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# Optimizing Vibrations Dynamic Absorbers (I)

Tănase Dinu

Universitatea Petrol-Gaze din Ploiesti, Bd. Bucuresti 39, Ploiesti
e-mail: tdinu@mail.upg-ploiesti.ro

## Abstract

*The two papers present a general algorithm for determining the dynamic response of a structure with any type of damping, based on the Galerkin method. The elaborated algorithms are transposed into computer programmes. The paper presents a new method for determining the parameters of dynamic absorbers of vibrations, using a C.R.D. programme.*

**Key words:** *pulsations, damping, simple dynamic absorber, inertial matrix, matrix of rigidity, disturbance force, residue*

## Introduction

The dynamic calculation of structures is achieved in most cases on the basis of dynamic models with discrete masses which constitute systems with a finite number of freedom degrees. For such a system the equations of motion are written under the matrix form:

$$M \cdot \ddot{\eta} + B \cdot \dot{\eta} + R \cdot \eta = F(t) \qquad (1)$$

where:

$M$ is the inertial matrix ( $M \in \mathcal{M}_{n,n}$ );

$B$ is the matrix of damping coefficients; ( $B \in \mathcal{M}_{n,n}$ );

$R$ is the matrix of rigidity coefficients ( $R \in \mathcal{M}_{n,n}$ );

$\eta(t)$ is the motion vector ( $\eta(t) \in \mathcal{M}_{n,1}$ ),

$F(t)$ the vector of disturbance forces, where $F(t) = [F_1(t), F_2(t), ..., F_n(t)]^T$ .

In order to obtain the dynamic response in the case of a structure with a finite number of degrees of freedom, alongside with equation (1) there must also be given the initial *conditions*:

$$\begin{cases} \eta(0) = \eta_0 ; \\ \dot{\eta}(0) = v_0 \ . \end{cases} \quad (2)$$

In order to determine the dynamic response along the time interval $[0, \Delta t]$ where $\Delta t = t - t_0$, we choose the simplest vector function able to approximate the vector of displacements

$$\eta(t) = A_1 + t \cdot A_2 + t^2 \cdot A_3 + t^3 \cdot A_4 , \quad (3)$$

where $A_1, A_2, A_3, A_4$ are known vectors with $n$ dimensions, which are determined from the conditions set at the beginning and the ending of the $[0, \Delta t]$ time interval in displacements and velocities. At the beginning of the $t = 0$ interval, these conditions are given by (2). By replacing we obtain : $A_1 = \eta_0$ and $A_2 = v_0$. At the ending of the $t_1 = \Delta t$ time interval these conditions are not known and will therefore be noted as follows:

$$\eta(t_1) = \eta_t \text{ and } \dot{\eta}(t_1) = \dot{\eta}_t . \quad (4)$$

Conditions (4) imposed on the function (3) allow obtaining vectors $A_3$ and $A_4$. Consequently the displacement vector (3) becomes:

$$\eta(t) = (1 - \frac{3 \cdot t^2}{(\Delta t)^2} + \frac{2 \cdot t^3}{(\Delta t)^3}) \cdot \eta_0 + (t - \frac{2 \cdot t^2}{\Delta t} + \frac{t^3}{(\Delta t)^2}) \cdot v_0 +$$
$$+ (\frac{3 \cdot t^2}{(\Delta t)^2} - \frac{2 \cdot t^3}{(\Delta t)^3}) \cdot \eta_t + (- \frac{t^2}{\Delta t} + \frac{t^3}{(\Delta t)^2}) \cdot \dot{\eta}_t \quad (5)$$

Replacing the displacement vector (5) in the equation (1) generates the residue:

$$\varepsilon = M \cdot \ddot{\eta} + B \cdot \dot{\eta} + R \cdot \eta - F(t) \quad (6)$$

The determination of vectors $\eta_t$ and $\dot{\eta}_t$ (at the moment $t_1 = \Delta t$) will be realized out of the conditions which must be fulfilled by residue with the basic functions $\Phi_1(t)$ and $\Phi_2(t)$, where:

$$\begin{cases} \Phi_1 = 3 \frac{t^2}{(\Delta t)^2} - 2 \frac{t^3}{(\Delta t)^3} \\ \Phi_2 = - \frac{t^2}{\Delta t} + \frac{t^3}{(\Delta t)^2} \end{cases} , \quad (7)$$

which multiply in (5) the unknown vectors $\eta_t$ and $\dot{\eta}_t$. Using the Galerkin method of ortogonalizing of the residue with functions (7) we obtain the *conditions* :

$$\begin{cases} \int_0^{\Delta t} \Phi_1(t)\cdot\varepsilon(t)\cdot dt = 0 \ , \\ \int_0^{\Delta t} \Phi_2(t)\cdot\varepsilon(t)\cdot dt = 0 \end{cases} , \tag{8}$$

out of which there result parameters $\eta_t$ and $\dot{\eta}_t$. In order to determine the dynamic response of problem (1) with the initial conditions (2) along a $[0,T]$ time interval, this interval is divided in a $m$ sub-intervals $[t_i, t_{i+1}]$ with $i \in \{0,1,2,...,m-1)$ which may be equidistant or not. The algorithm presented in the first stage is applied along the $[t_0, t_1]$ interval with $\Delta t = t_1 - t_0$ and there results $\eta(t_1)$, $\dot{\eta}(t_1)$ and $\eta(t)$ for $(\forall)$ $t \in [t_0, t_1]$. In the next stage we consider $t_0 = t_1$, $t_1 = t_2$ and $\Delta t = t_2 - t_1$ which has the initial conditions obtained in the previous stage and the algorithm is resumed.

There results $\eta(t_2)$, $\dot{\eta}(t_2)$ and $\eta(t)$ for $(\forall)$ $t \in [t_1, t_2]$. The same procedure is applied up to the $[t_{m-1}, t_m]$ interval with $t_m = T]$ and there results $\eta(t_m)$, $\dot{\eta}(t_m)$ and $\eta(t)$ for $(\forall)$ $t \in [t_{m-1}, t_m]$. In order to directly obtain the dynamic response of a structure there has been established an iterative, incremental procedure based on the Galerkin method. Within a $\Delta t = t_2 - t_1$ time interval, knowing the initial conditions (displacement and velocities at $t_1$) there can be determined displacements and velocities at $t_2$, setting the condition of orthogonalizing the residue with two weight functions chosen accordingly. On the basic of displacements and velocities at moments $t_1$ and $t_2$ there can be expressed displacements and velocities for any $(\forall)$ $t \in [t_1, t_2]$.

The algorithms elaborated for proper pulsations with damping and for the direct dynamic response, have been transposed into computer programmes written in Turbo-Pascal, which allows for the results thus obtained to be easily implemented in engineering practice noted C.R.D. 9.

## Contents

### The simple dynamic absorber

We take into account the two degree-of-freedom system from Figure 1, which is composed of the primary oscillatory system with the elastic constant *k* and the mass *m* and the attached oscillatory system with the elastic constant $k_a$ and the mass $m_a$. When a disturbance force $F_1 \cdot e^{i \cdot w \cdot t}$ is applied to the primary mass *m*, the system starts to vibrate.

The differential equations of the movement are:

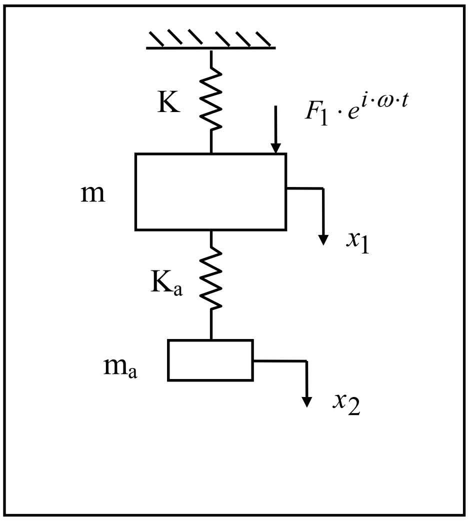


**Fig. 1.** The two degree-of-freedom system

$$\begin{cases} m\dfrac{d^2x_1}{dt^2} + (k + k_a)\cdot x_1 - k_a \cdot x_2 = F_1 \cdot e^{i\cdot\omega\cdot t} \\ m_a \dfrac{d^2x_2}{dt^2} + k_a \cdot (x_2 - x_1) = 0 \end{cases}, \qquad (9)$$

where $x_1(t)$ and $x_2(t)$ are the two movements of the masses. The solution of the system (9) is in the form:

$$x_1(t) = a_1 \cdot e^{i\cdot\omega\cdot t} \text{ si } x_2(t) = a_2 \cdot e^{i\cdot\omega\cdot t}, \qquad (10)$$

where $a_1$ and $a_2$ are the amplitudes of the movement of the two masses.

By replacing (10) in (9) we can obtain:

$$\begin{cases} (k + k_a - m\cdot\omega^2)\cdot a_1 - k_a \cdot a_2 = F_1 \\ -k_a \cdot a_1 + (k_a - m_a \cdot \omega^2)\cdot a_2 = 0 \;. \end{cases} \qquad (11)$$

The following *notations* are made:

$x_{st} = \dfrac{F_1}{k}$ (the static movement of the primary mass);

$P_0 = \sqrt{\dfrac{k}{m + m_a}}$ (the specific pulsation of the primary system with $m_a$ attached to the mass $m$);

$P_a = \sqrt{\dfrac{k_a}{m_a}}$ (the specific pulsation of the attached system);

$\mu = \dfrac{m}{m + m_a}$ (the ratio between masses);

$n = \dfrac{P_a}{P_0}$ (the ratio between the pulsation of the disturbing force and the specific pulsation of the primary system with the attached auxiliary mass $m_a$);

$\Omega = \dfrac{\omega}{P_0}$ (the ratio between the pulsation of the disturbing force and the specific pulsation of the primary system with the attached auxiliary mass $m_a$).

Through complex calculus which is specific to every type of absorber[2], we can determine the following ratios $\left|\dfrac{a_1}{x_{st}}\right|$ and $\left|\dfrac{a_2}{x_{st}}\right|$.

Figure 2 represents the variation curve of the ratio $\left|\dfrac{a_1}{x_{st}}\right|$ and, in a similar way of the ratio $\left|\dfrac{a_2}{x_{st}}\right|$ according to $\Omega$ in case $n = 1 (P_a = P_0)$ and $\mu = 0{,}8$ ( the auxiliary mass $m_a$ is a quarter of the mass $m$, which means $m_a$ = 0, 25 $m$).

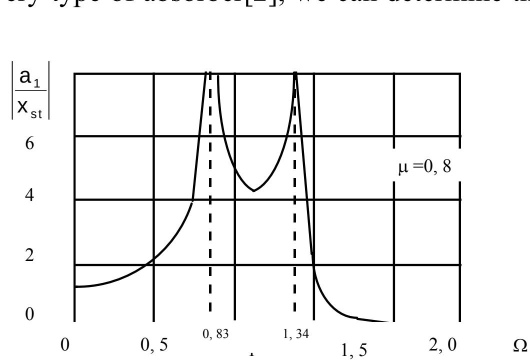


**Fig. 2.** The variation curve

We can see that, by attaching the dynamic absorber, the primary system turns into a two degree-of-freedom system
having the specific pulsations $P_1 = 0{,}83 \cdot P_0$ and $P_2 = 1{,}34 \cdot P_0$.

From the way in which the ratio $\left|\frac{a_1}{x_{st}}\right|$ varies, it results that the simple dynamic absorber is useful only when the pulsation of the disturbing force is actually constant. For the machines that have a varied speed, this type of absorber is useless, because, by attaching it, the primary system turns from a one-degree-of-freedom system into a two degree-of-freedom system with two resonances. In these cases it is necessary to use a dynamic absorber with dampers. If the auxiliary system has no damping devices, it functions as a dynamic absorber with a knot at the linking point. The auxiliary mass must be high enough so that its movement should not have very large amplitude. It is easy to determine the auxiliary mass for a dynamic absorber connected to the primary system in the point where the excitation applies.

As the primary mass stays fixed, the force exerted by the absorber, for an amplitude $u_0$ of the movement of the auxiliary mass, is equal and opposed in direction to the exiting force $F$ where:

$$F = m_a \cdot \omega^2 \cdot u_0 \quad \text{(a)}$$

Taking into account that the pulsation is known, the mass and the movement amplitude, needed for counterbalancing the influence of the given exciting force, are determined by the relation (a)

The elastic constant of the arch from the auxiliary system is determined under the condition that the frequency of this system should be equal to the one of the exciting force, which means:

$$k_a = m_a \cdot \omega^2 \quad \text{(b)}$$

Although the idea of harmonizing a dynamic absorber seems simple, there are some practical issues that make the precise harmonizing of a system very difficult. When the auxiliary mass is small as compared to the one of the primary system, the efficiency depends on the precision of harmonizing the frequencies. If the harmonizing is not perfect, the adding of the auxiliary mass can cause the composed system (made of the primary and the auxiliary systems together) to resonate with the exciting force.

## The dynamic absorber with viscous damper

The scheme of a dynamic absorber with a damping device attached to the primary system without damping is presented in Figure 3. The differential equations for the movement of the two masses are:

$$\begin{cases} m \cdot \ddot{x}_1 + c_a \cdot (\dot{x}_1 - \dot{x}_2) + (k + k_a) \cdot x_1 - k_a \cdot x_2 = F_1 \cdot e^{i \cdot \omega \cdot t} \\ m_a \cdot \ddot{x}_2 - c_a \cdot (\dot{x}_1 - \dot{x}_2) - k_a \cdot (x_1 - x_2) = 0 \end{cases}, \quad (12)$$

where $c_a$ is the damping constant of the dynamic absorber. The solution of the system (12) being in the form (10), then: $\dot{x}_1 = i \cdot \omega \cdot x_1$, $\dot{x}_2 = i \cdot \omega \cdot x_2$, $\ddot{x}_1 = -\omega^2 \cdot x_1$ and $\ddot{x}_2 = -\omega^2 \cdot x_2$ which are replaced in (12) to result:

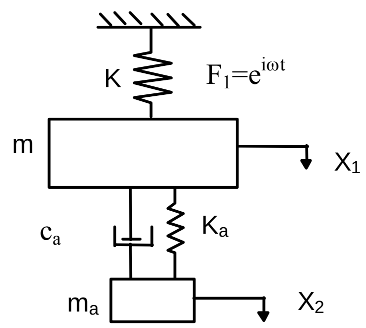

**Fig. 3.** The scheme of a dynamic absorber

$$\begin{cases} x_2 = \dfrac{x_1 \cdot (k_a + i \cdot \omega \cdot c_a)}{(k_a - \omega^2 \cdot m_a + i \cdot \omega \cdot c_a)} \\ x_1 \cdot [k + k_a - \omega^2 \cdot m + i \cdot \omega \cdot c_a - \dfrac{(k_a + i \cdot \omega \cdot c_a)^2}{(k_a - \omega^2 \cdot m_a + i \cdot \omega^2 \cdot c_a)}] = F_1 \cdot e^{i \cdot \omega \cdot t}. \end{cases} \tag{13}$$

From the relation above, by using the notes from the simple dynamic absorber and considering $\gamma_a = \dfrac{c_a}{m_a \cdot P_0}$, we can obtain the ratio $\dfrac{k \cdot x_1}{F_1 \cdot e^{i \cdot \omega \cdot t}} = \dfrac{R_1 + i \cdot I_1}{R_2 + i \cdot I_2}$, where:

$R_1 = n^2 - \Omega^2$ ; $R_2 = \mu \cdot \Omega^4 - (1 + n^2) \cdot \Omega^2 + n^2$ ; $I_1 = \Omega \cdot \gamma_a$ ; $I_2 = \Omega \cdot \gamma_a \cdot (1 - \Omega^2)$.

The transmissibility that defines the ratio between the transmitted force and the disturbing force will have the expression:

$$\left| \frac{k \cdot x_1}{F_1 \cdot e^{i \cdot \omega \cdot t}} \right| = \sqrt{\frac{R_1{}^2 + I_1{}^2}{R_2{}^2 + I_2{}^2}}. \tag{14}$$

When there is no damping, ($\gamma_a = 0$)for the auxiliary system, then the transmissibility is the same as:

$$T = \left| \frac{R_1}{R_2} \right| = \left| \frac{a_1}{x_{st}} \right|, \tag{15}$$

and if the damping is infinite ($\gamma_a = \infty$) the result is:

$$T = \pm \frac{1}{1 - \Omega^2}, \tag{16}$$

and the oscillatory system can turn into a one degree-of-freedom system, whose mass is the sum of the masses *m* and $m_a$. Figure 4 presents the variation curves of transmissibility according to the **Ω** ratio that can be obtained with the help of the relations (15) and (16).

The two curves referring to the absence of damping ($\gamma_a = 0$) and infinite damping ($\gamma_a = \infty$) are intersected in the points *P* and *Q* with the abscissas $\Omega_P$ and $\Omega_Q$ which are fixed. All the curves of transmissibility pass through these points, regardless of the value of the damping ratio $\gamma_a$ So, at the frequencies $\Omega_P$ and $\Omega_Q$ transmissibility does not depend on damping.

As (15) can be written in the form:

$$T^2 = \frac{A + B \cdot \gamma_a{}^2}{C + D \cdot \gamma_a{}^2} \tag{17}$$

where: $A = \Omega^2$ ; $B = \Omega^2$; $C = R_2{}^2$ ; $D = \Omega^2 \cdot (1 - \Omega^2)$, then the expression (17) does not depend on $\gamma_a$ if the ratio of the coefficients is a constant, that is $\dfrac{A}{C} = \dfrac{B}{D}$. This condition leads to the relation: $(n^2 - \Omega^2) \cdot (1 - \Omega^2)^2 = (\mu \cdot \Omega^4 - (1 + n^2) \cdot \Omega^2 + n^2)^2$ which has the roots $\Omega_P$ and $\Omega_Q$. (18)

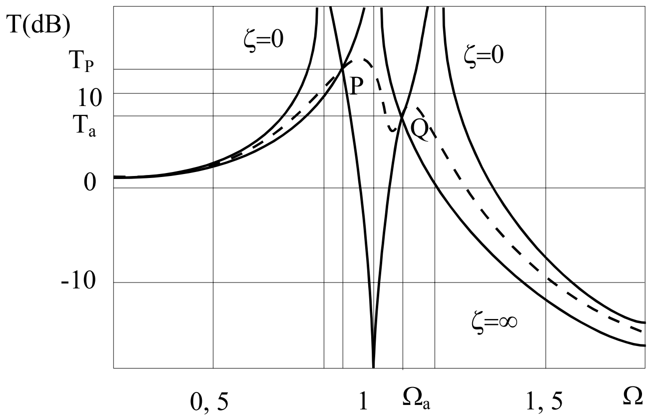


**Fig. 4.** The variation curves of transmissibility

## The optimal harmonization of the absorber

From Figure 4 we can deduce that the functioning of the dynamic absorber is becoming more efficient if transmissibility has the same value $(T_P = T_Q)$ in points *P* and *Q*.

When the coordinates of the two points are equal, we can say that the absorber is *optimally harmonized*. From $T_P = T_Q$ and taking into account the relations (16), (17), (18), we deduce the relation:

$$\Omega_{P,Q}^2 = 1 \pm \sqrt{\frac{1-\mu}{1+\mu}}, \; T_{P,Q} = \sqrt{\frac{1+\mu}{1-\mu}} \text{ si } n_0 = \sqrt{\mu} \tag{19}$$

which is the expression of transmissibility when the absorber is optimally harmonized.

For a mass ratio $\mu > 0{,}5$, the optimal damping coefficient, noted as $\gamma_{aom}$, can be calculated with sufficient precision using the following relation:

$$\gamma_{aom} = \sqrt{\frac{3 \cdot \mu \cdot (1-\mu)}{2}} \tag{20}$$

By introducing a damping system in the auxiliary system, the amplitudes of the primary system are diminished to pulsations that are equal to the specific ones. This is the reason why the dampers with an auxiliary mass are commonly used to diminish pressures and the amplitudes of the vibrations.

From the two theoretical examples that are presented, simple as they are from the mechanical point of view, but very complex in means of calculations, we can infer that the harmonizing of these vibration absorbers becomes a very delicate mathematic problem when the structure is more complex (it has many masses). Keeping in mind that the C.R.D. programme determines also the maximum movement of the structural components, then the optimal parameters of a dynamic absorber which is attached to a structure can be easily determine by simulating these parameters with the C.R.D. programme. The procedure consists in giving various values to the parameters and keeping the combination for which the mass movements are the smallest. We will present further an example of harmonizing a dynamic absorber for a one-degree-of-freedom structure. The determination of the optimal parameters is achieved due to both the theory presented above and to the simulation through the C.R.D. programme.

## A Calculation Example

Given an oscillatory one-degree-of –freedom system (Figure 4) with the mass $m$ = 50 Kg, the elastic constant $k$ = 20 *KN/m*, which is under the stress of a harmonic disturbance force with the amplitude of 30 *N*. To this system, we attach a conventional dynamic absorber with viscous damping which causes the vibrations of the primary mass to be smaller than 8 *mm*, for every value the pulsation of the disturbance force might have. We are asked to determine the optimal parameters of this absorber. (To be continued in second part)

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# Optimizarea absorbitorilor dinamici (I)

## Rezumat

*Aceste două lucrări prezintă algoritmul general al răspunsului dinamic al unei structuri cu amortizare oarecare folosind metoda Galerkin. Acest algoritm a fost transpus pe calculator în programul care dă răspunsul dinamic (C.R.D.). Lucrarea prezintă o nouă metodă de determinare a parametrilor absorbitorilor dinamici de vibrații, utilizând programul C.R.D.*