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On the Calculus of the Equilibrium Moment in the Case of a Plane Mechanism

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Abstract

In this paper the influence of some parameters (technological forces and moments, inertial forces and moments, the weight of the component elements) on the variation of the equilibrium moment in the case of a plane mechanism with a single degree of freedom is analyzed. A computer program that simulates the mechanism functioning has been developed. Finally, some simulation results are presented.

Key words: *mechanism, dynamic analysis, equilibrium moment*

Introduction

In many cases, the design of the mechanisms used in industrial applications requires a rigorous analysis of their running behavior. In this scope, the achievement of computer programs that establish the variations curves of different positional, cinematic or dynamical parameters corresponding to the mechanisms functioning is necessary. In this paper a methodology for the dynamic analysis of plane mechanisms structures is presented. The main purpose of this method is to highlight the influence of some parameters (technological forces and moments, inertial forces and moments, the weight of the component elements) on the variation of the equilibrium moment in the case of a plane mechanism with a single degree of freedom. A simulation program has been developed. Finally, some simulation results are presented.

Theoretical Considerations and Verification Results

Generally, for a plane mechanism with a single degree of freedom, the equilibrium moment verifies the following relation [2]:

$$\overline{M}_e \cdot \overline{\omega}_1 + \sum_j \overline{G}_j \cdot \overline{v}_j + \sum_j \left(\overline{F}_{ru,j} \cdot \overline{v}_j + \overline{M}_{ru,j} \cdot \overline{\omega}_j\right) + \sum_j \left(\overline{F}_{ij} \cdot \overline{v}_j + \overline{M}_{ij} \cdot \overline{\omega}_j\right) = 0 \qquad (1)$$

where: $\overline{M}_e$ is the equilibrium moment, $\overline{G}_j$ is the weight force corresponding to the *j* link, $\overline{F}_{ru,j}$ and $\overline{M}_{ru,j}$ are the resultant technological force and moment which act on the *j* link and which are reduced in the mass centre of this link, $\overline{F}_{ij}$ and $\overline{M}_{ij}$ are the resultant inertia force and the resultant inertia moment corresponding to the *j* link, $\overline{v}_j$ is the speed of the mass centre of the *j* link, $\overline{\omega}_j$ is the angular speed of the *j* link.

From (1), the variation of the equilibrium moment can be determined with the following relation:

$$M_e = -\frac{1}{\omega_1} \cdot \left(m_e^g + m_e^{ru} + m_e^{f_i} + m_e^{m_i} \right) \tag{2}$$

where:

$$m_e^g = \sum_j \overline{G}_j \cdot \overline{v}_j \tag{3}$$

$$m_e^{ru} = \sum_j \left(\overline{F}_{ru,j} \cdot \overline{v}_j + \overline{M}_{ru,j} \cdot \overline{\omega}_j \right) \tag{4}$$

$$m_e^{f_i} = \sum_j \overline{F}_{ij} \cdot \overline{v}_j \tag{5}$$

$$m_e^{m_i} = \sum_j \overline{M}_{ij} \cdot \overline{\omega}_j \tag{6}$$

For obtaining the variation of the equilibrium moment, the positional and cinematic analysis of the mechanism has to be accomplished. In the stage of the positional analysis the position of the component links are determined depending on the position of the leader link. With this end in view, some analytical methods [1] can be used: the bars method, the method of the independent contours with representation in complex plane, the method of the projection of the independent vector circuits etc. In the phase of the cinematic analysis the following parameters can be determined: the angular speeds and accelerations of the component links, the speed and acceleration of the main points on the mechanism (the center of the joints, the points where the technological forces and moments are applied). With this end in view, we can derive the variation functions of the corresponding position parameters or apply some analytical methods [1]: the method of the independent cycles, the method of the transfer function etc.

The influence of the technological forces and moments, inertial forces and moments and the weight of the component elements on the variation of the equilibrium moment have been analyzed in the case of the plane mechanism with a single degree of freedom and two independent contours represented in figure 1.

For this mechanism the following elements are considered to be known:

- the dimensions of the component links: *OA*=0.06m; *AB*=0.45m; *BC*=*BD*=*CD*=0.3m; *DE*=0.75m; *OC*=0.48m; *MC*=0.1m. The mass centers: C_1, C_2, C_4 are on the middle of the corresponding links and C_3 is on the mass centre of the triangle *BCD*.

- the mass of the component links: $m_1 = 1{,}5\text{kg}$; $m_2 = 7\text{kg}$; $m_3 = 9\text{kg}$; $m_4 = 12\text{kg}$; $m_5 = 2\text{kg}$.

- the moments of inertia of the links: $I_{C_1} = 0{,}001\ \text{kgm}^2$; $I_{C_2} = 0{,}12\ \text{kgm}^2$; $I_{C_3} = 0{,}25\ \text{kgm}^2$; $I_{C_4} = 0{,}56\ \text{kgm}^2$. The value of I_{C_5} is neglected.

- the technological forces and moments: $F_{ru}^{dr} = 500\ \text{N}$; $F_{ru}^{st} = 50\ \text{N}$; $M_{ru} = 60\ \text{Nm}$.

- the nominal angular speed of the leader link of the mechanism: $\omega_1 = 20\text{rad/s}$.

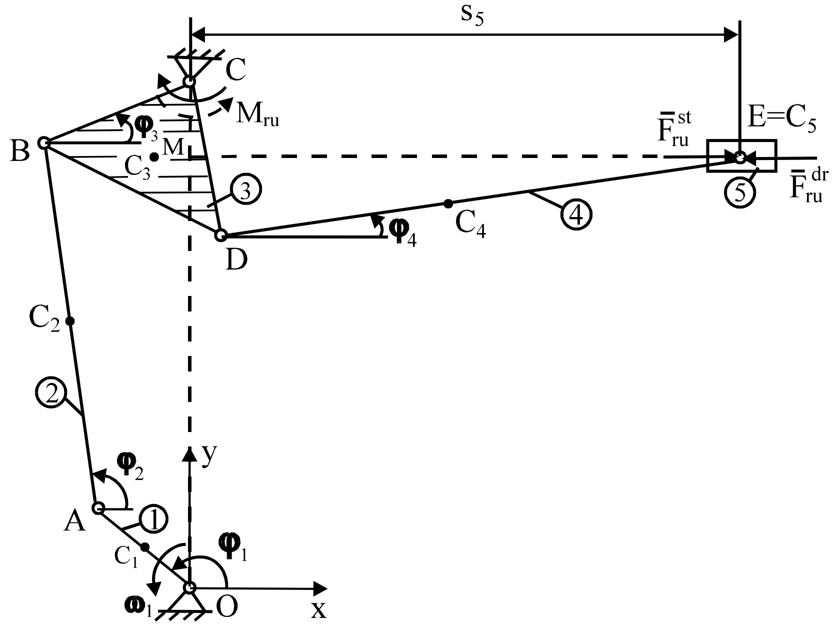

Fig. 1. Plane mechanism with two independent contours

The method of the projection of the independent vector circuits has been used for the positional analysis. The mechanism has two independent contours: $O-A-B-C-O$ and $C-D-E-C$.

By projecting the vector circuits corresponding to these independent contours on the *x* and *y* axes (fig. 1), the following systems of equations are obtained:

$$\begin{cases} OA\cdot\cos\varphi_1 + AB\cdot\cos\varphi_2 + BC\cdot\cos\varphi_3 = 0 \\ OA\cdot\sin\varphi_1 + AB\cdot\sin\varphi_2 + BC\cdot\sin\varphi_3 - OC = 0 \end{cases} \tag{7}$$

$$\begin{cases} CD\cdot\cos\varphi_{3'} + DE\cdot\cos\varphi_4 - s_5 = 0 \\ CD\cdot\sin\varphi_{3'} + DE\cdot\sin\varphi_4 + MC = 0 \end{cases} \tag{8}$$

where: $\varphi_{3'} = \varphi_3 + \frac{4\pi}{3}$.

By solving the systems of equations (7) and (8), the unknown parameters: $\varphi_2, \varphi_3, \varphi_4$ and s_5 can be calculated from the following relations:

$$A_2\cdot\cos\varphi_2 + B_2\cdot\sin\varphi_2 = C_2 \tag{9}$$

where:

$$\begin{cases} A_2 = 2\cdot OA\cdot AB\cdot\cos\varphi_1 \\ B_2 = 2\cdot OA\cdot AB\cdot\sin\varphi_1 - 2\cdot AB\cdot OC \\ C_2 = BC^2 - OA^2 - AB^2 - OC^2 + 2\cdot OA\cdot OC\cdot\sin\varphi_1 \end{cases} \tag{10}$$

$$\begin{cases} \cos\varphi_3 = -\dfrac{1}{BC}\cdot(OA\cdot\cos\varphi_1 + AB\cdot\cos\varphi_2) \\ \sin\varphi_3 = -\dfrac{1}{BC}\cdot(OA\cdot\sin\varphi_1 + AB\cdot\sin\varphi_2 - OC) \end{cases} \tag{11}$$

$$\sin\varphi_4 = -\frac{1}{DE}\cdot(CD\cdot\sin\varphi_{3'} + CM) \tag{12}$$

$$s_5 = CD\cdot\cos\varphi_{3'} + DE\cdot\cos\varphi_4 \tag{13}$$

The angular and linear speeds and accelerations distributions have been determined by deriving the variation functions of the corresponding position parameters. Then, the variation of the equilibrium moment M_e has been obtained using the relation (2).

The analysis has been transposed into a computer program using the Maple software.
In figure 2 the variation of the equilibrium moment M_e is presented.

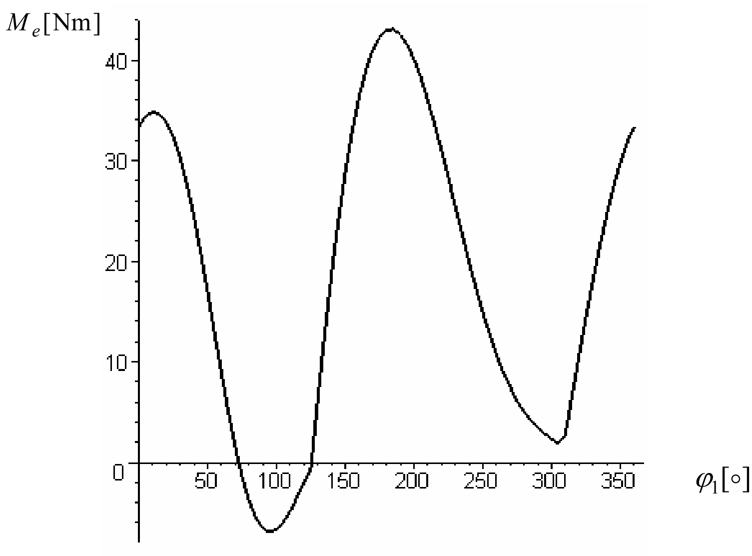

Fig. 2. The variation of the equilibrium moment

In figures 3÷6 the variation of the components: $-(m_e^g/\omega_1)$, $-(m_e^{ru}/\omega_1)$, $-(m_e^{fi}/\omega_1)$ and $-(m_e^{mi}/\omega_1)$ are represented.

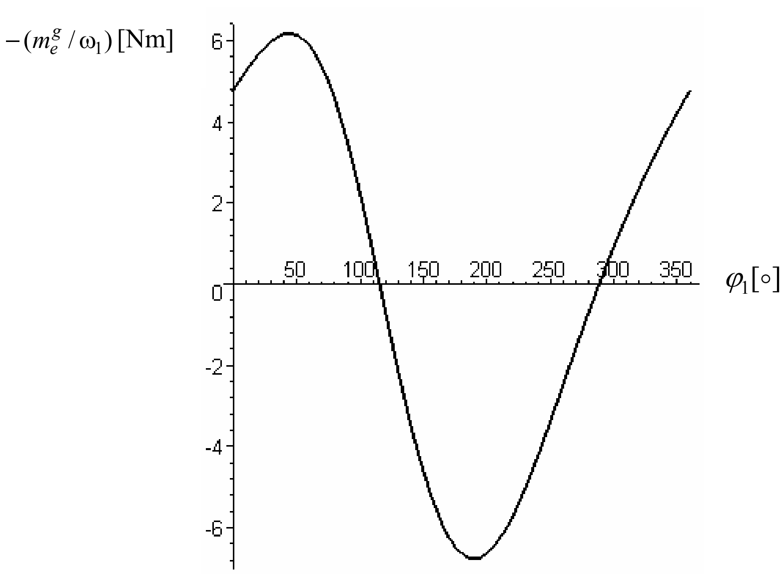

Fig. 3. The variation of the $-(m_e^g/\omega_1)$ component

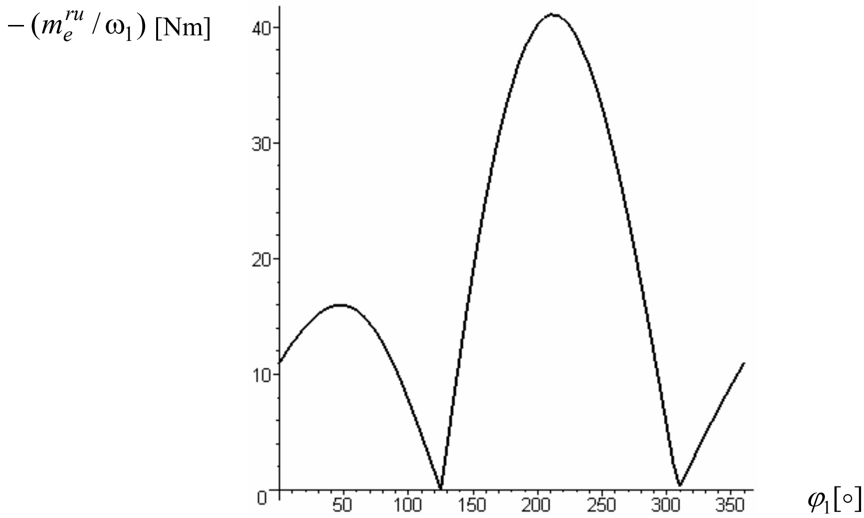

Fig. 4. The variation of the $-(m_e^{ru}/\omega_1)$ component

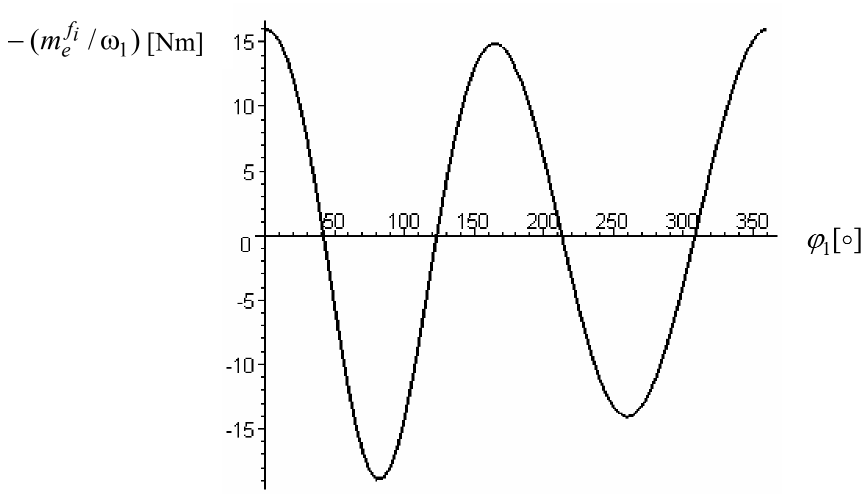

Fig. 5. The variation of the $-(m_e^{f_i}/\omega_1)$ component

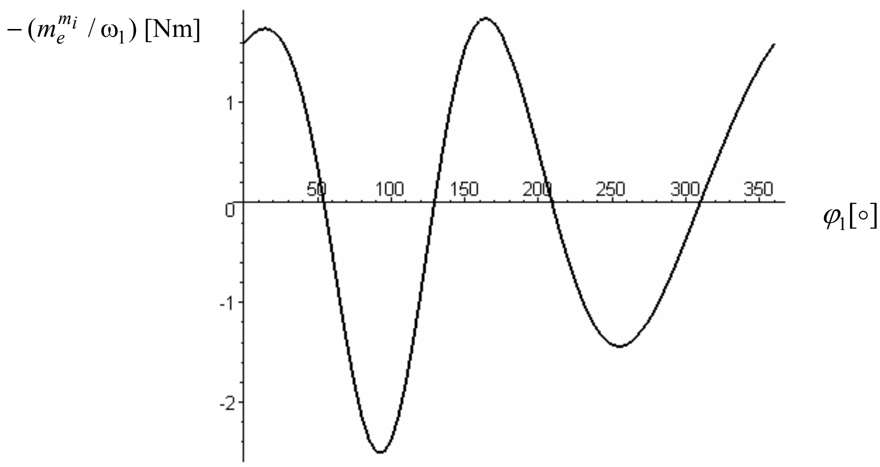

Fig. 6. The variation of the $-(m_e^{m_i}/\omega_1)$ component

Conclusions

In this paper a methodology for the analysis of a plane mechanism structure is presented. Positional, cinematic and dynamical stages are analyzed. It was highlighted the influence of some parameters (technological forces and moments, inertial forces and moments, the weight of the component elements) on the variation of the equilibrium moment. A simulation program has been developed. Finally, some simulation results have been presented. In the case of the analyzed mechanism the technological forces and moments and the inertial forces have the most significant influence on the variation of the equilibrium moment. The weight of the component elements and the inertial moments have a much smaller influence on the variation of the equilibrium moment.

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Asupra calculului momentului de echilibrare în cazul unui mecanism plan

Rezumat

In articol se analizează influenţa anumitor parametri (momentele şi forţele tehnologice, momentele şi forţele de inerţie, greutatea elementelor componente) asupra variaţiei momentului de echilibrare în cazul unui mecanism plan cu un singur grad de libertate. A fost dezvoltat un program de calculator care simulează funcţionarea mecanismului. In final, sunt prezentate o serie de rezultate ale simulărilor.