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Some Aspects Regarding the Isolating Method for Elastic Bodies

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Abstract

In the paper, another possibility of isolating the elastic bodies from a solidified system by using the common displacements from the isolating point is presented. This could be more convenient when the displacements are known or easier to be calculated. The efforts from a current section are expressed against the end displacements of a beam element and the results obtained are analysed in a calculus example.

Key words: *deflection, slope, bending, shear.*

General Equations

A beam element subjected to bending has the end efforts M_o, T_o, M_1, T_1 and the end displacements v_o, φ_o, v_1, φ_1 (fig.1). The beam has the bending rigidity EI and the length l.

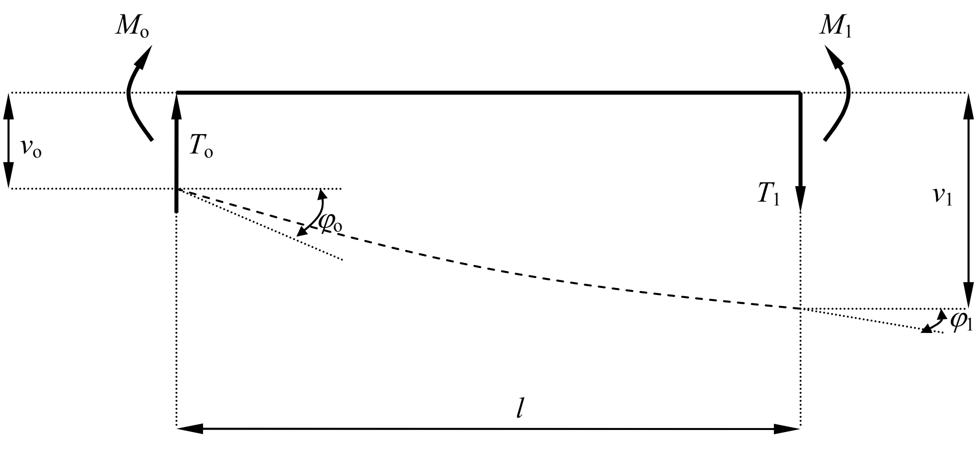

Fig. 1. Beam element with end parameters

The efforts and displacements from right end can be expressed function of the analogue parameters from left end by using the so called equations from the origin parameters method [1]:

$$v_1 = v_o + \varphi_o \cdot l - \frac{M_o}{2EI} \cdot l^2 - \frac{T_o}{6EI} \cdot l^3 \tag{1}$$

$$\varphi_1 = \varphi_o - \frac{M_o}{EI} \cdot l - \frac{T_o}{2EI} \cdot l^2 \tag{2}$$

$$M_1 = M_o + T_o \cdot l \tag{3}$$

$$T_1 = T_o \tag{4}$$

In the case that the element 0-1 is subjected to some supplementary external loads, the above equations have to be completed with some particular terms that could be obtained by using a translation of the starting point.

The first two equations (1) and (2) can be considered as a system with the unknowns M_o and T_o. Solving this system of linear equations it results:

$$M_o = -\frac{6EI}{l^2} v_1 + \frac{2EI}{l} \cdot \varphi_1 + \frac{6EI}{l^2} v_o + \frac{4EI}{l} \varphi_o \tag{5}$$

$$T_o = \frac{12EI}{l^3} v_1 - \frac{6EI}{l^2} \varphi_1 - \frac{12EI}{l^3} v_o - \frac{6EI}{l^2} \varphi_o \tag{6}$$

Replacing (5) and (6) into (3) and (4) the expressions of the unknowns M_1 and T_1 are written under the forms :

$$M_1 = \frac{6EI}{l^2} v_1 - \frac{4EI}{l} \cdot \varphi_1 - \frac{6EI}{l^2} v_o - \frac{2EI}{l} \varphi_o \tag{7}$$

$$T_1 = \frac{12EI}{l^3} v_1 - \frac{6EI}{l^2} \varphi_1 - \frac{12EI}{l^3} v_o - \frac{6EI}{l^2} \varphi_o \tag{8}$$

The (5), (6), (7) and (8) relations express the efforts from the ends of the beam in function of the displacements and slopes from the ends.

The above relations can be arranged also in a matrix form :

$$\begin{bmatrix} M_1 \\ T_1 \\ M_o \\ T_o \end{bmatrix} = \begin{bmatrix} \frac{6EI}{l^2} & -\frac{4EI}{l} & -\frac{6EI}{l^2} & -\frac{2EI}{l} \\ \frac{12EI}{l^3} & -\frac{6EI}{l^2} & -\frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ -\frac{6EI}{l^2} & \frac{2EI}{l} & \frac{6EI}{l^2} & \frac{4EI}{l} \\ \frac{12EI}{l^3} & -\frac{6EI}{l^2} & -\frac{12EI}{l^3} & -\frac{6EI}{l^2} \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ \varphi_1 \\ v_o \\ \varphi_o \end{bmatrix} \tag{9}$$

The (9) relation proves that when some parts of a structure have to be isolated, in the common points it could be introduced the displacements instead of efforts. This observation may be more convenient because there are many cases (especially when the finite elements method is used) when the displacements can be introduces easier than the forces and moments.

It is also important to notice that, in the common points is possible to introduce the reactions (forces and moments) or the common displacements, because the displacements generate de common reactions.

Looking to the form of the (9) relation it can be noticed that it is not possible to expressed the column matrix of displacements in function of the column matrix of efforts because the intermediate matrix (4 x 4) is a singular one.

It is possible instead to express from the first and the third equations from (9) the slopes from the ends in function of the bending moments and displacements from the ends :

$$\varphi_o = \frac{l}{6EI}\left(2M_o + M_1\right) + \frac{v_1 - v_o}{l} \tag{10, a}$$

$$\varphi_1 = -\frac{l}{6EI}\left(M_o + 2M_1\right) - \frac{v_0 - v_1}{l} \tag{10, b}$$

When the element 0-1 is externally loaded with some concentrated forces, moments or pressure the above relations have to be completed with some particular solutions.

A Numerical Example

In order to exemplify the above observations regarding the isolation of the elastic bodies a beam loaded with a concentrated force is considered (fig. 2):

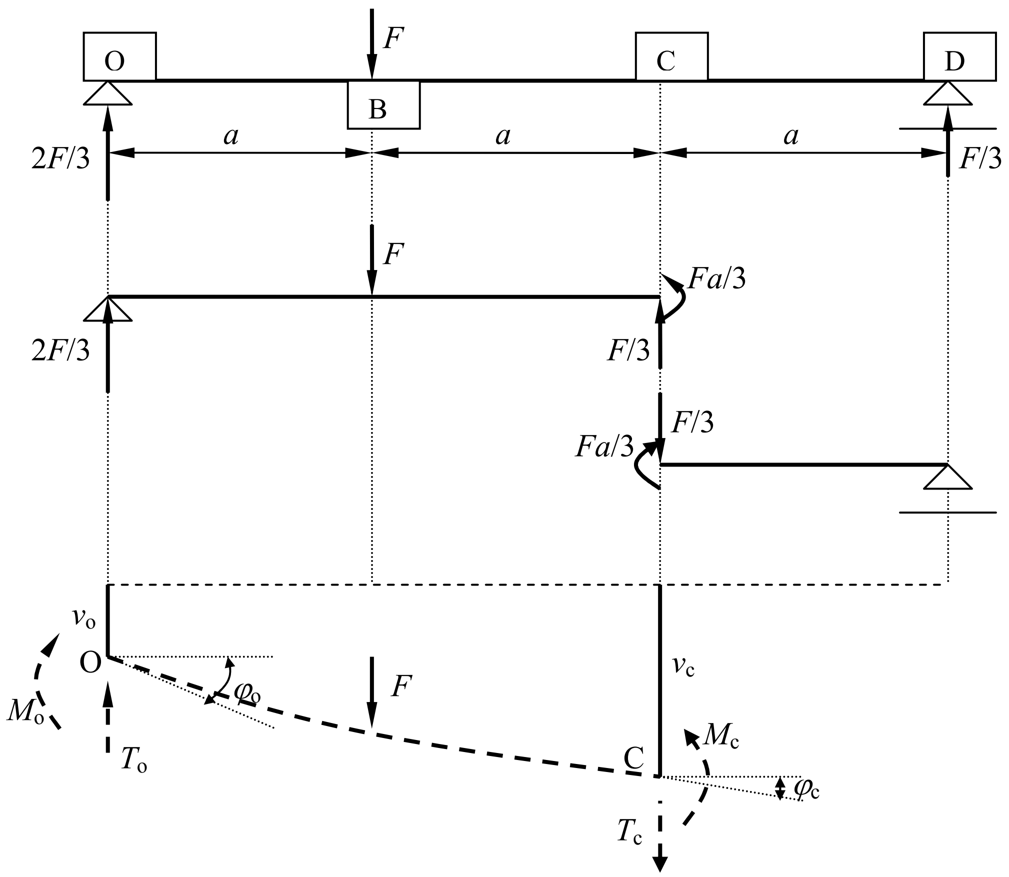

Fig. 2. The isolation of elastic body by introducing the end displacements

If the classical principle of isolation of bodies is applied the beam can be sub structured in two parts, for example *O-B-C* and *C-D* (fig. 2b). In the common point it is necessary to introduce the common reactions : $T_C = -F/3$ and $M_C = F \cdot a/3$.

If the results of this paper are used, in the common points and in origin is necessary to introduce the displacements: $v_c = \frac{7}{18}\frac{Fa^3}{EI}$, $\varphi_C = -\frac{5}{18} \cdot \frac{Fa^2}{EI}$, $v_o = 0$, $\varphi_o = \frac{5}{9}\frac{Fa^2}{EI}$. Using the above displacements and the equations (5), (6), (7) and (8) it results the following reactions at the ends of the *O-B-C* body : $M_o = 0$, $T_o = 2F/3$, $M_C = Fa/3$ and $T_C = -F/3$. It can be noticed that the end displacements introduced at the isolating process create the same reactions at the ends of the element. This expected result proves that the isolation of the two parts may be realised only by introducing the common displacements at the ends (in the isolation points).

Conclusions

In the paper is presented another possibility of isolating some parts of a structure introducing the common displacements in the common section of the substructures. The displacements produce at the ends the same reaction forces and moments as those produced by the classical method of isolation of bodies. The method of introducing the common displacements (instead of common reactions) is more convenient when the finite elements method is used. This observation is analysed in a calculus example.

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Aspecte privind metoda izolării corpurilor elastice

Rezumat

În lucrare se prezintã o metodã de izolare a corpurilor elatice, prin introducerea deplasãrilor comune în secțiunea comunã. Aceste deplasãri produc aceleaşi reacțiuni ca cele produse prin clasica metodã de izolare a corpurilor. Metoda introducerii deplasãrilor comune este de preferat uneori, în special la utilizarea metodei elementelor finite în calculul structurilor. Observația de mai sus este exemplificatã pe un exemplu de calcul.