	0.126	0.949	0.190	**1.265**	0.253	**1.581**	0.316	**1.897**	0.379
45	0.671	0.134	1.006	0.201	1.342	0.268	1.677	0.335	2.012	0.402
50	0.707	0.141	1.061	0.212	1.414	0.283	1.768	0.354	2.121	0.424
55	0.742	0.148	1.112	0.222	1.483	0.297	1.854	0.371	2.225	0.445
60	0.775	0.155	**1.162**	0.232	**1.549**	0.310	**1.936**	0.387	**2.324**	0.465
65	0.806	0.161	1.209	0.242	1.612	0.322	2.016	0.403	2.419	0.484
70	0.837	0.167	1.255	0.251	1.673	0.335	2.092	0.418	2.510	0.502
75	0.866	0.173	1.299	0.260	1.732	0.346	2.165	0.433	2.598	0.520
80	0.894	0.179	**1.342**	0.268	**1.789**	0.358	**2.236**	0.447	**2.683**	0.537
85	0.922	0.184	1.383	0.277	1.844	0.369	2.305	0.461	2.766	0.553

The values of l_c are independently by the water struration and $\frac{\rho_R}{\rho_{ai}}$.

In table 2 was selected the same values for porosity and water saturations and show in tables 3.1, 3.2, 3.3, 3.4, 3.5.

Considering the fact that in the specialized bibliography the tortuosity has values between 1 and 3, in the above mentioned tables there was chosen as maximum limit the value $\overline{l_c} \geq 3$, the values marked in the above mentioned tables.

Table 2. Lenght of capilar determination function of porosity

$\frac{\rho_R}{\rho_{ai}}$	Φ =0.10		Φ =0.15		Φ =0.20		Φ =0.25		Φ =0.30	
	Sa = 1	Sa =0.2	Sa =1	Sa =0.2	Sa =1	Sa =0.2	Sa =1	Sa = 0.2	Sa = 1	Sa =0.2
	lc1	lc2	lc1	lc2	lc1	lc2	lc1	lc2	lc1	lc2
90	0.949	0.190	1.423	0.285	1.897	0.379	2.372	0.474	2.846	0.569
95	0.975	0.195	1.462	0.292	1.949	0.390	2.437	0.487	2.924	0.585
100	**1.000**	0.200	**1.500**	0.300	**2.000**	0.400	**2.500**	0.500	**3.000**	0.600
110	1.049	0.210	1.573	0.315	2.098	0.420	2.622	0.524	3.146	0.629
120	**1.095**	0.219	**1.643**	0.329	**2.191**	0.438	**2.739**	0.548	**3.286**	0.657
130	1.140	0.228	1.710	0.342	2.280	0.456	2.850	0.570	3.421	0.684
140	**1.183**	0.237	**1.775**	0.355	**2.366**	0.473	**2.958**	0.592	**3.550**	0.710
150	1.225	0.245	1.837	0.367	2.449	0.490	3.062	0.612	3.674	0.735
200	**1.414**	0.283	**2.121**	0.424	**2.828**	0.566	**3.536**	0.707	**4.243**	0.849
250	1.581	0.316	2.372	0.474	3.162	0.632	3.953	0.791	4.743	**0.949**
300	1.732	0.346	2.598	0.520	3.464	0.693	4.330	0.866	5.196	1.039
350	1.871	0.374	2.806	0.561	3.742	0.748	4.677	0.935	5.612	1.122
400	2.000	0.400	3.000	0.600	4.000	0.800	5.000	1.000	6.000	1.200
450	2.121	0.424	3.182	0.636	4.243	0.849	5.303	1.061	6.364	1.273
500	2.236	0.447	3.354	0.671	4.472	**0.894**	5.590	**1.118**	6.708	**1.342**
600	2.449	0.490	3.674	0.735	4.899	0.980	6.124	1.225	7.348	1.470
700	2.646	0.529	3.969	0.794	5.292	1.058	6.614	1.323	7.937	1.587
800	2.828	0.566	4.243	0.849	5.657	1.131	7.071	1.414	8.485	1.697
900	3.000	0.600	4.500	0.900	6.000	1.200	7.500	1.500	9.000	1.800
1000	3.162	0.632	4.743	0.949	6.325	**1.265**	7.906	**1.581**	9.487	**1.897**
1100	3.317	0.663	4.975	0.995	6.633	1.327	8.292	1.658	9.950	1.990
1200	3.464	0.693	5.196	1.039	6.928	1.386	8.660	1.732	10.392	2.078
1300	3.606	0.721	5.408	1.082	7.211	1.442	9.014	1.803	10.817	2.163
1400	3.742	0.748	5.612	1.122	7.483	1.497	9.354	1.871	11.225	2.245
1500	3.873	0.775	5.809	**1.162**	7.746	**1.549**	9.682	**1.936**	11.619	**2.324**
1600	4.000	0.800	6.000	1.200	8.000	1.600	10.000	2.000	12.000	2.400
1700	4.123	0.825	6.185	1.237	8.246	1.649	10.308	2.062	12.369	2.474
1800	4.243	0.849	6.364	1.273	8.485	1.697	10.607	2.121	12.728	2.546
1900	4.359	0.872	6.538	1.308	8.718	1.744	10.897	2.179	13.077	2.615
2000	4.472	0.894	6.708	**1.342**	8.944	**1.789**	11.180	**2.236**	13.416	**2.683**
2100	4.583	0.917	6.874	1.375	9.165	1.833	11.456	2.291	13.748	2.750
2200	4.690	0.938	7.036	1.407	9.381	1.876	11.726	2.345	14.071	2.814
2300	4.796	0.959	7.194	1.439	9.592	1.918	11.990	2.398	14.387	2.877
2400	4.899	0.980	7.348	1.470	9.798	1.960	12.247	2.449	14.697	2.939
2500	5.000	**1.000**	7.500	**1.500**	10.000	**2.000**	12.500	**2.500**	15.000	**3.000**
3000	5.477	**1.095**	8.216	**1.643**	10.954	**2.191**	13.693	**2.739**	16.432	**3.286**
3500	5.916	**1.183**	8.874	**1.775**	11.832	**2.366**	14.790	**2.958**	17.748	**3.550**
4000	6.325	1.265	9.487	1.897	12.649	2.530	15.811	3.162	18.974	3.795
4500	6.708	1.342	10.062	2.012	13.416	2.683	16.771	3.354	20.125	4.025
5000	7.071	**1.414**	10.607	**2.121**	14.142	**2.828**	17.678	**3.536**	21.213	**4.243**

Table 3.1.

$\frac{\rho_R}{\rho_{ai}}$	Φ = 0.10	
	Sa =1	Sa =0.2
	lc1	lc2
20	0.45	0.09
40	0.63	0.13
60	0.77	0.15
80	0.89	0.18
100	**1.00**	0.20
120	**1.10**	0.22
140	**1.18**	0.24
200	**1.41**	0.28
250	1.58	0.32
500	2.24	0.45
1000	3.16	0.63
1500	3.87	0.77
2000	4.47	0.89
2500	5.00	**1.00**
3000	5.48	**1.10**
3500	5.92	**1.18**
5000	7.07	**1.41**

Table 3.2

$\frac{\rho_R}{\rho_{ai}}$	Φ = 0.15	
	Sa =1	Sa =0.2
	lc1	lc2
	0.67	0.13
	0.95	0.19
60	**1.16**	0.23
80	**1.34**	0.27
100	**1.50**	0.30
120	**1.64**	0.33
140	**1.77**	0.35
200	**2.12**	0.42
250	2.37	0.47
500	3.35	0.67
1000	4.74	0.95
1500	5.81	**1.16**
2000	6.71	**1.34**
2500	7.50	**1.50**
3000	8.22	**1.64**
3500	8.87	**1.77**
5000	10.61	**2.12**

Table 3.3.

$\frac{\rho_R}{\rho_{ai}}$	Φ =0.2	
	Sa =1	Sa =0.2
	lc1	lc2
20	**0.89**	0.18
40	**1.26**	0.25
60	**1.55**	0.31
80	**1.79**	0.36
100	**2.00**	0.40
120	**2.19**	0.44
140	**2.37**	0.47
200	**2.83**	0.57
250	3.16	0.63
500	4.47	**0.89**
1000	6.32	**1.26**
1500	7.75	**1.55**
2000	8.94	**1.79**
2500	10.00	**2.00**
3000	10.95	**2.19**
3500	11.83	**2.37**
5000	14.14	**2.83**

Table 3.4.

$\frac{\rho_R}{\rho_{ai}}$	Φ =0.25	
	Sa =1	Sa =0.2
	lc1	lc2
20	**1.12**	0.22
40	**1.58**	0.32
60	**1.94**	0.39
80	**2.24**	0.45
100	**2.50**	0.50
120	**2.74**	0.55
140	**2.96**	0.59
200	3.54	0.71
250	3.95	0.79
500	5.59	**1.12**
1000	7.91	**1.58**
1500	9.68	**1.94**
2000	11.18	**2.24**
2500	12.50	**2.50**
3000	13.69	**2.74**
3500	14.79	**2.96**
5000	17.68	3.54

Table 3.5.

	Φ =0.30	
	Sa =1	Sa =0.2
$\frac{\rho_R}{\rho_{ai}}$	lc1	lc2
20	**1.34**	0.27
40	**1.90**	0.38
60	**2.32**	0.46
80	**2.68**	0.54
100	**3.00**	0.60
120	3.29	0.66
140	3.55	0.71
200	4.24	0.85
250	4.74	0.95
500	6.71	**1.34**
1000	9.49	**1.90**
1500	11.62	**2.32**
2000	13.42	**2.68**
2500	15.00	**3.00**
3000	16.43	3.29
3500	17.75	3.55
5000	21.21	4.24

With the selected values of $\bar{l}_c$, there was recalculated the formation factor, F and the saturation in water S_a, the results being presented in table 4.

Table 4. The determination of the formation factor and of the saturation in water

Φ = 0.15			Φ = 0.20			Φ = 0.25			Φ = 0.30		
F	lc	Sa	F	lc	Sa	F	lc	Sa	F	lc	Sa
									10	1.34	0.40
			20	0.89	0.57	20	1.12	0.57	20	1.90	0.57
			40	1.26	0.81	40	1.58	0.81	40	2.32	0.81
60	1.16	0.99	60	1.55	0.99	60	1.94	0.99	60	2.68	0.99

Also, from diagrams 2 and 3 it results that for the same porosity, the values of F vary in wide limits which also involves the variation of the saturation in water in the interval 0.4 – 1.

The experimental data showed that for small porosity values the formation factor is almost constant, irrespective of the value of the formation water resistivity, which is also confirmed by the values obtained in the calculation, respectively Φ < 1.

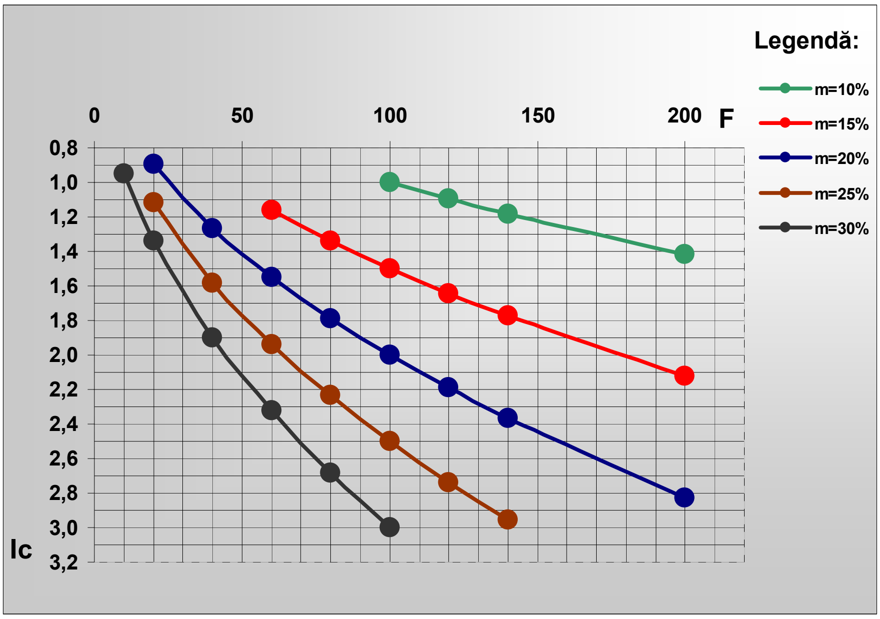

Fig. 2. The variation of theformation factor depending on lc

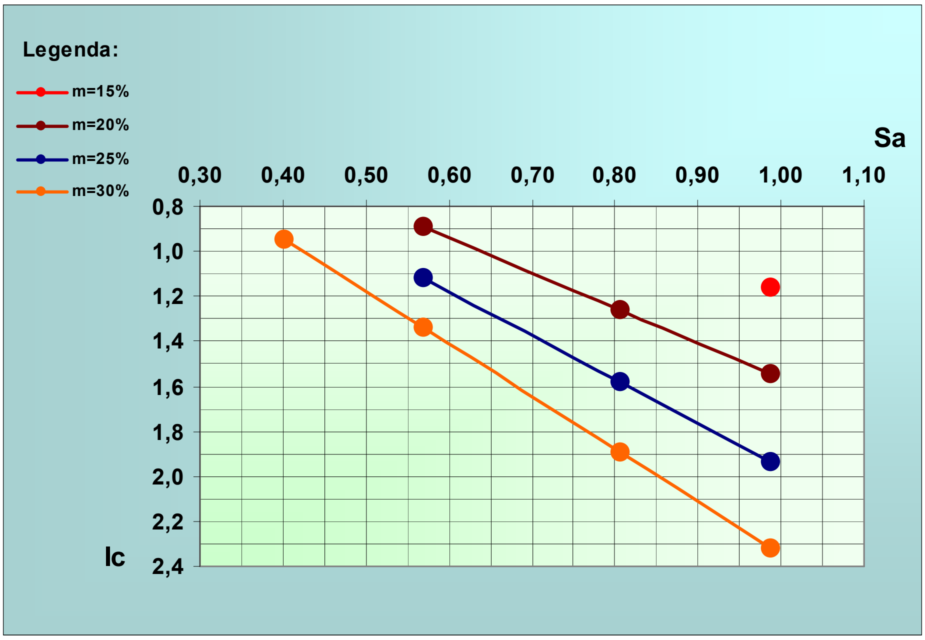

Fig. 3. The variation of the saturation in water depending on lc

Conclusions

Given the above mentioned results it is obvious the need to reduce the variation field of the structural index as well as that of the lithological coefficient of Archie's formula. The aleatory selection of the values of the two coefficients leads, as there can be noticed, to significant deviations of the saturation determined from the geophysical log.

The fact that the capillary length has values between 1 and 3 (as an example we mention that the tortuosity for a granular fictive rock, with cubic embedment, is given in the specialized literature as 1,4), and that it cannot have values smaller than 1, leads us to the conclusion that this paper raises a (an old) problem which is yet unsolved.

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Relaţia de dependenţă dintre factorul de formaţie şi tortuozitate

Rezumat

Factor de rezistivitate al formaţiei este determinat de porozitate şi de tortuozitate. Tortuozitatea este o proprietate petrofizică care nu poate fi determinată în laborator. De aceia factorul de rezistivitate al formaţiei pe scurt factor de rezistivitate a fost în general determinat pe baza măsurătorilor de laborator. În această lucrare s-a încercat o analiză a dependenţei factor de formaţie – tortuozitate pe un model simplificat şi aspecte teoretice.