BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LXII No. 1/2010	35 - 38	Seria Tehnică

The Construction of Intersection Points between a Straight Line and an Ellipsoid

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Abstract

In this paper, the drawing of the intersection between a straight line and an ellipsoid, using auxiliary graphic constructions, is presented. This kind of intersection demands many preparatory steps and auxiliary graphic drawings, as the finding of the tangent planes. The draft has been obtained using both the means offered by the descriptive geometry and the AutoCAD program. So, a very precise construction of the intersection points coordinates is guaranteed. Ellipsoidal shells are used in the chemical and petrochemical industry, as ellipsoidal heads for: pressure vessels, heat exchangers, posts etc. On these heads, some fittings can be installed and their axes can have various angles to the horizontal or vertical axis.

Key words: *ellipsoid, straight line, intersection points.*

Theoretical Considerations

A very interesting problem, but rarely broached, is the intersection between a straight line and a non-degenerate quadric surface.

The following surfaces belong to the non-degenerate quadrics category: sphere, hyperboloid of one sheet, hyperboloid of two sheets, ellipsoid, hyperbolic paraboloid and elliptic paraboloid. These quadrics are, from a mathematical point of view, a second order surfaces.

In the present paper, the construction of the intersection between a line and an ellipsoid, using auxiliary graphic constructions, is presented. In contrast with the intersection between a line and a polyhedron, which has a relatively simple graphic solution, the intersection between a straight line and an ellipsoid demands many more preparatory steps and many auxiliary graphic constructions as the finding of the tangent planes, by example. To find the tangent plane demands also some auxiliary constructions. The theoretical instruments given by the descriptive geometry can be successfully used to draw, with a maximum precision, all these kind of intersections. The necessary steps can be used to draw this intersection, utilizing a computer program, as AUTOCAD 2008, therefore the intersection points coordinates can be found with maximum precision.

The Graphic Construction of the Intersection between a Line and an Ellipsoid

In this paper, the graphic construction (the draft) between a given straight line *(MN)* and an ellipsoid *(E)* is presented. The ellipsoid is formed by revolving an ellipse about its major axis and the major axis is perpendicular to the vertical coordinate plane *[V]*.

The graphic construction is presented in the figure 1, both in horizontal and vertical coordinate planes, as a draft.

As it is known, the ellipsoid is the geometric locus of all space points whom the coordinates respect the following equation:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \qquad (1).$$

So, the horizontal projection of the ellipsoid in the draft will be an ellipse having the following equation:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad (2),$$

and the projection to the vertical coordinate plane is a circle, having the radius equal to the semi-minor axis of the ellipse described by (2).

Also, it is well known that the focus of an ellipse is given by:

$$f=\sqrt{a^2-b^2} \qquad (3),$$

where *a* and *b* are the semi-major and the semi-minor axis of the ellipse described by (2).

In horizontal projection in figure 1, the ellipsoid becomes the ellipse with the aa_1 and bb_1 as semi-major and semi-minor axis and in the vertical projection is a circle with the diameter $b`b`_1= bb_1$. To find the entrance and exit points of the given line which pierces the ellipsoid, named J_1 $(j_1,j`_1)$ and $J_2$ $(j_2,j`_2)$, some steps must be followed.

We must include the line *(MN)* *(mn, m`n`)* in a vertical plane, named *[P]*, thus the horizontal trace of this plane coincides with *mn* (the horizontal trace of a plane is the line in which the plane pierces the horizontal coordinate plane): $p_H= mn$. We can identify the foci of the ellipse using (3), meaning, in the draft, F $(f, f`)$, $F_1$ $(f_1, f`_1)$. It must be found, as a preliminary, the intersection between the plane *[P]* and the ellipsoid.

If E $(e, e`)$ is the ellipsoid's centre, then $e=a=a_1$. The director circle, lettered *(C)*, must be drawn. This construction is made as it follows: a point *p* on the ellipse is taken and the tangent *t* in this point is drawn. To do this, it is used the method shown in figure 2.

So, it is necessary to draw a circle (C_1), having a given radius, with the centre lying on p_H and passing through f_1; by example, a circle having the centre in *m*.

A perpendicular from f_1 to *t* is drawn and the distance f_1s is determined, as the double of the perpendicular segment line from f_1 to *t*. The circle *(C)*, having the centre in *f* and the radius *fs*, is drawn. The line *fm* intersects the perpendicular line from f_1 to p_H in point *c*. Now, the points *d* and *g* can be drawn, as tangency points of two lines drawn from *c* to the circle *(C)*. The lines *fd* and *fg* pierce p_H in the points d_1 and d_2. These points are the vertices of an ellipse, which is the result of the intersection between the vertical plane *[P]* and the given ellipsoid.

We draw a new plane *[Q]* parallel to the vertical coordinate plane, which contains the ellipse minor axis, so q_H //*(ox)*, where *(ox)* is also called the ground line. The lines D_1B_1 and D_2B

intersect in the common point *I(i, i`)*. Through *i* we draw a parallel to *mn*, which intersects $q_H$ in the point *l*. The vertical projection, *l`*, is obtained by letting raise a perpendicular to the ground line until it intersects the parallel to *m`n`* through *i`*. Let it be a given point *U(u, u`)* lying on the line *(MN)*. Its vertical projection *i`u`* intersects the perpendicular raised from *b* in the point v_1`. It's obvious that the horizontal projection of $v_1$` is $v_1=b$.

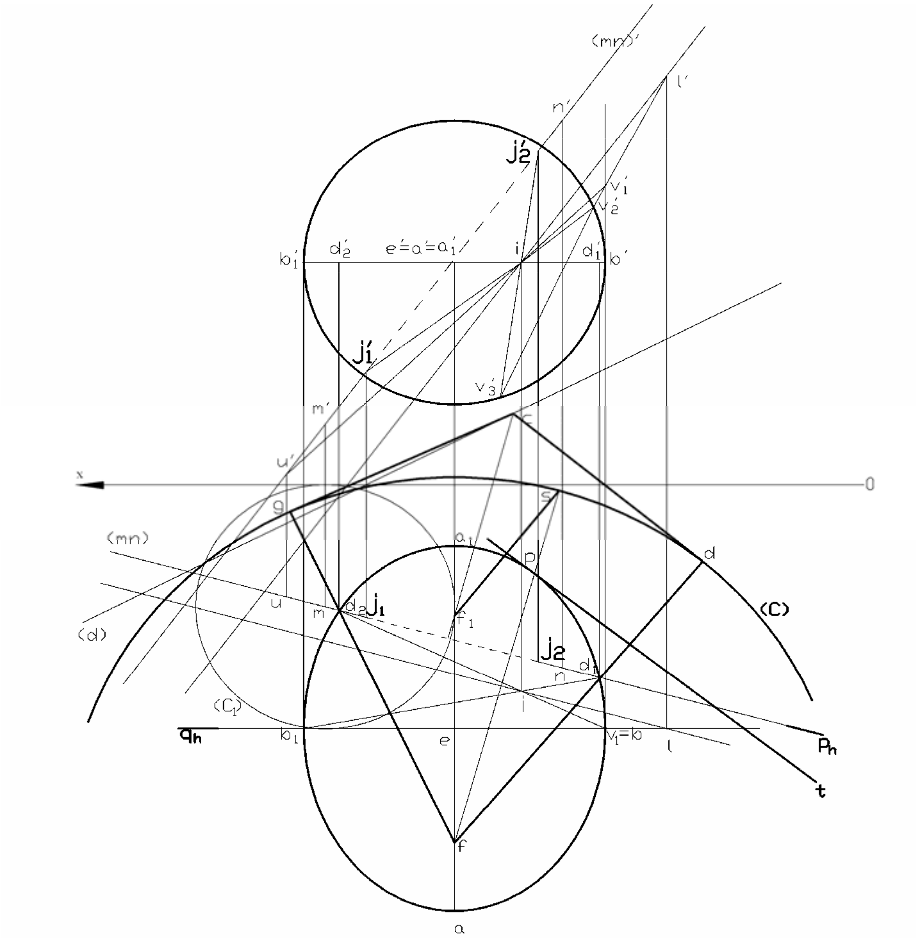

Fig. 1. The draft of the intersection

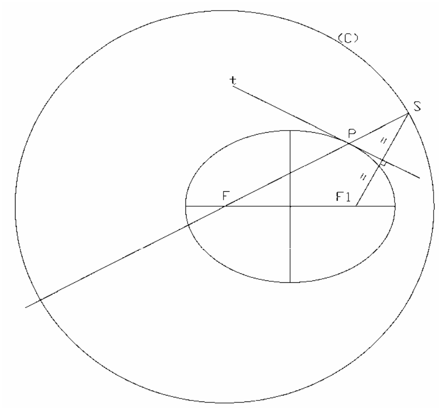

Fig. 2. The construction of the tangent line to the ellipse

The line $l`v_1`$ intersects a circle having the diameter $b`b_1`$ in the points $v_2`$ and $v_3`$. The lines $i`v_2`$ and $i`v_3`$ intersect the line $m`n`$ in the demanded points, having the vertical projections in $j_1`$ and $j_2`$. Then, their horizontal projections j_1 and j_2 are found by letting fall perpendicular lines to the ground line until they intersect *mn*, the horizontal projection of *(MN)*. So, the two intersection points between the line *(MN)* and the ellipsoid, J_1 $(j_1,j`_1)$ and $J_2$ $(j_2,j`_2)$, are found.

Conclusions

In the present paper it is described a way to obtain the intersection points between a straight line and an ellipsoid having the major axis perpendicular to the vertical coordinate plan. The draft needs to use many auxiliary drawings and it is obtained using both the AutoCAD program and the means offered by the descriptive geometry. Using the AutoCAD program, a very precise construction of the intersection points coordinates is guaranteed. We mention that the ellipsoidal shells are used in the chemical and petrochemical industry, as ellipsoidal heads for: pressure vessels, heat exchangers, posts etc. On these heads, some fittings can be installed and their axes can have various angles to the horizontal or vertical axis.

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Construcția punctelor de intersecție dintre o dreaptă şi un elipsoid

Rezumat

In această lucrare este prezentată intersecția dintre o dreaptă şi un elipsoid, folosind construcții grafice auxiliare. Acest tip de intersecție necesită mai mulți paşi pregatitori şi construcții grafice suplimentare, cum ar fi găsirea unor plane tangente. Epura a fost obținută utilizând atât mijloacele oferite de geometria descriptivă, cât şi programul AutoCAD. Astfel, este obținută o construcție foarte precisă a coordonatelor punctelor de intersecție. Învelişurile elipsoidale sunt utilizate în industria chimică şi petrochimică sub denumirea de funduri şi capace elipsoidale. Echipamentele tehnologice din a căror componență fac parte acestea sunt: vase sub presiune, schimbătoare de căldură, coloane, amestecătoare etc. Pe aceste capace elipsoidale sunt montate racorduri având diverse unghiuri de incidență. Axele acestor racorduri pot fi asimilate dreptei de intersecție din lucrarea prezentată.