BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LIX No. 1/2007	63- 68	Seria Tehnică

The Calculus of the Extensible Cables Externally Loaded with Different Uniform Forces

Şerban Vasilescu

Universitatea Petrol-Gaze din Ploieşti, Bd. Bucureşti 39, Ploieşti, România
e-mail: vserban @mail.upg-ploiesti.ro

Abstract

In the paper are presented two calculus methodologies in order to study an extensible cable loaded with different external forces : first of one start with the hypothesis that the externalforces are uniform distributed on a horizontal length and the second one admits the hypothesis that the forces are uniform distributed directly on a length of the cable. The results obtained are analysed in a calculus example.

Keywords: *extensible cable, tension and suspension points*

General Equations

It is considered an extensible cable (fig. 1) having the suspension points A and D. The horizontal distance between A and D is l and the vertical one is h. The cable is loaded with three uniform external pressures located respectively in the intervals [0, a], [a, b] and [b, l].

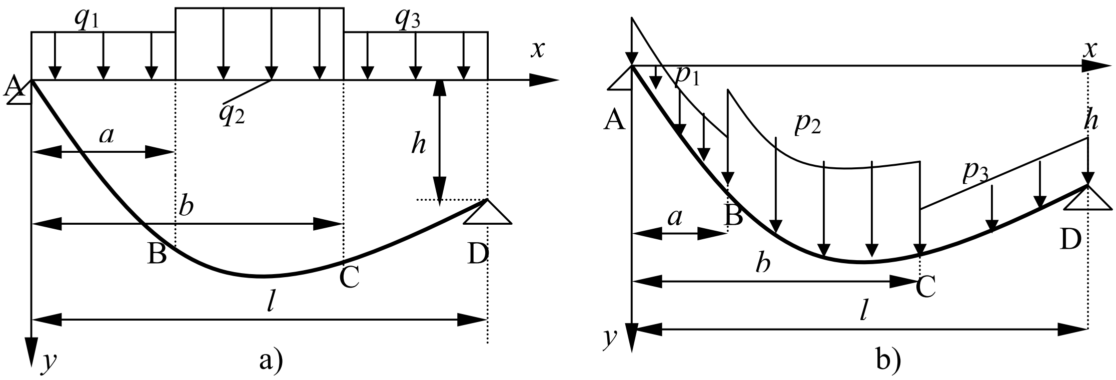

Fig. 1. *The cable supported at the ends*

The horizontally distributed forces are q_1, q_2, q_3 (fig. 1a) and the forces distributed directly on the cable are p_1, p_2, p_3 (fig 1b).

In the case that the external forces are horizontally distributed (fig. 1a), the general equations of the deflection and of the slope of the cable are [1] :

$$y(x)=\begin{cases}-\dfrac{q_1}{2H}x^2+C_1\cdot x+C_2, x\in[0,a) & \text{a)}\\ -\dfrac{q_2}{2H}x^2+C_3\cdot x+C_4, x\in[a,b) & \text{b)}\\ -\dfrac{q_3}{2H}x^2+C_5\cdot x+C_6, x\in[b,l] & \text{c)}\end{cases} \tag{1}$$

$$y'(x)=\begin{cases}-\dfrac{q_1}{H}x+C_1, x\in[0,a) & \text{a)}\\ -\dfrac{q_2}{H}x+C_3, x\in[a,b) & \text{b)}\\ -\dfrac{q_3}{H}x+C_5, x\in[b,l] & \text{c)}\end{cases} \tag{2}$$

In the above relations, H represents the horizontal component of the force from the cable.

The limit conditions have to respect the imposed displacements in A and D and to allow the continuity of the deflection and of the slope respectively in B and C :

$$x=0\Rightarrow y_A=0\Rightarrow C_2=0 \qquad \text{a)}$$

$$x=a\Rightarrow y_{B-\varepsilon}=y_{B+\varepsilon}\Rightarrow -\frac{q_1}{2H}a^2+C_1\cdot a=-\frac{q_2}{2H}a^2+C_3\cdot a+C_4 \qquad \text{b)}$$

$$x=a\Rightarrow y'_{B-\varepsilon}=y'_{B+\varepsilon}\Rightarrow -\frac{q_1}{H}a+C_1=-\frac{q_2}{H}a+C_3 \qquad \text{c)}$$

$$x=b\Rightarrow y_{C-\varepsilon}=y_{C+\varepsilon}\Rightarrow -\frac{q_2}{2H}b^2+C_3\cdot b+C_4=-\frac{q_3}{2H}b^2+C_5\cdot b+C_6 \qquad \text{d)} \tag{3}$$

$$x=b\Rightarrow y'_{C-\varepsilon}=y'_{C+\varepsilon}\Rightarrow -\frac{q_2}{H}b+C_3=-\frac{q_3}{H}b+C_5 \qquad \text{e)}$$

$$x=l\Rightarrow y_3=h\Rightarrow -\frac{q_3}{2H}l^2+C_5\cdot l+C_6=h \qquad \text{f)}$$

Usually in the theory of cables it is imposed also the condition that the deflection at the middle of the horizontally distance between the suspension points to be f. If the middle point is considered to be on the B-C arc of the cable it results :

$$x=\frac{l}{2}\Rightarrow -\frac{q_2}{2H}x^2+C_3\cdot x+C_4=f \tag{4}$$

Solving the (3) system of equations the constants C_i (i=1,6) can be written function of the unknown H, under the forms :

$$C_1 = \frac{2f}{l} + \frac{q_2 - q_1}{H} \cdot \frac{a^2}{l} + \frac{q_2 l}{4H} - \frac{q_2 - q_1}{H} a \qquad \text{a)}$$

$$C_2 = 0 \qquad \text{b)}$$

$$C_3 = C_1 + \frac{q_2 - q_1}{H} a \qquad \text{c)}$$

$$C_4 = -\frac{q_2 - q_1}{2H} a^2 \qquad \text{d)} \qquad (5)$$

$$C_5 = C_1 + \frac{q_2 - q_1}{H} a + \frac{q_3 - q_2}{H} b \qquad \text{e)}$$

$$C_6 = -\frac{q_2 - q_1}{2H} a^2 - \frac{q_3 - q_2}{2H} b^2 \qquad \text{f)}$$

Replacing all the above constants in (4) the expression of the horizontal component of the force of the cable is :

$$H = \frac{2q_3\left(l^2 + b^2 - 2bl\right) - q_2\left(l^2 + 2b^2 - 4bl + 2a^2\right) + 2q_1 \cdot a^2}{4(2f - h)} \qquad (6)$$

The difference between the final and the initial length of the cable can be calculated with the expression [1] :

$$\Delta l = \frac{H}{EA}\left[\int_0^a \left(1 + y_1'^2\right)dx + \int_a^b \left(1 + y_2'^2\right)dx + \int_b^l \left(1 + y_3'^2\right)dx\right] \qquad (7)$$

The final length of the cable is expressed by the relation [3] :

$$L = \int_0^a \sqrt{1 + y_1'^2} \cdot dx + \int_a^b \sqrt{1 + y_2'^2} \cdot dx + \int_b^l \sqrt{1 + y_3'^2} \cdot dx \qquad (8)$$

If the external forces are distributed directly on the cable (fig. 1b) the equations of the deflection and of the slope (first derivative of the deflection y in respect with x) can be written as :

$$y(x)=\begin{cases} y_1=-\dfrac{H}{p_1}ch\left(-\dfrac{p_1}{H}x+C_1\right)+C_2, x\in[0,a) & \text{a)}\\ y_2=-\dfrac{H}{p_2}ch\left(-\dfrac{p_2}{H}x+C_3\right)+C_4, x\in[a,b) & \text{b)}\\ y_3=-\dfrac{H}{p_3}ch\left(-\dfrac{p_3}{H}x+C_5\right)+C_6, x\in[b,l] & \text{c)}\end{cases} \tag{9}$$

$$y'(x)=\begin{cases} y'_1=sh\left(-\dfrac{p_1}{H}x+C_1\right), x\in[0,a)\\ y'_2=sh\left(-\dfrac{p_2}{H}x+C_3\right), x\in[a,b)\\ y'_3=sh\left(-\dfrac{p_3}{H}x+C_5\right), x\in[b,l]\end{cases} \tag{10}$$

Using the (3b,c,d,e,f) limit conditions the constants C_2, C_3, C_4, C_5, C_6 can be expressed as functions of C_1 and H, under the forms :

$$C_3=C_1+\frac{p_2-p_1}{H}a \qquad a)$$

$$C_5=C_1+\frac{p_2-p_1}{H}a+\frac{p_3-p_2}{H}b \qquad b)$$

$$C_6=h+\frac{H}{p_3}ch\left(-\frac{p_3}{H}l+C_1+\frac{p_2-p_1}{H}a+\frac{p_3-p_2}{H}b\right) \qquad (c) \tag{11}$$

$$C_4=C_6+\frac{H}{p_2}ch\left(-\frac{p_2}{H}b+C_3\right)-\frac{H}{p_3}ch\left(-\frac{p_3}{H}b+C_5\right) \qquad d)$$

$$C_2=C_4+\frac{H}{p_1}ch\left(-\frac{p_1}{H}a+C_1\right)-\frac{H}{p_2}ch\left(-\frac{p_2}{H}a+C_3\right) \qquad e)$$

Using the (3a) and (4) limit conditions the following system of equation can be written :

$$C_2=\frac{H}{p_1}chC_1 \qquad a)$$

$$\tag{12}$$

$$C_4=f+\frac{H}{p_2}ch\left(-\frac{p_2 l}{2H}+C_3\right) \qquad b)$$

Replacing (11) in (12) the (12) system of equation will have only the unknowns C_1 and H in an implicit form. The numerically solving of the above system can be made by fixing an initial value for C_1 (for example zero), calculating the value for H as the first positive root of (12a) and verifying the (12b) equation. If (12b) is not identically satisfied, the initial value for C_1 is increased until the (12b) relation will be verified.

A Numerically Example

A cable made from *OL 37* with the cross sectional area $A = 3{,}36\ cm^2$ is considered. The cable is loaded with three uniform external forces $q_1 = p_1 = 29\ N/m$, $q_2 = p_2 = 58\ N/m$, $q_3 = p_3 = 29\ N/m$, each of them acting on a third of horizontal abscissa. The horizontal and vertical distances between the suspension points are respectively $l = 150\ m$ and $h = 10\ m$. The imposed deflection at the middle of the horizontally distance between the suspension points is $f = 40\ m$.

Solving numerically this example using (6), (7) and (8) relations the following values are obtained : $H = 3625\ \mathrm{N}; L_o = 168.577\ \mathrm{m}$

In order to solve numerically the (12) system of equations a specialised programme has been used and the following values for the unknowns have been obtained : $C_1 = 0{,}8169816$; $H = 3827.55\ \mathrm{N}$. Using the (7) and (8) relations the initial length of the cable is $L_o = 168.774\ \mathrm{m}$.

Analysing the results obtained above it can be noticed that the both methods of calculation reach nearly the same results.

However when the imposed deflection at the middle of the horizontal distance between the suspension points is higher ($f / l \geq 1/3$) the differences between the two methods of calculation became significant.

Conclusions

In the paper are presented two calculus methodologies that can be used for extensible cables loaded with different external forces. Thirst of them admits the hypothesis that the external forces are distributed on the horizontal distance between the suspension points. The second method of calculation admits the hypothesis that the external forces are distributed directly on the cable.

The both methods admit also the hypothesis that each of the external forces is uniform distributed. The results obtained are analysed in a calculus example. From the presented example it can be noticed that for a current distance between the suspension points (150 m), a vertical distance between the same points (10 m) and an imposed deflection in the cable displacement (40 m), the initial lengths of the cable are nearly the same.

The differences between the results obtained become significant if the fraction *f/l* is higher than 1/3.

In the same time the second method contains a numerical algorithm that make the calculus very efficient.

It can be also appreciated that the second method allows the calculation of an exact length of the cable that is an important issue. The methodology that is presented in the paper can be applied in any situation, for any dimensions of the cable and any geometry of the suspension points.

References

1. P o s e a N ., *Rezistența materialelor* , Editura Didactică şi Pedagogică, Bucureşti, 1979.
2. R ă d o i M . , D e c i u E . , *Mecanica*, Edit. Didactică şi Pedagogică, Bucureşti, 1981.
3. V a s i l e s c u Ş . , *A Method of Calculation of the Tension and Displacements in an Extensible Cable Loaded with Uniform External Pressure,* Buletinul U.P.G. Ploieşti, Seria Tehnică, LVIII(4), Ploieşti 2006.
4. V a s i l e s c u Ş ., T a l l e V . , *Rezistența materialelor-solicitări fundamentale*, sub tipar, Editura U.P.G., Ploieşti 2007.

Calculul firelor extensibile solicitate cu diferite forțe exterioare uniforme

Rezumat

În lucrare se prezintă două metodologii de calcul al cablurilor solicitate cu mai multe forței exterioare uniforme. Prima metodă admite ipoteza conform căreia forțele exterioare sunt distribuite in lungul deschiderii orizontale, pe mai multe tronsoane. Cea de-a doua metodă utilizează ipoteza în conformitate cu care, forțele exterioare sunt distribuite uniform pe mai multe tronsoane situate în lungul deformatei cablului. Ambele metode de calcul sunt analizate pe un exemplu de calcul, din carese observă că rezultaele sunt diferite numai dacă raportul dintre săgeata maximă impusă la mijlocul deschiderii este mai mare decât o treime din deschiderea orizontală a cablului.

În plus, metoda în care forțele exterioare sunt distribuite direct pe cablu furnizează un algoritm numeric extrem de eficient, care permite determinarea necunoscutelor prin utilizarea unui program specializat de analiza numerică.