

# Research Concerning the Dynamic Analysis of Plane Mechanisms Structures

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## Abstract

*In the paper a methodology for the analysis of plane mechanisms structures is presented. Positional, cinematic and dynamical stages are analyzed. With this end in view, a simulation program has been developed. Finally, some simulation results are presented. Some interesting 3D results regarding the variation of the equilibrium moment can then be used for optimum design of the analyzed mechanism.*

**Key words:** *mechanism, joints, position, equilibrium moment*

## Introduction

Generally, for an optimum design of the mechanisms used in industrial applications, a rigorous analysis of their running behavior has to be realized. In many cases, it is necessary to establish the variations curves of different positional, cinematic or dynamical parameters. In this paper a methodology for the analysis of plane mechanisms structures is presented. Positional, cinematic and dynamical stages are analyzed. The method is applied in the case of a plane mechanism with two independent contours. A simulation program has been developed. Finally, some simulation results are presented. Some interesting 3D results regarding the variation of the equilibrium moment can then be used for optimum design of the analyzed mechanism.

## Theoretical Considerations and Verification Results

For the analysis of a plane mechanism, the following elements are considered to be known: the dimensions of the component links, the masses of the links and their moments of inertia, the values of the technological forces and moments, the nominal angular speed of the leader link of the mechanism:  $\omega_1 = \omega_n$ .

The first stage of the analysis is the positional analysis. In this stage the position of the component links are determined depending on the position of the leader link. With this end in view, some analytical methods [1] can be used: the bars methods, the method of the independent contours with representation in complex plane, the method of the projection of the independent vector circuits etc.

The second stage is the cinematic analysis. In this stage the following parameters can be determined: the angular speeds and accelerations of the component links, the speed and

acceleration of the main points on the mechanism (the center of the joints, the points where the technological forces and moments are applied). With this end in view, we can derive the variation functions of the corresponding position parameters or apply some analytical methods [1]: the method of the independent cycles, the method of the transfer function etc.

The third stage of the analysis is the dynamic analysis. In this stage a lot of parameters and their variation can be calculated: the motor moment, the variation of the equilibrium moment, the variation of different synthesis parameters (the reduced moment, the reduced force, the reduced mass and the reduced moment of inertia), the moment of inertia of the fly wheel etc. In this paper, we will focus on the variation of the equilibrium moment for a cinematic cycle of functioning. The variation of the equilibrium moment can be determined from the following relation [2]:

$$\bar{M}_e \cdot \bar{\omega}_1 + \sum_j (\bar{F}_j \cdot \bar{v}_j + \bar{M}_j \cdot \bar{\omega}_j) + \sum_j (\bar{F}_{ij} \cdot \bar{v}_j + \bar{M}_{ij} \cdot \bar{\omega}_j) = 0 \quad (1)$$

where:  $\bar{M}_e$  is the equilibrium moment,  $\bar{F}_j$  and  $\bar{M}_j$  are the resultant force and the resultant moment corresponding to the external forces and moments which act on the  $j$  link and which are reduced in the mass centre of the  $j$  link,  $\bar{F}_{ij}$  and  $\bar{M}_{ij}$  are the resultant inertia force and the resultant inertia moment corresponding to the  $j$  link,  $\bar{v}_j$  is the speed of the mass centre of the  $j$  link,  $\bar{\omega}_j$  is the angular speed of the  $j$  link.

The three stages of analysis have been applied to the plane mechanism represented in figure 1.

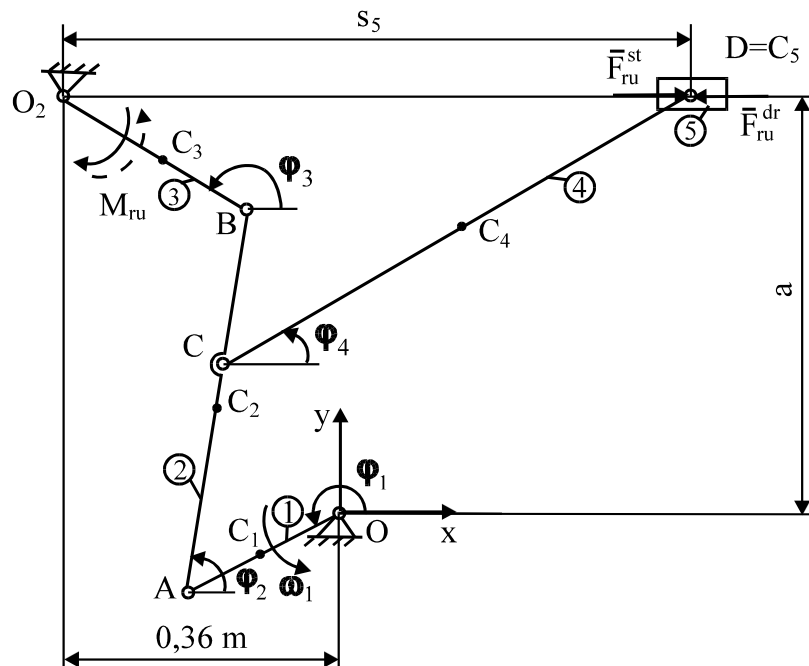


Fig. 1. Plane mechanism with two independent contours

For this mechanism the following elements are considered to be known:

- the dimensions of the component links:  $OA=0.075\text{m}$ ;  $AB=0.6\text{m}$ ;  $O_2B=0.35\text{m}$ ;  $BC=0.2\text{m}$ ;  $CD=0.75\text{m}$ . The mass centers:  $C_1, C_2, C_3, C_4$  are on the middle of the corresponding links. The dimension of  $a$  varies between 0.5 and 0.77.
- the mass of the component links:  $m_1=1.5\text{kg}$ ;  $m_2=10\text{kg}$ ;  $m_3=4.5\text{kg}$ ;  $m_4=12\text{kg}$ ;  $m_5=2\text{kg}$ .
- the moments of inertia of the links:  $I_{C_1}=0.004\text{kgm}^2$ ;  $I_{C_2}=0.3\text{kgm}^2$ ;  $I_{C_3}=0.045\text{kgm}^2$ ;  $I_{C_4}=0.562\text{kgm}^2$ . The value of  $I_{C_5}$  is neglected.
- the technological forces and moments:  $F_{ru}^{dr}=1000\text{[N]}$ ;  $F_{ru}^{st}=150\text{[N]}$ ;  $M_{ru}=120\text{[Nm]}$ .
- the nominal angular speed of the leader link of the mechanism:  $\omega_1=10\text{[rad/s]}$ .

The method of the projection of the independent vector circuits has been used for the positional analysis. The mechanism has two independent contours:  $O-A-B-O_2-O$  and  $O-A-C-D-O_2-O$ . By projecting the vector circuits corresponding to these independent contours on the  $x$  and  $y$  axes (fig. 1), the following systems of equations are obtained:

$$\begin{cases} l_1 \cdot \cos \varphi_1 + l_2 \cdot \cos \varphi_2 + l_3 \cdot \cos \varphi_3 + b = 0 \\ l_1 \cdot \sin \varphi_1 + l_2 \cdot \sin \varphi_2 + l_3 \cdot \sin \varphi_3 - a = 0 \end{cases} \quad (2)$$

$$\begin{cases} l_1 \cdot \cos \varphi_1 + AC \cdot \cos \varphi_2 + l_4 \cdot \cos \varphi_4 - s_5 + b = 0 \\ l_1 \cdot \sin \varphi_1 + AC \cdot \sin \varphi_2 + l_4 \cdot \sin \varphi_4 - a = 0 \end{cases} \quad (3)$$

where:  $l_1 = OA$ ;  $l_2 = AB$ ;  $l_3 = O_2B$ ;  $l_4 = CD$ ;  $b = 0.36\text{ m}$ .

By solving the systems of equations (2) and (3), the unknown parameters:  $\varphi_2, \varphi_3, \varphi_4$  and  $s_5$  can be calculated from the following relations:

$$\begin{cases} A_2 \cdot \cos \varphi_2 + B_2 \cdot \sin \varphi_2 = C_2 \\ A_3 \cdot \cos \varphi_3 + B_3 \cdot \sin \varphi_3 = C_3 \\ \sin \varphi_4 = (a - l_1 \cdot \sin \varphi_1 - AC \cdot \sin \varphi_2) / l_4 \\ s_5 = l_1 \cdot \cos \varphi_1 + AC \cdot \cos \varphi_2 + l_4 \cdot \cos \varphi_4 + b \end{cases} \quad (4)$$

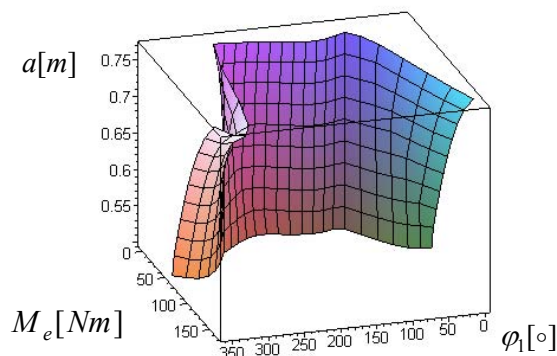
where:

$$\begin{cases} A_2 = 2 \cdot l_1 \cdot l_2 \cdot \cos \varphi_1 + 2 \cdot b \cdot l_2 \\ B_2 = 2 \cdot l_1 \cdot l_2 \cdot \sin \varphi_1 - 2 \cdot a \cdot l_2 \\ C_2 = l_3^2 - l_1^2 - l_2^2 - b^2 - a^2 - 2 \cdot b \cdot l_1 \cdot \cos \varphi_1 + 2 \cdot a \cdot l_1 \cdot \sin \varphi_1 \end{cases} \quad (5)$$

$$\begin{cases} A_3 = 2 \cdot l_1 \cdot l_3 \cdot \cos \varphi_1 + 2 \cdot b \cdot l_3 \\ B_3 = 2 \cdot l_1 \cdot l_3 \cdot \sin \varphi_1 - 2 \cdot a \cdot l_3 \\ C_3 = l_2^2 - l_1^2 - l_3^2 - b^2 - a^2 - 2 \cdot b \cdot l_1 \cdot \cos \varphi_1 + 2 \cdot a \cdot l_1 \cdot \sin \varphi_1 \end{cases} \quad (6)$$

The angular and linear speed and accelerations distributions have been determined by deriving the variation functions of the corresponding position parameters. Then, the variation of the equilibrium moment  $M_e$  has been obtained using the relation (1).

The analysis has been transposed into a computer program using the Maple software. In the figure 2 the variation of the equilibrium moment  $M_e$  when the parameter  $a$  varies between  $0.5 m$  and  $0.77 m$  is presented.



**Fig. 2.** The variation of the equilibrium moment

## Conclusions

In this paper a methodology for the analysis of plane mechanisms structures is presented. Positional, cinematic and dynamical stages are analyzed. The method is applied in the case of a plane mechanism with two independent contours. A simulation program has been developed. Finally, a 3D variation of the equilibrium moment is presented. This result can then be used for optimum design of the analyzed mechanism.

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## Cercetări privind analiza dinamică a structurilor mecanismelor plane

### Rezumat

*In articol se prezintă o metodologie de analiză a structurilor mecanismelor plane. Sunt analizate fazele de analiză pozițională, cinematică și dinamică. În acest scop a fost dezvoltat un program de calculator. În final, sunt prezentate rezultate ale simulărilor. Rezultate în reprezentare 3D ale variației momentului de echilibrare pot fi apoi folosite pentru proiectarea optimă a mecanismului analizat.*