BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LIX No. 1/2007	1- 8	Seria Tehnică

Mathematical Modelling of the PAM Control Algorithm for the Mono-Phase Inverter

Gheorghe Cremenescu

Universitatea Petrol – Gaze din Ploieşti, Bd. Bucureşti 39, Ploieşti, România
e-mail: gcremenescu@upg-ploiesti.ro

Abstract

The article presents a mathematical support of the control algorithm of the pulse amplitude modulation (PAM) for the synthesis of the mono-phase inverter output voltage, having as objectives the approaching of the effective value of the fundamental to the effective value of the proposed sinusoidal voltage at the terminals of the charge and diminishing of the weight of low frequency harmonics in the harmonic content of the voltage.. The commutation moments of the pulses are computed in the conditions when the fundamental of the synthetic voltage is equal to the proposed sinusoidal voltage, and the high harmonics up to order 4m – 2 are null. For the numerical simulation of the model, the Matlab toolbox was used. The results of the simulation are numerically and graphically presented; they confirm the validity of the mathematical support of the control algorithm PAM.

Key words: *pulse amplitude modulation, PAM control algorithm, mono--phase inverter, synthetic voltage.*

Introduction

In the adjustable electrical drives, the asynchronous motor challenges the direct current motor if the power supply is a frequency static converter, which performs the variation of frequency and, correspondingly, the variation of the output voltage. Depending on the strategy used to obtain variable frequency and voltage, two groups of frequency static converters could be identified [1]:

- one with constant direct voltage intermediate circuit, when the inverter functions after the principle of pulse width modulation (PWM);
- one with variable direct voltage, when the control functions of the output voltage and the frequency of the converter are divided between the commanded rectifier or the power chopper and the autonomous inverter with forced commutation. For a given voltage, the periods of conduction for the thyristors of the inverter are not fragmented and equal to $T/2$ (180 electrical degrees) or $T/3$ (120 electrical degrees), depending on the commutation program (T is the period of the given voltage).

In figure 1, one presents the output voltage waveform of a mono-phase inverter at frequencies f_1 and f_2 ($f_1 < f_2$), for a period of 180 electrical degrees; the amplitude of the voltage step U_1, correspondingly U_2 ($U_1 < U_2$) is correlated with the frequency f_1, respectively f_2. In the harmonic content of the rectangular output voltage, the low harmonics of order 3, 5, 7, 9, 11, 13 have high amplitudes (in the case of a three phased inverter, the line voltages do not contain harmonics multiple of 3).

At low frequency for the power supply of the asynchronous motor, these harmonics produce high oscillating couples, which give a pulsating characteristic to the electromagnetic couple and a jerky movement of the rotor [1, 7]. So, it is mandatory that the power supply of the motor would be done at a voltage close to a sinusoidal form, called synthetic voltage.

In this paper, one elaborates the mathematical model of the synthetic voltage approximated by a number of steps (the pulses in the interval 0 – $T/4$) and by numerical simulation one analyses the voltage synthesis (the commutation moments, the pulses amplitude, the spectral analysis, and the waveforms). Such a frequency converter functions after the principle of pulse amplitude modulation (PAM).

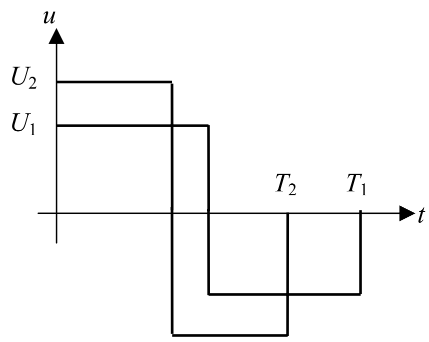

Fig. 1 The voltage shape for the non- modulated regime.

Mathematical Model of the Optimal Synthetic Voltage

At a frequency of 50 Hz, the voltage $u(t)$ at inverter terminals has a positive and negative rectangular pulse shape, like in figure 1, of amplitude U and length $T/2$, where U is the direct voltage in the intermediate circuit.

The Fourier series expansion of the voltage $u(t)$ contains only odd harmonics in sinus [1, 4, 5]:

$$u(t) = \frac{4}{\pi} U \sum_{n=1,3,5}^{\infty} \frac{1}{n} \sin n\omega t = \sum_{n=1,3,5}^{\infty} \sqrt{2} U_{efn} \sin n\omega t\,, \tag{1}$$

where U_{efn} is the effective value for the n^{th} harmonic.
The condition that the effective value of the fundamental (n = 1) equals the nominal voltage of the charge $U_{\mathrm{ef}\,1} = U_{\mathrm{nom}}$, gives the amplitude of the voltage pulse:

$$U = \frac{\sqrt{2}\pi}{4} U_{nom}\,. \tag{2}$$

The effective value of the n^{th} harmonic is $U_{\mathrm{ef}\,n} = U_{\mathrm{ef}\,1}/n$, and its frequency $f_n = 50n$. For the asynchronous motor, the 5th and 7th harmonics (with frequencies of 250 Hz and 350 Hz) have a small influence on the motor functioning and do not require the modulation of the voltage pulse.

Below nominal frequency, f < 50 Hz, the output voltage of the inverter has the shape of rectangular pulses [2, 5] with amplitude U_k and length equal to $\Delta t = T/4m$, where m is the number of steps (the pulses in the interval 0 – T / 4), and $T = 1/f$ is the period of the fundamental. The sinusoidal voltage proposed at the inverter terminals:

$$u_s = U_{\max} \sin n\omega t\,, \tag{3}$$

is approximated with the synthetic voltage defined for a period like this:

$$u(t) = U_k\,, \tag{4}$$

for $t_k \le t \le t_{k+1}$, k=1, 2,…m with limits t_1=0 and t_{m+1} =$T/4$;

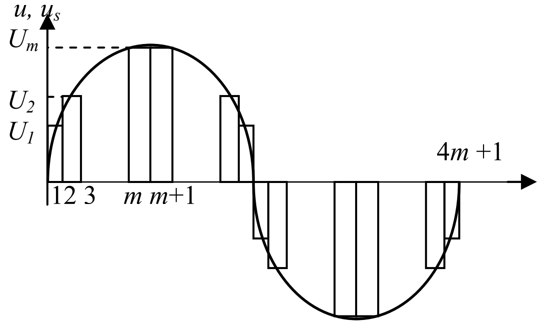

Fig. 2. The waveforms of the synthetic

$$u(t) = U_j = U_i \,, \tag{5}$$

for $t_j \le t \le t_{j+1}$, $j = m + k$, $i = m - k + 1$, $k = 1, 2, \ldots m$, with the limits $t_{m+1} = T/4$ and $t_{2m+1} = T/2$;

$$u(t) = U_j = -U_k \,, \tag{6}$$

for $t_j \le t \le t_{j+1}$, $j = 2m + k$, $k = 1, 2, \ldots 2m$ with the limits $t_{2m+1} = T/2$ and $t_{4m+1} = T$.

The voltage graphs $u_s(t)$ and $u(t)$ in the interval $0 - T$ are presented in figure 2; the notations on the time axis being the indexes of the commutation moments.

The Fourier series expansion of the synthetic voltage $u(t)$ contains only odd harmonics in sinus [1, 4, 6]:

$$u(t) = \sum_{n=1,3,5}^{\infty} b_n \sin n\omega t = \sum_{n=1,3,5}^{\infty} \sqrt{2} U_{efn} \sin n\omega t \,, \tag{7}$$

where the series coefficients are:

$$b_n = \frac{2}{T}\int_0^T u(t)\sin n\omega t \,\mathrm{d}t = \frac{4}{n\pi}\sum_{k=1}^{m} U_k\left(\cos n\omega t_k - \cos n\omega t_{k+1}\right). \tag{8}$$

From the conditions that the fundamental equals the proposed voltage and that the harmonics 3, 5, … 2 m – 1 nullify, the following system of equations can be inferred:

$$\begin{cases} b_1 = \dfrac{4}{\pi}\sum\limits_{k=1}^{m} U_k\left(\cos\omega t_k - \cos\omega t_{k+1}\right) = U_{\max} \\ b_3 = \dfrac{4}{3\pi}\sum\limits_{k=1}^{m} U_k\left(\cos 3\omega t_k - \cos 3\omega t_{k+1}\right) = 0. \\ . \\ b_{2m-1} = \dfrac{4}{(2m-1)\pi}\sum\limits_{k=1}^{m} U_k\left(\cos(2m-1)\omega t_k - \cos(2m-1)\omega t_{k+1}\right) = 0 \end{cases} \tag{9}$$

with the unknowns the voltage steps $U_1, U_2, \ldots U_m$.

One note with $u_k = U_k / U_{\max}$ the relative voltage and the system of equations becomes:

$$\begin{cases} \sum\limits_{k=1}^{m} u_k\left(\cos\omega t_k - \cos\omega t_{k+1}\right) = \dfrac{\pi}{4} \\ \sum\limits_{k=1}^{m} u_k\left(\cos 3\omega t_k - \cos 3\omega t_{k+1}\right) = 0. \\ . \\ \sum\limits_{k=1}^{m} u_k\left(\cos(2m-1)\omega t_k - \cos(2m-1)\omega t_{k+1}\right) = 0. \end{cases} \tag{10}$$

The linear system of algebraic equations (10) could be solved numerically and the voltage step values are obtained.
The synthetic voltage is optimal because the first $2m - 1$ harmonics after the fundamental are nullified (the even harmonics are included, even if these are null due to the symmetry of $u(t)$). For a well-defined number p of nullified harmonics, the number of voltage steps is equal to $\mathrm{int}\left(\frac{p+1}{2}\right)$, where *int* is the integer part of the number.

Spectral Analysis of the Synthetic Voltage

The effective values of voltage harmonics and the distortion coefficients are given by the following formulas [1, 4, 5, 8]:

° the effective values of the n^{th} harmonic :

$$U_{efn} = \frac{b_n}{\sqrt{2}},\ n = 1,\ 3,\ 5,...\ ; \tag{11}$$

° the total effective value:

$$U_{ef.t} = \left[\frac{1}{T}\int_0^T u^2(t)\,\mathrm{d}t\right]^{\frac{1}{2}} = \left[\frac{4}{T}\sum_{k=1}^{m} U_k^2\left(t_{k+1} - t_k\right)\right]^{\frac{1}{2}}; \tag{12}$$

or, depending on the components:

$$U_{ef.t} = \left[\frac{1}{T}\int_0^T\left(\sum_{n=1,3,5,}^{\infty} b_n \sin n\omega t\right)^2 \mathrm{d}t\right]^{\frac{1}{2}} = \left[\frac{1}{2}\sum_{n=1,3,5,}^{\infty} b_n^2\right]^{\frac{1}{2}}; \tag{13}$$

one takes into account that $\int_0^T \sin m\omega t\ \sin n\omega t\,\mathrm{d}t$ is equal to 1/2 when $m = n$ and 0 when $m \neq n$;

° the total effective value of the high harmonics :

$$U_{ef.t.a} = \left[\frac{1}{T}\int_0^T\left(\sum_{n=3,5,..}^{\infty} b_n \sin n\omega t\right)^2 \mathrm{d}t\right]^{\frac{1}{2}} = \left[\frac{1}{2}\sum_{n=3,5,..}^{\infty} b_n^2\right]^{\frac{1}{2}} = \left[U_{ef.t}^2 - U_{ef.1}^2\right]^{\frac{1}{2}}; \tag{14}$$

° the distortion coefficient k_{d1}, defined as the square root of the ratio between the conducting power in harmonics and the conducting power in the fundamental:

$$k_{d1} = \frac{U_{ef.t.a}}{U_{ef.1}} = \left[\frac{U_{ef.t}^2}{U_{ef.1}^2} - 1\right]^{\frac{1}{2}}; \tag{15}$$

° the distortion coefficient k_{d2}, defined as the square root of the ratio between the conducting power in harmonics and the conducting power in the synthetic voltage:

$$k_{d2} = \frac{U_{ef.t.a}}{U_{ef.t}} = \frac{k_{d1}}{\left(1 + k_{d1}^2\right)^{\frac{1}{2}}} \tag{16}$$

Table1. The results of the numerical simulation.

Input data / Output data	Umax [V]	20						
	f [Hz]	5						
	m	1	2	3	4	5	6	7
The relative amplitude of pulse	u_1	0,785	0,393	0,262	0,196	0,157	0,131	0,112
	u_2		0,948	0,715	0,560	0,456	0,384	0,331
	u_3			0,977	0,837	0,710	0,610	0,533
	u_4				0,987	0,895	0,796	0,709
	u_5					0,992	0,926	0,848
	u_6						0,994	0,946
	u_7							0,996
The pulses amplitude [V]	U_1	15,70	7,85	5,24	3,93	3,14	2,62	2,24
	U_2		18,96	14,30	11,18	9,12	7,67	6,62
	U_3			19,54	16,74	14,20	12,21	10,66
	U_4				19,74	17,90	15,91	14,17
	U_5					19,84	18,53	16,97
	U_6						19,88	18,92
	U_7							19,92
Pulses commutation moments in 0 – T/4 [ms]	t_1	0	0	0	0	0	0	0
	t_2	50	25	17	12	10	8	7
	t_3		50	33	25	20	17	14
	t_4			50	37	30	25	21
	t_5				50	40	33	29
	t_6					50	42	36
	t_7						50	43
	t_8							50
Amplitude/effective value of the first 25 harmonics [V]	b_1	20	20	20	20	20	20	20
	$U_{ef\,1}$	14,14	14,14	14,14	14,14	14,14	14,14	14,14
	b_3	6,66	0	0	0	0	0	0
	$U_{ef\,3}$	4,71	0	0	0	0	0	0
	b_5	4,0	0	0	0	0	0	0
	$U_{ef\,5}$	2,83	0	0	0	0	0	0
	b_7	2,86	2,86	0	0	0	0	0
	$U_{ef\,7}$	2,02	2,02	0	0	0	0	0
	b_9	2,22	2,22	0	0	0	0	0
	$U_{ef\,9}$	1,57	1,57	0	0	0	0	0
	b_{11}	1,82	0	1,82	0	0	0	0
	$U_{ef\,11}$	1,28	0	1,28	0	0	0	0
	b_{13}	1,54	0	1,54	0	0	0	0
	$U_{ef\,13}$	1,09	0	1,09	0	0	0	0
	b_{15}	1,33	1,33	0	1,33	0	0	0
	$U_{ef\,15}$	0,94	0,94	0	0,94	0	0	0
	b_{17}	1,18	1,18	0	1,18	0	0	0
	$U_{ef\,17}$	0,83	0,83	0	0,83	0	0	0
	b_{19}	1,05	0	0	0	1,05	0	0
	$U_{ef\,19}$	0,74	0	0	0	0,74	0	0
	b_{21}	0,95	0	0	0	0,95	0	0
	$U_{ef\,21}$	0,67	0	0	0	0,67	0	0
	b_{23}	0,87	0,87	0,87	0	0	0,87	0
	$U_{ef\,23}$	0,62	0,62	0,62	0	0	0,62	0
	b_{25}	0,80	0,80	0,80	0	0	0,80	0
	$U_{ef\,25}$	0,56	0,56	0,56	0	0	0,56	0
Total effective value[V]	$U_{ef.t}$	15,71	14,51	14,30	14,23	14,20	14,18	14,17
Tot.eff.val.of harm. [V]	$U_{ef.t.a}$	6,84	3,26	2,15	1,61	1,29	1,07	0,92
Distortion coefficients	k_{d1}	0,483	0,230	0,152	0,114	0,091	0,076	0,065
	k_{d2}	0,435	0,244	0,150	0,113	0,090	0,075	0,065

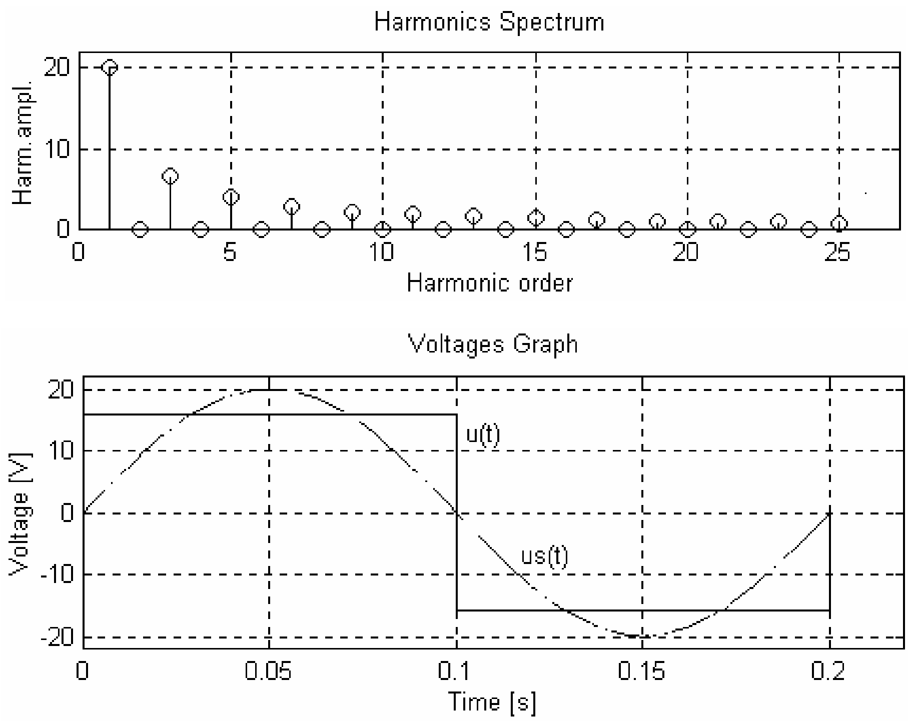

Fig. 3. The graphical results of the simulation for m

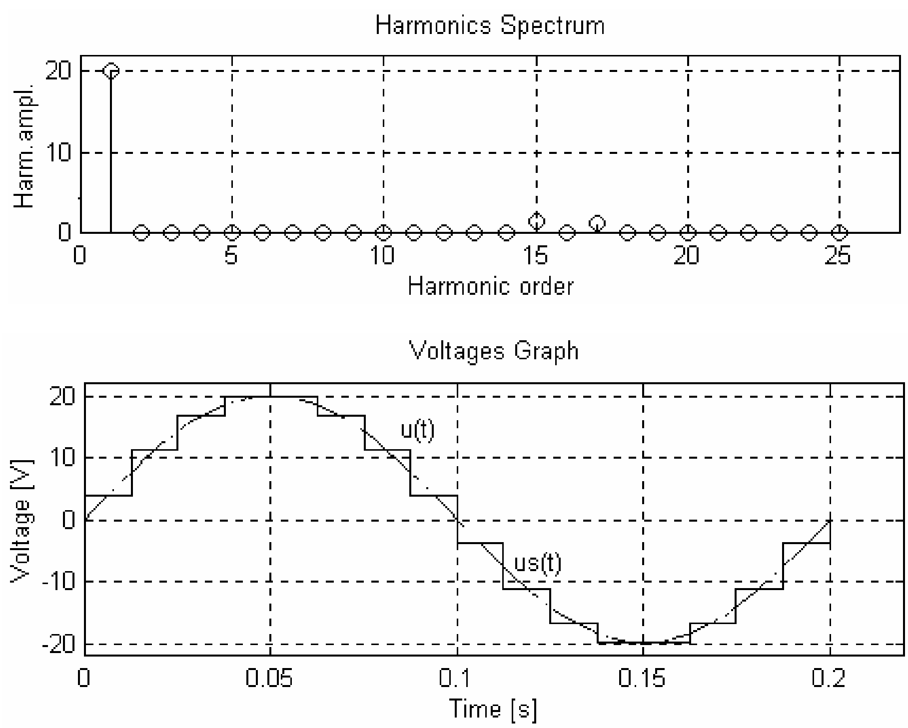

Fig. 4. The graphical results of the simulation for m

Results of the Numerical Simulation

For the numerical simulation of the voltage synthesized from amplitude modulated pulses one has used the Matlab toolbox [3], which has facilities for solving linear systems of algebraic equations (10), for the spectral analysis of the synthetic voltage with relations (11,..16), for the construction of time-voltage vectors with relations (4, 5, 6), for the graphical representations of the synthetic voltage and of the frequency spectrum.

The input data are: U_{max}, f, m.

The output data are:
- the amplitude of the voltage steps $U_1, U_2, \ldots U_m$;
- the commutation moments $t_1, t_2, \ldots t_{m+1}$ in the interval $0 - T/4$;
- the amplitude and the effective value of the first 25 harmonics;
- the distortion coefficients k_{d1}, k_{d2};
- the graphs of the synthetic voltage and the harmonics spectrum.

The numerical results of the simulation of the voltage synthesized from $m = 1, 2,...7$ approximation steps are presented in table 1. The graphical representations of the synthetic voltage and the harmonics spectrum obtained by simulation for $m = 1$ and $m = 4$ are presented in figure 3 and 4.

For $m = 1$, the inverter in non-modulated regime, one could observe the high weight of low harmonics (of order 3, 5, 7, 9, 11) in the harmonic content of the voltage.

In the case of the inverter with the output voltage in m steps (in modulated regime), only the harmonics of order $4km \pm 1$, $k = 1, 2, 3, ...$ are presented, the others being nulls.

So, for a functioning of the asynchronous motor at low frequencies of 5 –10 Hz in conditions close to a sinusoidal source power supply, 4 – 7 approximation steps are needed for the voltage of the inverter.

The work [2] gives the general scheme of an inverter with synthetic voltage from 3 steps obtained in the secondary of a transformer, but presents the disadvantage that the number of steps cannot be modified.

Conclusions

The mathematical model of the pulse amplitude modulation presented in the paper is based on the requirement of a harmonic content of the synthetic voltage, according to relations (9). The results obtained through numerical simulation on the model, using the Matlab toolbox, show that the fundamental of the synthetic voltage is identical to the proposed sinusoidal voltage and that the high harmonics, except those of order $4km \pm 1$, $k = 1,2,3,..$, are null

The method for synthesis of the voltage based on principle of the pulses amplitude modulation could be implemented in software on a microprocessor, which could command the rectifier or the chopper and the autonomous inverter.

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Modelarea matematică a algoritmului de comandă PAM al invertorului monofazat

Rezumat

Articolul prezintă un suport matematic al algoritmului de comandă a modulației pulsurilor in amplitudine (PAM, acronimele de la Pulse Amplitude Modulation) pentru sintetizarea tensiunii de ieşire a invertorului monofazat, având ca obiective aproximarea valorii efective a fundamentalei cu valoarea efectivă a tensiunii sinusoidale propuse la bornele sarcinii şi diminuarea ponderii armonicelor de joasa frecvența în conținutul armonic al tensiunii. Momentele de comutație a pulsurilor sunt calculate în condițiile în care fundamentala tensiunii sintetice este egală cu tensiunea sinusoidală propusă şi armonicele până la ordinul 4m – 2 sunt nule. Pentru simularea numerică a modelului s-a folosit pachetul Matlab. Rezultatele simulării sunt prezentate numeric şi grafic; se confirmă validitatea suportului matematic al algoritmului de comandă a modulației pulsurilor in amplitudine.