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The Physical Shaping of the Precision Oxygen Analyzer

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Abstract

In this work we are proposing as a topic of discussion the shape of the oxygen analyzer of precision by checking the experimental results regarding the linear concentration of oxygen depending of the torsion balance. The concentration of oxygen influences the magnetic susceptibility of the average in which the torsion balance exists and the bringing back in initial 0 positions can be realized by an electric current producing an electromagnetic force in the opposite direction. The constants which appear in the dependence relation in this way can be expressed depending on the specific physics measures also indicating external factors which can influence the precision of the measurements.

Key words: *magnetic susceptibility, magnetic forces, torsion, concentration*

Fundamental Principles

An element of small dimensions, in comparison with the distances which an inhomogeneous magnetic field, has appreciable variations, diving into a medium of a different magnetic susceptibility is submitted to an action of a force given by the relation

$$\vec{F}_{\text{Faraday}} = (\chi_1 - \chi_2)v\ H \operatorname{grad} H \tag{1}$$

where v represents the body volume, χ_1 the magnetic susceptibility of the element, χ_2, the magnetic susceptibility of the medium and H the intensity of the magnetic field.

In the case of our model, there are used two identical small spheres, like in Fig. 3, by introducing them into the inhomogeneous magnetic field they are going to interact with this according to the difference of the susceptibilities (which characterizes the medium and the bodies). If there is used a pendulum of torsion, the Faraday force torque can be balanced by the torque of torsion forces from the wire

$$|\vec{F}_{\text{torsiune}}| = k(\alpha - \alpha_0) \tag{2}$$

where k is a torsion constant and $\Delta\alpha = \alpha - \alpha_0$ the angle of torsion of the wire. The angle α_0 represents the initial position of the wire in comparison with the chosen initial system of reference.

In order to bring back the torsion wire into the equilibrium position we must use an external force, electromagnetic force, for example. In this way we can find a direct correlation between a measurable quantity and the magnetic susceptibility.

In the case of electromagnetic force towards the length element of the conductor act an elementary force

$$\mathrm{d}\vec{F}_{\text{electromagnetică}} = I\left(\mathrm{d}\vec{l} \times \vec{B}\right). \tag{3}$$

The total force can be evaluated through an integral

$$\vec{F}_{\text{electromagnetică}} = \int_{\Gamma} \mathrm{d}\vec{F}_{\text{electromagnetică}} = \int_{\Gamma} I\left(\mathrm{d}\vec{l} \times \vec{B}\right). \tag{4}$$

In the case of oxygen the magnetic susceptibility is more different that the one of nitrogen, reason for which in a mixture of oxygen and nitrogen, the relative concentration will be strongly dependent of the susceptibility of the mixture, in a first approximation a weighted average

$$\chi_{2} = c_{\mathrm{O}_2}\chi_{\mathrm{O}_2} + c_{\mathrm{N}_2}\chi_{\mathrm{N}_2}. \tag{5}$$

By combining the relations (1),(4) and (5) we can find a direct correlation between the concentration of oxygen and the intensity of the electric current.

Initial Considerations

If we accept the values that are proposed in the work [1], $\chi_{\mathrm{O}_2} = +139 \cdot 10^{-9}\,\mathrm{cm}^{-1}$ and $\chi_{\mathrm{N}_2} = -0{,}49 \cdot 10^{-9}\,\mathrm{cm}^{-1}$ in units CGS, the dependence of the medium susceptibility of concentration will have the indicated dependence in Fig. 1.

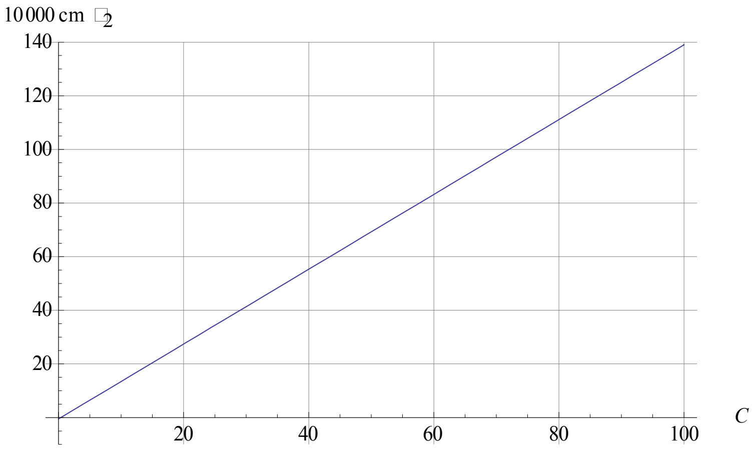

Fig. 1. The concentration dependence by the susceptibility

The 3D model of an inhomogeneous magnetic field, also according to the work **[1]**, can be realized with a good approximation with the help with some more functions. Hereby, we are proposing trigonometric functions. In the rotation plan (plan $x\mathrm{O}y$) we are supposing that the intensity of the magnetic field has a gradient only towards the axis $\mathrm{O}x$ described in the relation bellow,

$$H(x,y) = 12000\cos\left(\frac{y[\mathrm{mm}]}{5}\right)\left[\frac{\mathrm{A}}{\mathrm{m}}\right]. \tag{6}$$

The (6) function's diagram is represented in Fig. 2 and highlights a gradient of the magnetic field by the following differential equation

$$\mathrm{d}\,H(x,y)=\frac{\partial H}{\partial x}\mathrm{d}\,x+\frac{\partial H}{\partial y}\mathrm{d}\,y=-12000\cdot\frac{1}{5}\sin\left(\frac{y[mm]}{5}\right)\mathrm{d}\,y\,. \tag{7}$$

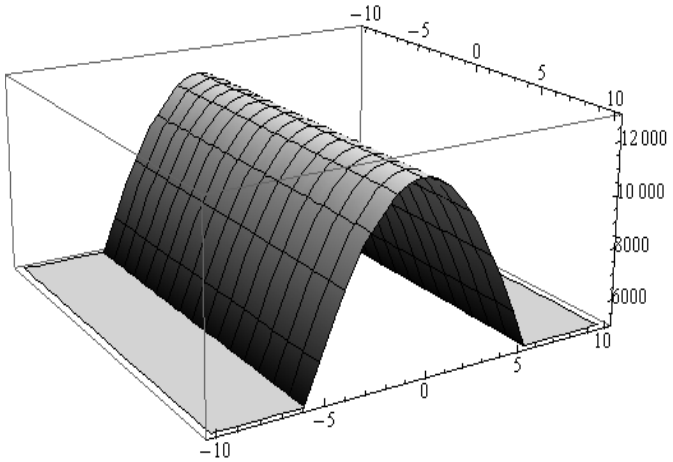

Fig. 2. The shaping of the inhomogeneous magnetic field

In the relations (6) and (7) we considered the field lines to be parallel and in the same direction (but not equidistant). The relation (6) can be also rewritten as,

$$H(x,y)=H_0\cos\left(\frac{y}{a}\right). \tag{8}$$

where $H_0=12000\,\mathrm{A/m}$ and $a=5\cdot10^{-3}\,\mathrm{m}$.

In the Fig. 3 there are shaped the forces which are taking place towards the detection system.

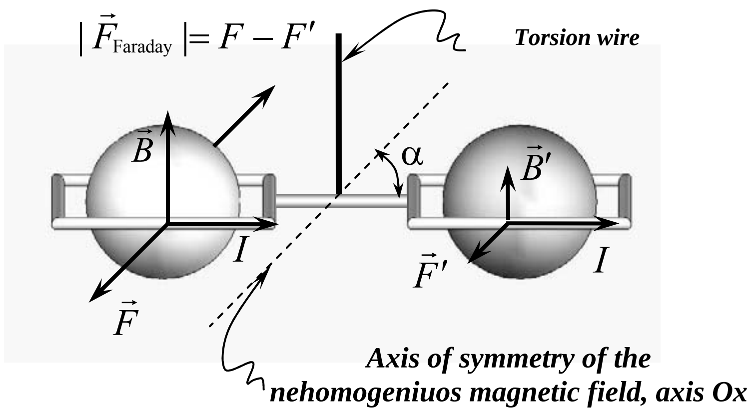

Fig. 3. Experimental 3D model of the torsion pendulum

For a rectangular frame the total electromagnetic force which takes action towards it is null in the case of a homogeneous magnetic field. By using an inhomogeneous magnetic field, Fig. 2, it can be obtained a nonzero total force. In Fig. 4, we have chosen a referential system, in such way as the centre of rotation of the system to be exactly in the origin and the inhomogeneous magnetic field to have the gradient across the $\mathrm{O}y$, axis, suggested by relation (6).

As it can be noticed in Fig. 4 (here it was taken into consideration only a symmetrical part of the detector, the electromagnetic diagrams' forces being identical also for the other part) the DC side of the frame exists closer to $\mathrm{O}x$,axis, meaning the place where the magnetic field has maximum value, reason for which the force that takes action towards the DC side will be higher than the force which takes action towards the AB side, $F_{AB} < F_{DC}$. The forces which take action towards the AD and BC sides also won't be equal but they are not producing a clear moment of torsion the action the direction of the rotation's pole.

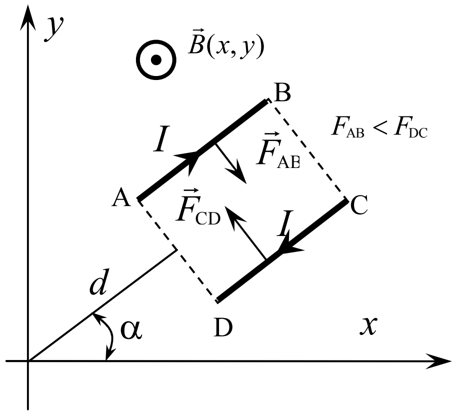

Fig. 4. The model of electromagnetic interaction in the inhomogeneous magnetic field

In order to estimate the moment of force we will use the relation (4) under a scalar form (the induction of the magnetic field is perpendicular to the plan's frame) to find out the force which acts across a side, for example AB .

$$F_{AB} = \int_{A}^{B} I \left| d\vec{l} \times \vec{B} \right| = I \int_{A}^{B} B(l)\, dl \, . \tag{9}$$

The resultant electromagnetic force will be

$$F_{em} = F_{CD} - F_{AB} \, , \tag{10}$$

and the moment of the average force is (where L = AB = CD):

$$M_{em} = 2\left(d + \frac{L}{2} \right) F_{em} \, . \tag{11}$$

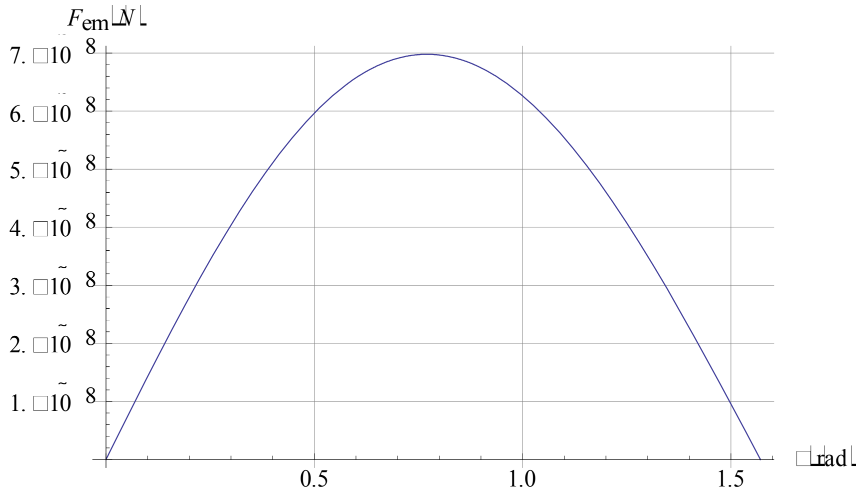

Fig. 5. The diagram of the electromagnetic force depending on the angle

Solving the integral (9) and using the $\vec{B} = \mu \vec{H}$ relation, it is obtained

$$F_{em}(\alpha) = \frac{C}{\sin(\alpha)} \sin\left(\frac{l\cos(\alpha)}{2a} \right) \sin\left(\frac{L\sin(\alpha)}{2a} \right) \sin\left(\frac{(2d+L)\sin(\alpha)}{2a} \right) \tag{12}$$

where $C = 4aH_0\mu_0(\chi+1)I$.

The diagram's form shows that the maximum value of the force is reached across the $45°$ angle. Certainly this value depends on the constants from the (12), relation, in our case $\chi = 0$ $I = 20\,\text{mA}$, $\mu_0 = 4\pi\cdot10^{-7}\,\text{H/m}$ and $l = L = \text{d} = 2\,\text{mm}$.

These values can be modified according to the situation and the curve can be also modified. The right establishment of the constants will eliminate the situation with the multiple equilibrium positions. For example if the a constant's values are lowered there can be obtained chaotic oscillations of the F_{em} force value, see Fig. 6. In the following stages we will keep the values which helped the acquirement of Fig. 5.

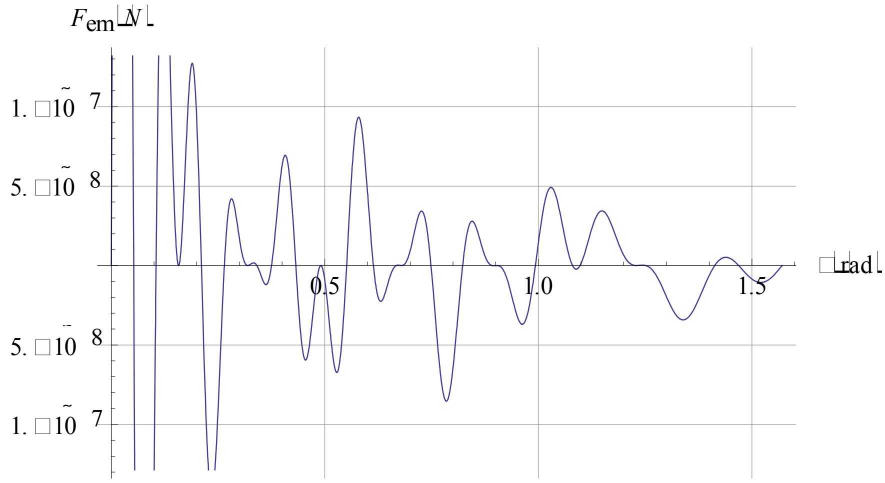

Fig. 6. The diagram of the magnetic force depending on the angle modifying the α parameter

The Acquirement of the Concentration Intensity Dependence of the Current

In order to reduce the relations complexities we assume that in the environment exist only two types of gazes nitrogen and oxygen. Rewriting the relation (5), only depending on the concentration of oxygen we obtain

:
$$\chi_2 = c_{O_2}\chi_{O_2} + (1 - c_{O_2})\chi_{N_2} . \tag{13}$$

We are replacing the relation above with (1)

$$\vec{F}_{\text{Faraday}} = \left(\chi_1 - c_{O_2}\chi_{O_2} - (1 - c_{O_2})\chi_{N_2}\right)v\ H\,\text{grad}\,H \tag{14}$$

And we evaluate the intensity of the H field and also the gradient at the point where is the centre of a sphere with volume v (Fig. 3) that represent also the centre of rectangular frame modeled on Fig. 4.

$$H(\alpha) = H\left(\left(d + L/2\right)\cos(\alpha), \left(d + L/2\right)\sin(\alpha)\right) . \tag{15}$$

As we can notice from this relation the Faraday force also depends on the angle between the disposition axis and the symmetry axis of the magnetic field distribution.

We are suggesting that the disposition is reduced into the equilibrium position (initially settled the in absence of the oxygen) so the electromagnetic force is rebalancing only the Faraday force via their moments

$$2\left(d + L/2\right)F_{\text{Faraday}}\cos(\alpha) = 2\left(d + L/2\right)F_{\text{em}} . \tag{16}$$

It is multiplied with 2 because we have two spheres and $\cos(\alpha)$ appears because of the projection of the force on a perpendicular direction on the axis of the system.

By simplifying we get to the equality of the medium forces which take an extra action in rebalancing the system

$$F_{\mathrm{Faraday}}(\alpha)\cos(\alpha) = F_{\mathrm{em}}(\alpha)\,. \tag{17}$$

Solving the equation it is obtained the concentration depending on the rebalancing current as a linear dependence

$$c_{\mathrm{O_2}} = m\,I + n\,. \tag{18}$$

In relation (18) the constants are

$$n = \frac{-\chi_1 + \chi_{\mathrm{N_2}}}{\chi_{\mathrm{N_2}} - \chi_{\mathrm{O_2}}}\,, \tag{19}$$

$$m = A\,\mathrm{cosec}(2\alpha)\sin\left(\frac{l\cos(\alpha)}{2a}\right)\sin\left(\frac{L\sin(\alpha)}{2a}\right)\sec\left(\frac{(2d+L)\sin(\alpha)}{2a}\right), \tag{20}$$

$$A = -\frac{8a^2\mu_0(\chi_1+1)}{H_0 v\left(\chi_{\mathrm{N_2}} - \chi_{\mathrm{O_2}}\right)}\,. \tag{21}$$

Conclusions

By relation (18) we obtain the dependence in a non electrical measure (the concentration og oxygen) and an electrical measure (the intensity of the electrical current). By using the values reminded bellow and the other constants as it follows: $\alpha \cong 45°$, $\chi_1 = -0{,}2\,10^{-9}\,\mathrm{cm}^{-1}$, $r = 1\,\mathrm{mm}$ (radius of the spheres) it is obtained a direct relation

$$c_{\mathrm{O_2}}(I) = 7.8\,I[\mathrm{mA}] + 0.2[\%]\,. \tag{22}$$

The representation of the bellow expression shows the linear dependence and for a maximum (100%) concentration the currents won't exceed $12-13\,\mathrm{mA}$.

References

1. M u n d a y C. W. - *A Precision oxygen analyzer for chemical plants*, Proc. Int. Conf. Sponsored by SIT. Swansea, pp. 104-117, 1958.

Modelarea fizică a analizorului de oxigen de precizie

Rezumat

În această lucrare propunem modelarea analizorului de oxigen de precizie prin verificarea rezultatelor experimentale privind dependența liniară a concentrației de oxigen în funcție de curentul de reechilibrare a balanței de torsiune. Concentrația de oxigen influențează susceptibilitatea magnetică a mediului în care se află balanța de torsiune iar readucerea în poziția de 0 inițială se poate realiza prin intermediul unui curent electric generând o forță electromagnetică de sens contrar. Constantele care intervin în relația de dependență în acest fel pot fi exprimate în funcție de mărimi fizice caracteristice indicând şi factorii externi care pot influența precizia măsurătorilor.