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Algorithm for Estimating the Area of Complex 3D Surfaces from 2D Images

Zoltan Borsos, Octavian Dinu, Grigore Ruxanda, Stancu Mihaela

Petroleum – Gas University of Ploieşti, 39 Bucureşti Blvd., Ploieşti, ROMÂNIA
e-mail: borsos.zoltan@gmail.com

Abstract

There are two ways of estimating the area of a complex 3D surface. One the one hand, one may use the exact function that describes the surface; on the other hand, one may use the Cartesian coordinates of each point of that surface. In the first case, if the exact function is known the calculated surface area is considered an exact value, otherwise the obtained value is approximate. In this paper, we suggest an algorithm of estimating the area of such surfaces for three-dimensional nanostructures using two-dimensional grayscale images and linear and nonlinear elevation methods, in order to apply to 2D SEM images of nanowalls. A particular case is studied, when the image is taken from a perpendicular direction and the 3D surface is illuminated with diffuse light for the best correlation between height and grayscale colors.

Key words: *2D image, non-linear elevation, surface area.*

Introduction

In some situations there is only a two-dimensional (2D) picture of a spatial (three-dimensional or 3D) image: terrain satellite images[1], old photographs of different objects, SEM (Scanning Electron Microscope[2]) or TEM (Transmission Electron Microscopy[3]) images of nanostructures, real-time registration of street traffic [1], etc. The aim of the present paper is to create 3D models for such pictures using specialized software and calculate the surface area. These techniques are already used by some graphics editing programs like Adobe Photoshop, Corel Bryce, MacDEM, and World Construction, which are specialized in exploring the 3D world geometrical properties without the possibility to calculate the obtained 3D surface area.

In the next paragraphs, after creating the linear or non-linear elevation model of a two-dimensional space, two numerical methods will be used to estimate the area of the 3D surface, with arbitrary complexity. In the first case, there is used a discrete method, determined by the resolution of the 2D picture and in the second case this digitization is eliminated using interpolation functions $f(x,y)$, where x and y are the picture length and width.

This information, i.e. the 3D surface area, is useful in some special cases: evaluation of the terrain water absorbance, heat transfer of the electrical circuits into air, specific surface area for

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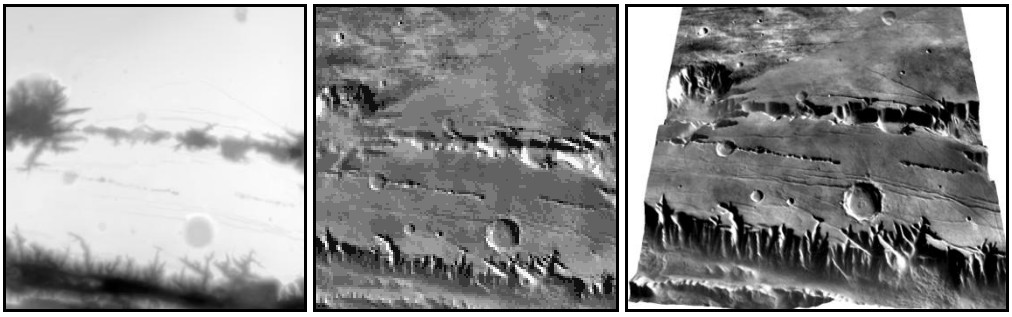
[1] http://www.satimagingcorp.com/svc/exploration.html
[2] http://en.wikipedia.org/wiki/Scanning_electron_microscope
[3] http://en.wikipedia.org/wiki/Transmission_electron_microscope

complex structures etc. In order to model the 3D elevation (linear or non-linear) and calculate the surface area the *Mathematica*® software is used for the implementation of the algorithm. The specific functions are presented in the following paragraph.

The algorithm for estimating the area of the surface of 3D images created from 2D picture

In order to create a 3D model from two-dimensional pictures the *Digital Elevation Model* (DEM) is frequently used. This technique is useful in case of grayscale pictures because in the case of 256 levels there are 256 elevation heights for the 3D image. In order to show the diversity of the domains of application, an example is represented in Fig. 1.: a) there is grayscale image for DEM, while in b) and c) a 3D model is created using this method, in b) there is a perpendicular view and in c) there is a perspective view.



a) **b)** **c)**

Fig. 1. 3D view of elevation model of Tithonium Chasma, from http://en.wikipedia.org/wiki/File:Mtm-05277e_3d.png (April 2009)[4].

This method was developed in some cases for the implementation in other imaging software [2-4] without a direct algorithm for estimating the surface area. In *Mathematica*, where line instructions are used step by step, without recalibrations of distances, the algorithm is reduced to a few lines. The case of study is focused on a grayscale SEM image of nanocavities (see Fig. 2.) on the top surfaces of ZnO microcrystals [5] and the first step is to import and transform the 2D image into a matrix, i.e. *m* in the algorithm (all variables are italic and the functions are bold), with the same size as the picture. This method will be further used to estimate the specific surface of different nanostructures from 2D SEM images. The matrix elements are numerically equal, in case of the linear elevation method (LEM), with grayscale level numbers 0 for black and 255 for white.

$$m_{ij} = \mathrm{Color}(Grayscale\ level)_{ij} \tag{1}$$

```
ln[1]:= fin = "2.bmp"; calea = "D:\\ ";
ln[2]:= elem = Import[calea <> fin, "Data"];
es = Import[calea <> fin, "ImageSize"];
m = N@Map[255 - Mean[#] &, elem, {2}]; ArrayPlot[m]
```

[4] This image is in the *public domain* because it contains materials that originally came from the United States Geological Survey, an agency of the United States Department of Interior. For more information, see the official USGS copyright policy (http://www.usgs.gov/visual-id/credit_usgs.html#copyright).

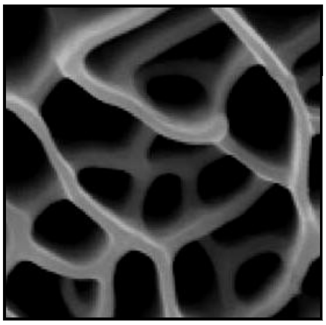


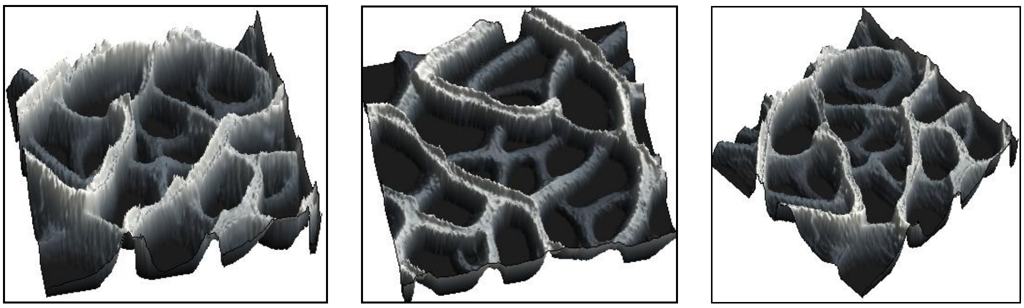
Fig. 2. SEM image of nanocavities on the top surfaces of ZnO microcrystals after its import in Mathematica® with dimensions 293x282 pixels (px) and characteristic area 82 626 px^2.

In the case of non-linear elevation method (NLEM) an additional function $nlf = nlf(x)$ is used where $x \in \{0,1,...,255\}$ and the associated matrix is determined by this function through equation (1)

$$m_{ij} = nlf\left(\text{Color}(Grayscale\ level)_{ij}\right). \tag{2}$$

The 3D image can be viewed using the **ListPlot3D** function and for different viewpoints it is represented in Fig. 3.

```
ln[3]:= ListPlot3D[Transpose[255 - m], Mesh→None,
      ColorFunction→"GrayTones", Axes → False, Boxed→False,
      ViewPoint→{5, -1, 10}, AspectRatio→1, PlotRange→{0, 300}]
```



a) **b)** **c)**

Fig. 3. 3D image of nanocavities obtained using DEM for different viewpoints:
a) ViewPoint→{1, -5, 5}; b) ViewPoint→{5, -1, 10}; c) ViewPoint→{5, 5, 20}.

The analytical expression of a 2D surface area, defined by $f(x, y)$, is

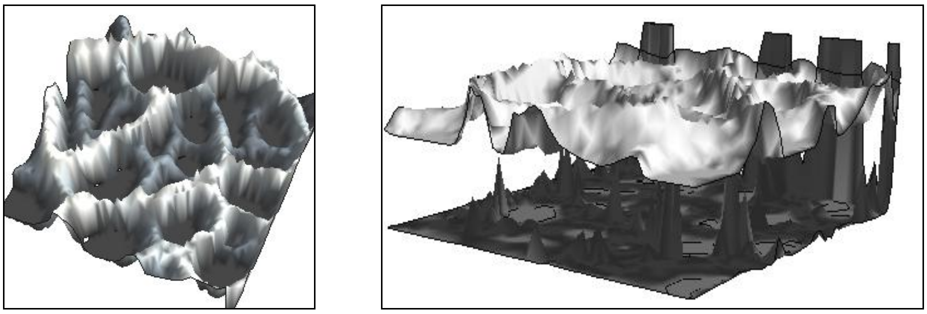
$$S = \iint_{x\,y} \sqrt{1 + \left(\frac{\partial \text{f}(x,y)}{\partial x}\right)^2 + \left(\frac{\partial \text{f}(x,y)}{\partial y}\right)^2}\,\text{d}x\text{d}y\,. \tag{3}$$

In order to estimate analytically the surface area using formula (3), the interpolation function is created using the function **Interpolation**.

```
ln[4]:= ptInterpolare =  Flatten[Table[{{i, j}, m[[i]][[j]]},
      {i, 1, es[[2]]}, {j, 1, es[[1]]}], 1];
fint = Interpolation[ptInterpolare];
```

In order to compare the solution with the original image, the surface and its derivative are represented on Fig. 4.

```
ln[5]:= Plot3D[256 - fint[x, y], {x, 1, es[[2]]}, {y, 1, es[[1]]},
      Mesh→None, ColorFunction→"GrayTones", Axes→False,
      Boxed→False, PlotRange→{0, 255}]
```



a) **b)**

Fig. 4. 3D graphics using interpolating function with two variables and its derivative: a) the 3D surface; b) both the 3D surface above and the derivative under – the darker surface.

Similar to line "*ln[5]:=*" one may create the 3D graphics in Fig. 4.b, "*ln[6]:=*". Using formula (3) through line 7 (*ln[7]:=*), the surface area is calculated.

```
ln[7]:= NIntegrate[Evaluate[Sqrt[1 +
      (\!\(\*SubscriptBox[\(\[PartialD]\),
      \(x\)]\(fint[x, y]\)\))^2 +
      (\!\(\*SubscriptBox[\(\[PartialD]\),
      \(y\)]\(fint[x, y]\)\))^2]],
      {x, 1, es[[2]] - 1}, {y, 1, es[[1]] - 1}, Method→"MonteCarlo"]
```

The result in this case is a number with area units determined by a rescaled picture (eventually $\mathrm{pixel}^2 = \mathrm{px}^2$, if one unit for grayscale level is considered as 1 pixel), $S = 561443(\mathrm{px}^2)$.Using information from Fig. 2., we obtain a characteristic surface ratio of 6.79, a dimensionless number that characterizes the size of the 3D surface related to the projected 2D surface.

Conclusions

The proposed algorithm may be used (with recalibrations, correlation between 2D picture dimensions, grayscale deep and real dimensions), in order to obtain numerical results with units about complex surface areas in a large variety of research fields. The authors will further use this algorithm in order to estimate the specific surface area for nanowalls using SEM images and the specific volume of nanostructures, from AFM (Atomic Force Microscope) imaging.

References

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Algoritm pentru evaluarea ariei suprafeţelor 3D complexe folosind imagini 2D

Rezumat

Evaluarea ariei unei suprafeţe 3D se poate realiza prin două metode: în primul caz se poate utilizând funcţii exacte ce descriu suprafaţa studiată iar în cel de al doilea caz un număr finit de coordonate ale punctelor de pe suprafaţă. În primul caz aria poate fi calculată exact iar în cel de al doilea se obţin numai valori aproximative. În lucrare prezentăm un algoritm pentru evaluarea ariei suprafeţelor unor nanostructuri tridimensionale folosind imagini bidimensionale (SEM) ale acestora în nuanţe de gri utilizând metoda extrudării liniare şi neliniare în vederea aplicării acestuia în cazul nanopereţilor. Se studiază un caz particular în care imaginea este realizată după o direcţie perpendiculară iar iluminarea este difuză (fără umbre) pentru a permite o corelare cât mai precisă între înălţime şi nuanţele de gri.

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