

# On the Positional Analysis of a Mitsubishi Robot System with Six Degrees of Freedom

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## Abstract

*The paper presents a method that permits the positional analysis of the Mitsubishi Melfa RV-1A robot system that contains six rotation modules. With this end in view, Khalil-Kleinfinger parameters have been used. The verification of the results obtained with the positional model has been done using the robot arm controller. Finally, some simulation results are presented.*

**Key words:** robot, rotation module, position, orientation.

## Introduction

Today the robots are widely used in different operations of manipulation that are part of an industrial process. In order to program the tasks of these manipulator robots the human operator has to know the relative position and orientation between the component modules and between the grasping module and the base of the robot during its movement. The positional analysis of the mechanisms of these robots has to be very precise. The results can then be verified using the control system of the robot.

In this paper a method that allows the positional analysis of the Mitsubishi Melfa RV-1A robot system (fig. 1) is presented. In this scope, Khalil-Kleinfinger parameters have been used. The verification of the model has been done using the robot controller.

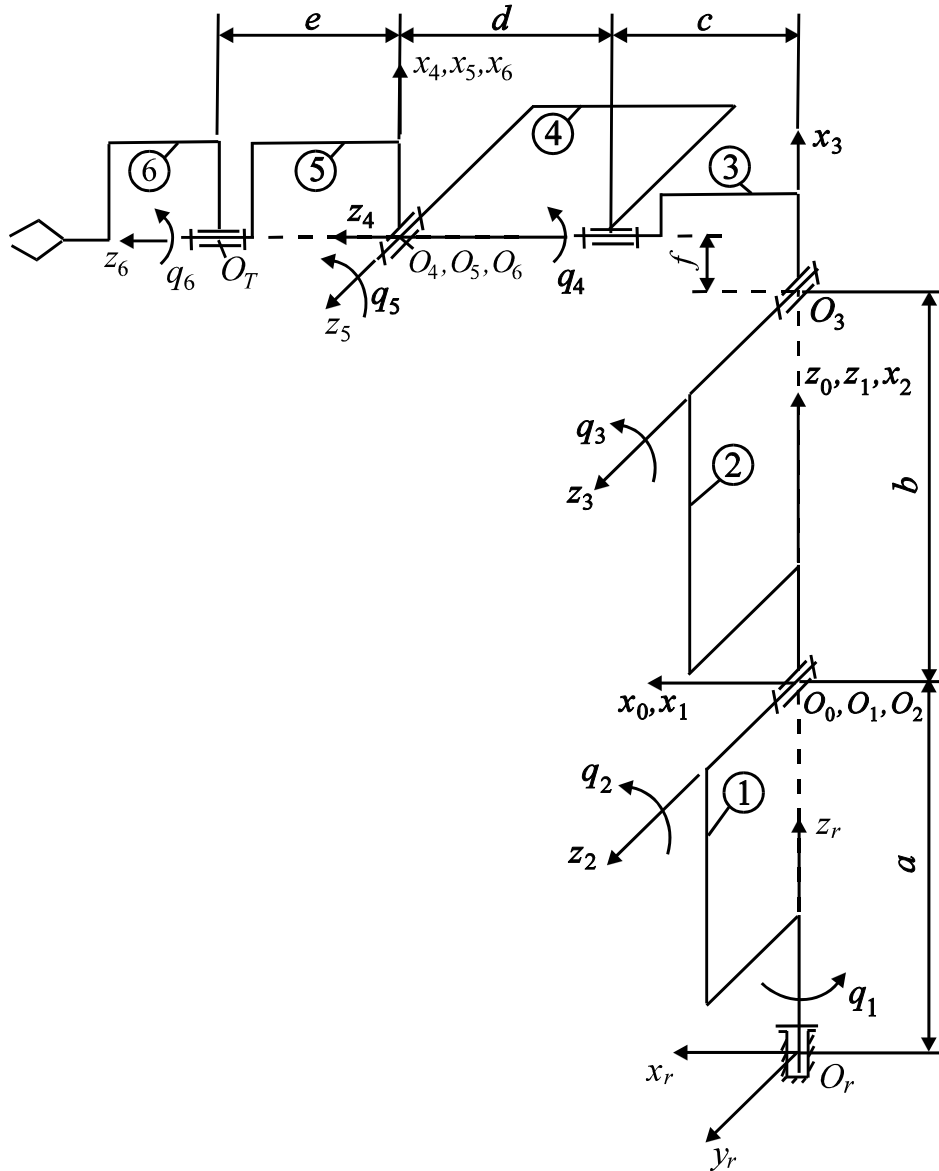
## Theoretical Considerations and Verification Results

In figure 1, the cinematic scheme of the mechanism of the Mitsubishi Melfa RV-1A robot system is presented. The systems of coordinates,  $(O_i x_i y_i z_i), i = \overline{0,6}$ , have been attached to each component module  $i, i = \overline{0,6}$ , using the Khalil-Kleinfinger method [4].

The axes,  $(O_i z_i), i = \overline{1,6}$ , have the direction of the joints  $(C_i)$  between the modules  $i-1$  and  $i$ , the axes  $(O_i x_i), i = \overline{1,5}$ , have the direction of the common perpendicular between the axes  $(O_i z_i)$  and  $(O_{i+1} z_{i+1})$  and the axes  $(O_i y_i), i = \overline{1,6}$ , are chosen in such a way that  $(O_i x_i y_i z_i), i = \overline{1,6}$ , are right-orthogonal systems of coordinate. In order to simplify the presentation of the cinematic scheme, the  $(O_i y_i)$  axes have not been represented.

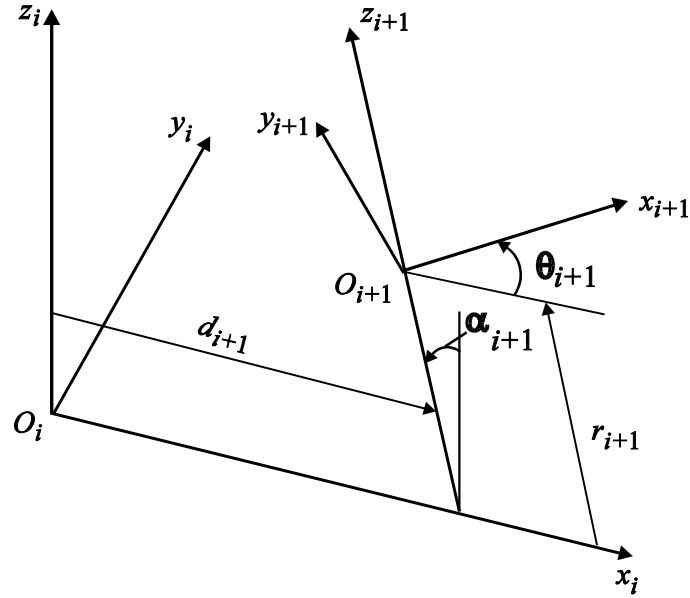
The fixed system of coordinate ( $T_0$ ) is chosen so that it coincides with ( $T_1$ ) when  $q_1 = 0$ ,  $q_1$  being the generalized coordinate corresponding to the joint ( $C_1$ ).

The axis ( $O_6x_6$ ) is chosen to be collinear with the axis ( $O_5x_5$ ) when  $q_6 = 0$  and the origin  $O_6$  will be at the intersection of the axes ( $O_5x_5$ ) and ( $O_6z_6$ ).



**Fig. 1.** Mitsubishi Melfa RV-1A robot mechanism

The systems of coordinates ( $T_{i+1}$ ) and ( $T_i$ ),  $i = \overline{0,5}$ , are relatively positioned using four parameters (fig. 2): the angle  $\alpha_{i+1}$  between the axes ( $O_i z_i$ ) and ( $O_{i+1} z_{i+1}$ ), the distance  $d_{i+1}$  between the same axes, the angle  $\theta_{i+1}$  between the axes ( $O_i x_i$ ) and ( $O_{i+1} x_{i+1}$ ) and the distance  $r_{i+1}$  between ( $O_i x_i$ ) and ( $O_{i+1} x_{i+1}$ ), measured on the positive direction of the axis ( $O_{i+1} z_{i+1}$ ).



**Fig. 2.** Khalil-Kleinfinger parameters

In table 1 the values corresponding to the Khalil-Kleinfinger parameters are presented.

**Table 1.** The values of the Khalil-Kleinfinger parameters

$i$	$\alpha_i$	$d_i$	$\theta_i$	$r_i$
1	0	0	$q_1$	0
2	$-90^\circ$	0	$q_2$	0
3	0	$b$	$q_3$	0
4	$-90^\circ$	$f$	$q_4$	$d+c$
5	$90^\circ$	0	$q_5$	0
6	$-90^\circ$	0	$q_6$	0

The homogeneous matrix corresponding to the relative position and orientation of the systems of coordinates  $(T_{i+1})$  and  $(T_i)$ ,  $i = \overline{0,5}$ , has the following general form [4]:

$${}^i T_{i+1} = \begin{bmatrix} {}^i R_{i+1} & {}^{(i)}O_i O_{i+1} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos \theta_{i+1} & -\sin \theta_{i+1} & 0 & d_{i+1} \\ \cos \alpha_{i+1} \cdot \sin \theta_{i+1} & \cos \alpha_{i+1} \cdot \cos \theta_{i+1} & -\sin \alpha_{i+1} & -r_{i+1} \cdot \sin \alpha_{i+1} \\ \sin \alpha_{i+1} \cdot \sin \theta_{i+1} & \sin \alpha_{i+1} \cdot \cos \theta_{i+1} & \cos \alpha_{i+1} & r_{i+1} \cdot \cos \alpha_{i+1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where:  ${}^i R_{i+1}$  is the rotation matrix corresponding to the relative orientation of the systems of coordinates  $(T_{i+1})$  and  $(T_i)$ .

The generalized coordinates,  $q_i$ ,  $i = \overline{1,6}$ , will be the angles  $\theta_i$  because all the component modules are of rotation.

By applying the relation (1) and taking into account the values of the Khalil-Kleinfinger parameters in the table 1, the following expressions for the homogeneous matrices,  ${}^i T_{i+1}$ ,  $i = \overline{0,5}$ , are obtained:

$$\begin{aligned} {}^0 T_1 &= \begin{bmatrix} c1 & -s1 & 0 & 0 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^1 T_2 = \begin{bmatrix} c2 & -s2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s2 & -c2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^2 T_3 = \begin{bmatrix} c3 & -s3 & 0 & b \\ s3 & c3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\ {}^3 T_4 &= \begin{bmatrix} c4 & -s4 & 0 & g \\ 0 & 0 & 1 & d+c \\ -s4 & -c4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^4 T_5 = \begin{bmatrix} c5 & -s5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s5 & c5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^5 T_6 = \begin{bmatrix} c6 & -s6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s6 & -c6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (2)$$

where:

$$\begin{cases} si = \sin q_i \\ ci = \cos q_i \end{cases} \quad i = \overline{1,6} \quad (3)$$

The position of the reference point  $O_T$  belonging to the flange of the grasping device (fig. 1) can be determined with the following relation:

$${}^{(0)}O_0 O_T = {}^0 T_5 \cdot {}^{(5)}O_5 O_T \quad (4)$$

where: the homogeneous matrix  ${}^0 T_5$  is given by:

$${}^0 T_5 = {}^0 T_1 \cdot {}^1 T_2 \cdot {}^2 T_3 \cdot {}^3 T_4 \cdot {}^4 T_5 \quad (5)$$

and the position vector  ${}^{(5)}O_5 O_T$ , expressed in a homogeneous form [1], has the following expression:

$${}^{(5)}O_5 O_T = [0 \quad e \quad 0 \quad 1]^T \quad (6)$$

The geometric parameters corresponding to the Mitsubishi robot mechanism have the following values:  $a = 300$  mm ;  $b = 250$  mm ;  $c = 43$  mm ;  $d = 117$ mm ;  $e = 72$  mm ;  $f = 90$  mm .

The relations above have been transposed in a computer program which allows calculating very easy and precisely the positional parameters of the Mitsubishi robot.

The results obtained with the computer program have been verified using the robot controller. The relations between the generalized coordinates,  $q_i$ ,  $i = \overline{1,6}$ , used in the positional analysis and those used by the controller and noted with  $q'_i$ ,  $i = \overline{1,6}$ , are the following:

$$\begin{cases} q_1 = q_1^r \\ q_2 = -90^\circ + q_2^r \\ q_3 = -90^\circ + q_3^r \\ q_4 = q_4^r \\ q_5 = q_5^r \\ q_6 = q_6^r \end{cases} \quad (7)$$

The relations between the coordinates of the reference point  $O_T$  calculated with the relation (4) and those given by the controller, calculated by taking into account the system of coordinates  $(O_r, x_r, y_r, z_r)$  (fig. 1) and noted with  $x_r$ ,  $y_r$  and  $z_r$  are:

$$\begin{cases} {}^{(0)}x_{O_T} = x_r \\ {}^{(0)}y_{O_T} = y_r \\ {}^{(0)}z_{O_T} + a = z_r \end{cases} \quad (8)$$

In the table 2 some results obtained with the Mitsubishi robot controller are presented.

**Table 2.** Results obtained with the robot controller

$q_1^r$ [°]	-13.33	20.65	40.13	-42.43	21.70
$q_2^r$ [°]	24.71	43.87	13.44	-15.11	23.19
$q_3^r$ [°]	84.14	63.60	131.22	120.33	96.86
$q_4^r$ [°]	-19.79	23.34	41.94	17.36	52.23
$q_5^r$ [°]	20.62	-24.12	-69.49	-27.18	-42.19
$q_6^r$ [°]	5.73	-43.01	32.55	7.02	35.47
$x_r$ [mm]	329.82	400.58	242.82	128.37	332.85
$y_r$ [mm]	-87.25	138.48	145.73	-130.64	91.30
$z_r$ [mm]	516.26	524.08	473.14	599.66	526.49

In the table 3 the results obtained with the computer program are presented.

**Table 3.** Results obtained with the computer program

${}^{(0)}x_{O_T}$ [mm]	329.824	400.577	242.813	128.371	332.838
${}^{(0)}y_{O_T}$ [mm]	-87.278	138.509	145.736	-130.637	91.314
${}^{(0)}z_{O_T} + a$ [mm]	516.236	524.085	473.131	599.670	526.505

## Conclusions

The paper presents a method that allows the positional analysis of the Mitsubishi Melfa RV-1A robot system. Khalil-Kleinfinger parameters have been used for obtaining the relative position and orientation between the component modules and between the grasping module and the base of the robot. The verification of the results has been done using the robot arm controller. The results presented in the table 3 show a very good accordance between the results obtained with the computer program and those given by the robot controller.

## References

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## Asupra analizei poziționale a unui sistem robot Mitsubishi cu șase grade de libertate

### Rezumat

*Articolul prezintă o metodă care permite analiza pozițională a sistemului robot Mitsubishi Melfa RV-1A care conține șase module de rotație. Pentru aceasta au fost folosiți parametrii Khalil-Kleinfinger. Verificarea rezultatelor obținute cu modelul pozițional s-a făcut folosind controlerul robotului. În final, sunt prezentate o serie de rezultate ale simulărilor.*