BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LXI No. 3/2009	287 - 292	Seria Tehnică

Considerations about MTTF and the Meaning of This Term to the Customer

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Abstract

This paper is addressed to the specialists needing to acquire equipment or spare parts, to fit in their business systems, considering also reliability factors. They usually take into consideration the MTTF / MTBF; however this reliability characteristic proves to have low relevance in a lot of situations. The scope of the paper is to propose and introduce a new reliability term which would be able to show some data relating to the expectations which someone would have regarding the product/spare part acquired. In this scope, the authors introduce the Reliable Life.

Key words: *reliable life, reliability distribution, MTTF, MTBF.*

Introduction: MTBF / MTTF – an important product feature

Mean time between failures (MTBF) – for repairable products and Mean time to failure (MTTF) for non-repairable products are probably the most known reliability terms (see [2, 3]). These product features are usually used by professionals in choosing the products.

A common mistake is to consider that a product would last at least a time equal to its MTBF. In order to avoid this kind of confusion, we propose to introduce a more relevant term, the reliable life, which we define as the functioning life until equipment reaches a certain probability to still be in service.

In this paper we first estimate the probability for an equipment to be still is in function, after a period t=MTTF, for the three most important distributions used in Reliability, the Weibull, the exponential and the normal distributions.

After this, we calculate the reliable life for the same distributions, in order to highlight the difference of meaning.

At the end of the paper we draw some conclusions based on the calculus made and we present some advantages of the term of Reliable Life.

We sustain the fact that the Reliable Life can be much more relevant for the customer.

Note: For simplification reasons, we will assimilate MTBF with MTTF.

The reliability for several distribution cases at the moment t=MTTF

In order to analyze this, let's look to several life distributions and estimate the MTTF.

We will take into consideration the most used three reliability distribution; however this exercise can be extended to other distributions.

Exponential Distribution

Consider the following notations:

λ = failure rate, considered as constant in this paper.

MTTF = Mean Time to Failure

t = time variable.

Considering the exponential distribution, we have the expression of the probability distribution function (pdf) as per [2, 4, 5]:

$$f(t) = \lambda \cdot e^{-\frac{1}{MTTF} \cdot t}. \quad (1)$$

The MTTF can easily be calculated:

$$MTTF = \int_0^\infty t \cdot f(t) \cdot dt = \int_0^\infty t \cdot \lambda \cdot e^{-\lambda \cdot t} \cdot dt = \frac{1}{\lambda}. \quad (2)$$

The reliability function for this case is:

$$R(t) = e^{-\lambda \cdot t} = e^{-\frac{t}{MTTF}} \ . \quad (3)$$

If we consider t=MTTF, we get to:

$$R(MTTF) = e^{-\frac{MTTF}{MTTF}} = e^{-1} = 36.7879\% \ . \quad (4)$$

This means that after a time interval equal to MTTF from putting an equipment into function, if the considered equipment follows an exponential distribution, there are less than 37% chances that the equipment would still work, or that from 100 equipments, only 37 would still work after a time equal with MTTF.

Weibull Distribution

We consider the following notations:

η = scale parameter;

γ = 0 = position parameter which we consider to be 0 for simplification;

β = shape parameter;

Γ(x) = gamma function.

Considering the Weibull distribution, the pdf is as per [2, 4, 5]:

$$f(t) = \frac{\beta}{\eta} \cdot \left(\frac{t-\gamma}{\eta}\right)^{\beta-1} \cdot e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}} = \lambda(t) \cdot e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}} \ . \quad (5)$$

Considering that the position parameter is 0, it results:

$$f(t) = \frac{\beta}{\eta} \cdot \left(\frac{t}{\eta}\right)^{\beta-1} \cdot e^{-\left(\frac{t}{\eta}\right)^{\beta}} = \lambda(t) \cdot e^{-\left(\frac{t}{\eta}\right)^{\beta}}. \quad (6)$$

The MTTF can easily be calculated:

$$\mathrm{MTTF} = \eta \cdot \Gamma\left(\frac{1}{\beta} + 1\right) = \frac{\eta}{\beta} \cdot \Gamma\left(\frac{1}{\beta}\right). \tag{7}$$

The reliability function for this case is:

$$\mathrm{R(t)} = e^{-\left(\frac{t}{\eta}\right)^{\beta}} \quad . \tag{8}$$

Considering that t = MTTF it results as shown in [4]:

$$\mathrm{R(MTTF)} = e^{-\left(\frac{\mathrm{MTTF}}{\eta}\right)^{\beta}} = e^{-\left(\frac{\eta \cdot \Gamma\left(\frac{1}{\beta}+1\right)}{\eta}\right)^{\beta}} = \left[e^{-\Gamma\left(\frac{1}{\beta}+1\right)}\right]^{\beta} = e^{-\left(\frac{1}{\beta}\right)^{\beta} \cdot \left[\Gamma\left(\frac{1}{\beta}\right)\right]^{\beta}} . \tag{9}$$

In the table 1 we have calculated the reliability for some particular values of the shape parameter. In the drawing bellow it can be seen how β influences the Reliability (fig.1).

Table 1. Calculated reliability, depending on the shape parameter

β	**1/β**	**Γ(1/β)**	**R(MTBF)**
0.50	2.00	0.14	0.5944
1.00	1.00	0.37	0.6922
1.50	0.67	0.51	0.8185
2.00	0.50	0.61	0.9121
3.00	0.33	0.72	0.9865
4.00	0.25	0.78	0.9986

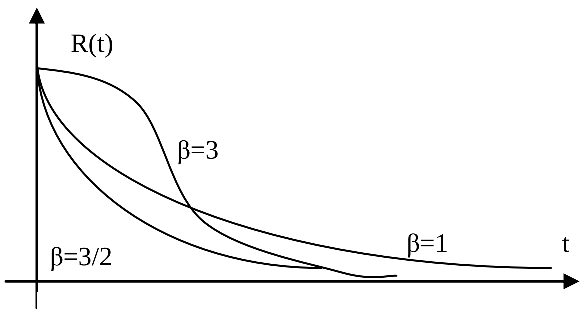

Fig.1. Reliability variation with β.

So depending on the Weibull shape parameter, the reliability can vary from 60% to more than 99%, so there is no much information if after t = MTTF the equipment will be still functioning.

Normal Distribution

For the normal distribution the pdf is shown bellow, as per [2, 4, 5]:

$$f(t) = \frac{1}{\sigma_t \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{1}{2} \cdot \left(\frac{t-\mu}{\sigma_t}\right)^2} . \tag{10}$$

The MTTF is equal to the mean, median and mode, part of this due to the distribution simetry:

$$\mu = \text{MTTF}. \tag{11}$$

The reliability results from the pdf relation:

$$R(t) = \int_t^{\infty} f(t) \cdot dt = \int_t^{\infty} \frac{1}{\sigma_t \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{1}{2} \cdot \left(\frac{t-\mu}{\sigma_t}\right)^2} \cdot dt\ . \tag{12}$$

For t = MTTF = μ it results:

$$R(\mu) = \int_\mu^{\infty} \frac{1}{\sigma_t \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{1}{2} \cdot \left(\frac{\mu-\mu}{\sigma_t}\right)^2} \cdot dt = \int_t^{\infty} \frac{1}{\sigma_t \cdot \sqrt{2 \cdot \pi}} \cdot dt = 0.5\ . \tag{13}$$

So, at t = MTTF, there are 50% chances the equipment will be up and running.

Introduction of the "reliable life", with some considerations regarding the above considered distributions

When a customer buys some parts or equipments, he has some underlying expectations from the product bought.

The expectations are related to how much time will work the equipment and how much the customer can rely on it. This can help to design a predictive maintenance system, with a lot of advantages like: less down time for the equipment; we can choose and plan the time when the equipment is stopped for periodical checks; we know how much spare parts stock we need etc.

We propose the term of "50% reliable life" for equipments, which we note with $RL_{50\%}$ and we define to be the time measured from the beginning of the equipment life (put in function) until there are 50% chances that the equipment still works or 50% chances for the equipment to be down (without repairs).

Translated into mathematical formulas, we have:

$$R(RL_{50\%}) = 50\%\ . \tag{14}$$

From the relation above we can calculate the reliable life $RL_{50\%}$ for the distributions considered in this paper, as per [1].

Exponential Distribution

$$R(t) = e^{-\lambda \cdot RL_{50\%}} = e^{-\frac{RL_{50\%}}{MTTF}} = 50\%\ . \tag{15}$$

By using the natural logarithm we get:

$$\ln\left(e^{-\lambda \cdot RL_{50\%}}\right) = -\lambda \cdot RL_{50\%} \cdot \ln(e) = -\lambda \cdot RL_{50\%} = \ln(0.5)\ . \tag{16}$$

So we finally get:

$$RL_{50\%} = \frac{0.69315}{\lambda} = 0.69315 \cdot MTTF\ . \tag{17}$$

Weibull Distribution

$$R(RL_{50\%}) = e^{-\left(\frac{RL_{50\%}}{\eta}\right)^{\beta}} = 50\%\ . \tag{18}$$

By using the logarithms we get:

$$-\left(\frac{\mathrm{RL}_{50\%}}{\eta}\right)^{\beta} = \ln(0.5)\ . \tag{19}$$

So finally we get:

$$\mathrm{RL}_{50\%} = \frac{\sqrt[\beta]{-\ln(0.5)}}{\eta}\ . \tag{20}$$

In table 2 there are given some values for $\mathrm{RL}_{50\%}$, considering different values for β.

Table 2. RL50%, for different values of β

β	η*$\mathrm{RL}_{50\%}$
0.50	0.48
1.00	0.69
1.50	0.78
2.00	0.83
3.00	0.88
4.00	0.91

Normal Distribution

In this situation it is much simpler as the normal distribution is symmetrical and we easily conclude that:

$$\mathrm{RL}_{50\%} = \mathrm{MTTF} = \mu\ . \tag{21}$$

Conclusions

Based on the calculations made, we can make the values from table 3.

Table 3. Calculated values

	R(MTTF)	**$\mathrm{RL}_{50\%}$**	**Notes**
Exponential Distribution	36.79%	0.69*MTTF	
Weibull Distribution	Range from 59.44% to 99.86%	(0.48 to 0.91)*η	Beta from 0.5 to 4.0
Normal Distribution	50%	MTTF	

It is obvious that at MTTF the probability to be in function for an equipment has some values which are not so relevant, having a wide range of variation, from less than 37% to more than 99%. As in a lot of cases the reliability of the products is not very explicit, the MTTF being the sometimes the only parameter given, without even mentioning the reliability distribution, it comes out that the MTTF can be misleading.

In order to improve the Reliability related relevance for the customer, it is much better to have from the manufacturer the value of the reliable life for the products. This single value gives an indication on how much the customer can rely on the product.

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Considerații asupra MTTF și înțelesul acestui termen pentru client

Rezumat

Această lucrare se adresează specialiștilor care trebuie să achiziționeze echipamente sau piese de schimb, pentru a le utiliza în sistemele lor, ținând cont și de factorii de fiabilitate și mentenanță. Aceștia în general iau în considerație MTTF-ul sau MTBF-ul echipamentelor; totuși aceste caracteristici de fiabilitate s-au dovedit a avea o relevanță mai mică în multe situații. Scopul lucrării este de a propune și a introduce un nou termen de fiabilitate care să poată arata date relevante pentru produsul sau piesa de schimb achiziționată. Astfel, autorii introduc noțiunea de „viață fiabilă”.