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An Improvable Approach for PWM Control Algorithm of the Mono-Phase Inverter

- Part A -

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Abstract

The paper presents a mathematical support of the PWM control algorithm for the improvable synthesis of the output voltage of the mono-phase inverter, having as objectives the approaching of the effective value of the fundamental to the effective value of the proposed sinusoidal voltage at the terminals of the charge and diminishing of the weight of low frequency harmonics in the harmonic content of the voltage.

The commutation moments of the pulses are computed, imposing that the fundamental of the synthetic voltage is equal to the proposed sinusoidal voltage and the high harmonics up to order 4m-2 are null. For the numerical simulation of the model, the Matlab toolbox was used. The results of the simulation are numerically and graphically presented; they confirm the validity of the mathematical support of the proposed PWM control algorithm. The goal of this paper is to obtain an inverter which to ensure a power supply close to a sinusoidal form. Some practical considerations are also presented.

The paper was split in two parts. Part A contains a brief introduction, the principle electric scheme of a mono-phase inverter, the commutation program of the mono-phase inverter with resistive-inductive charge, the mathematical design of proposed PWM control algorithm and the spectral analysis of the synthetic voltage.

Key words: *PWM (pulse width modulation) control algorithm, mono--phase inverter, synthetic voltage.*

Introduction

The performances of the adjustable electrical drives with asynchronous motors are dependent on the capability of inverters to ensure a power supply close to a sinusoidal form. The inverter yields non-sinusoidal currents and voltages, which determine a deformed regime in the motor and in the supply network. The high harmonics have negative effects on the functioning of the ensemble inverter – motor through: the increase of currents in the chain winding of the motor, the increase of the power loss, the apparition of oscillating couples and the worsening of commutation phenomena in the power semiconductor devices. The unfavourable effect of the oscillating couples shows up especially at low frequencies and consists in a jerky movement of the rotor, even a resonance phenomenon in the mechanical transmission of the drive being possible [1].

In order to eliminate these effects, the commutation program of the thyristors should be adjustable depending on the frequency of the fundamental of the voltage in the motor. This could be achieved by modulating the pulses in width (PWM) according to a sinusoidal function.

The modulation of the pulse width is applied at frequencies $f < 50$ Hz and has a double role: the variation of amplitude of the fundamental correlated with its frequency and the nullifying of low frequency harmonics. In the literature [1, 2, 3], several modulation methods are known: the comparison of a sinusoidal modulator signal with a high frequency triangular modulated signal, the sampling of the angular position of the spatial vector of the phase voltages, the equality between the area sampled from the proposed sinusoidal voltage and the area of the pulse voltage.

In this paper, one elaborates the mathematical model of PWM control algorithm for improvable synthesis of the voltage for a mono-phase inverter. By numerical simulation one analyses the voltage synthesis; the commutation moments of the thyristors, the width of the pulses, the waveforms and the spectral analysis of the inverter's voltage are determined.

Commutation Program of Inverter

The electric principle scheme of a mono-phase inverter for the voltage is presented in figure 1. The inverter consists of the mono-phase bridge with the thyristors T_1, T_2, T_3, T_4 and the bridge with the restoration diodes D_1, D_2, D_3, D_4 in a parallel assembly. The bridge with diodes functions in the case of a charge (an electric receptor) with an inductive character. This assures the current continuity through the charge during the time the reactive energy accumulated in the inductance of the charge unloads [1, 4].

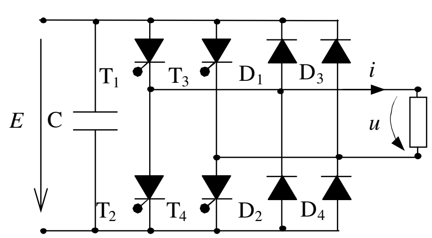

Fig. 1. The scheme of the inverter in mono-phase bridge.

At a frequency of 50 Hz, the inverter functions in non-modulated regime and the output voltage $u(t)$ has the shape of positive and negative rectangular pulses (a rectangular shape), and the current through the charge has an exponentially increasing and decreasing variation [3, 4]. The duration of a pulse is 10 ms, and the amplitude of the pulse is equal to *E,* the direct voltage of the intermediate circuit. The magnitude of that voltage can be computed from the equality between the effective values of the charge nominal voltage and the effective values of the fundamental of Fourier expansion of the inverter output voltage:

$$E = \frac{\sqrt{2}\pi}{4} U_{nom} = \frac{\sqrt{2}\,\pi}{4} U_{ef1}, \tag{1}$$

where: E is the direct voltage of the intermediate voltage; U_{ef1} – the effective value of the fundamental of the inverter voltage; U_{nom} – the effective value of the charge nominal voltage.

The time interval when a group of two thyristors is in conduction state is equal to $T/2$ (T =20 ms, is the period of the output voltage) and it is named tact; a period has two tacts. The inverter functions with two thyristors or two diodes in conduction; during the first tact the semiconductor devices T_1, T_4 or D_1, D_4 are in uninterrupted conduction on the positive alternation, and during the second tact the devices T_2, T_3 or D_2, D_3 are continuously conducting on the negative alternation.

At frequencies $f < 50$ Hz, the inverter functions in modulated regime, and the voltage at the terminals of the charge consists in a succession of non null pulse (positive or negative) – null pulse, depending on the conducting devices, with width modulated after a sinusoidal function. The non null pulses of voltage correspond to the conduction state of the semiconductor devices (thyristors or diodes) contained in the two groups of bridges (the devices T_1, T_4 or D_1, D_4 for the positive pulses and the devices T_2, T_3 or D_2, D_3 for the negative pulses), and the null pulses correspond to the conduction state of the semiconductor components (thyristor and diode) belonging to a single group of bridges (the devices T_1, D_3 or T_2, D_4 or T_3, D_1 or T_4, D_2). The commutation program for the semiconductor devices for one period is given in the table 1.

Table 1. The commutation program

Tact	Interval	Pulse	Thyristors in conduction	Diodes in conduction	Devices in conduction	Discharge energy
1	0 – T/4	null	T_1, T_3	D_1	T_3, D_1	x
				D_3	T_1, D_3	x
		positive	T_1, T_4	D_1, D_4	D_1, D_4	x
				-	T_1, T_4	-
	T/4 – T/2	positive	T_1, T_4	-	T_1, T_4	-
		null	T_2, T_4	D_2	T_4, D_2	x
2	T/2– 3T/4	null	T_2, T_4	D_2	T_4, D_2	x
				D_4	T_2, D_4	x
		negative	T_2, T_3	D_2, D_3	D_2, D_3	x
				-	T_2, T_3	-
	3T/4 - T	negative	T_2, T_3	-	T_2, T_3	-
		null	T_1, T_3	D_1	T_3, D_1	x

In the case of a resistive-inductive charge, when thyristors commute for passing from a tact to another, or from a non null pulse to a null pulse, one of the diodes (the one directly polarized) begins conducting; the reactive energy stored in the inductance of the charge discharges through it, thus maintaining the same sense of the current through the charge. The discharge current closes only through the charge, either and through the condenser C, or and through the source of direct current (if this is possible). The discharge current is decreasing and zeroes in non null pulses after a time t_D, called recovery time. After the reactive energy is unloaded, the current through the charge changes its sense and closes through the group of thyristors (T_1, T_4 or T_2, T_3) entered in the conduction state and increases in time. Thus, the current through the charge has an exponentially increasing and decreasing variation in time, depending of the power devices conduction.

Mathematical Design of Proposed PWM Control Algorithm

The inverter functions in modulated regime at frequencies less than 50 Hz, in order to reduce the weight of low harmonics. Even if the voltage E of the intermediate circuit is constant, the inverter adjusts not only the frequency, but also the effective value of the voltage fundamental. The principle of PWM control algorithm consists in the conductions of the power devices during intervals of time with the durations modulated by a sinusoidal function [1, 2, 5].

The technique of modulating the pulses width, presented in this paper, is based on the equality between the effective value of the fundamental of the synthetic voltage to the effective value of the proposed sinusoidal voltage at the terminals of the charge and nullifying (diminishing) of the weight of low frequency harmonics up to order $4m – 2$ (for m see relation (5)).

The proposed sinusoidal voltage at the output inverter is:

$$u_s(t) = A \sin \omega t = \sqrt{2}\, U \sin 2\pi f\, t \,, \tag{2}$$

where: A is the amplitude; U – the effective value; f – the frequency; $T = 1/f$ – the period.

The effective value of the proposed sinusoidal voltage is lower than the effective value of the output voltage fundamental of the inverter in non-modulated regime:

$$U < 2\sqrt{2}\, E / \pi = 0{,}9E \,, \tag{3}$$

where, E is the direct voltage of the intermediate circuit.

The proposed sinusoidal voltage, $u_s(t)$, can be approximated with a synthetic voltage (synthesized from pulses), $u(t)$, defined in a period T like this:

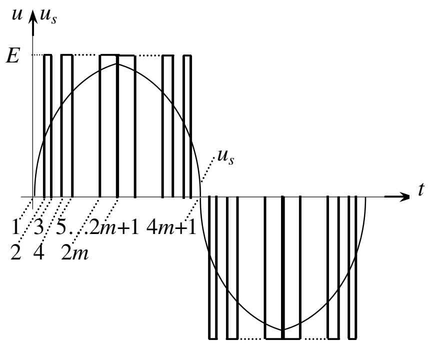

Fig. 2. The waveforms of the synthetic voltage and proposed voltage.

° during the first quarter of period:

$$u(t) = U_{2k-1} = 0, \text{ for } t_{2k-1} < t < t_{2k}, \quad (4)$$

$$u(t) = U_{2k} = E, \text{ for } t_{2k} < t < t_{2k+1}, \quad (5)$$

where: $k = 1, 2, \ldots m$ represents the number of pulses in the interval $0 - T/4$; $t_1 = 0$ and $t_{2m+1} = T/4$, being the limits of the interval;

° during the second quarter of period the pulses are symmetrical with respect to the moment $T/4$:

$$u(t) = U_j = U_i, \; t_{j+1} = T/2 - t_i, \quad (6)$$

for $t_j \le t \le t_{j+1}$, where: $j = 2m + k$, $i = 2m - k + 1$, $k = 1,2,\ldots 2m$; $t_{2m+1} = T/4$ and $t_{4m+1} = T/2$, being the limits of the interval:

° during the second semi period the pulses are negative and symmetrical with respect to the moment $T/2$:

$$u(t) = U_j = -U_k, \; t_{j+1} = T/2 + t_{k+1}, \quad (7)$$

for $t_j \le t \le t_{j+1}$, where: $j = 4m + k$, $k = 1, 2, \ldots 4m$; $t_{4m+1} = T/2$ and $t_{8m+1} = T$, being the limits of the interval.

In figure 2 the voltages $u_s(t)$ and $u(t)$ are graphically presented on the interval $0 - T$; the notations on the time axis are the indexes of the commutation moments.

The Fourier series expansion of the synthetic voltage contains only odd harmonics in sinus [6]:

$$u(t) = \sum_{n=1,3,5}^{\infty} b_n \sin n\omega t = \sum_{n=1,3,5}^{\infty} \sqrt{2} U_{efn} \sin n\omega t, \quad (8)$$

where U_{efn} is the effective value for the n^{th} harmonic and the series coefficients are:

$$b_n = \frac{2}{T} \int_0^T u(t) \sin n\omega t \, dt = = \frac{4E}{n\pi} \sum_{k=1}^{m} \left(\cos n\omega t_{2k} - \cos n\omega t_{2k+1} \right). \quad (9)$$

From the conditions that the fundamental equals the proposed sinusoidal voltage and that the first $2(m-1)$ odd harmonics (3^{rd}, 5^{th}, $\ldots(2m-1)^{th}$) nullify, the following system of equations is obtained:

$$\begin{cases} b_1 = A \\ b_3 = b_5 = \ldots = b_{4m-3} = 0 \end{cases} \quad (10)$$

From the relations (9) and (10) the following non-linear system of equations results:

$$\begin{cases} \sum_{k=1}^{m}\left(\cos\omega t_{2k} - \cos\omega t_{2k+1}\right) = \dfrac{\pi A}{4E} \\ \sum_{k=1}^{m}\left(\cos 3\omega t_{2k} - \cos 3\omega t_{2k+1}\right) = 0. \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \sum_{k=1}^{m}\left(\cos(4m-3)\omega t_{2k} - \cos(4m-3)\omega t_{2k+1}\right) = 0 \end{cases}, \tag{11}$$

with the commutations moments $t_2, t_3, \ldots t_{2m}$ as unknowns.

The non linear system of equations is solved numerically and the commutation moments from the interval 0 – T/4 are computed; the other commutation moments are found with relations (6) and (7). The synthetic voltage is completely defined.

For M an imposed number of harmonics (odd and even) nullified, the synthetic voltage contains m =1+Integer ((M+2)/4) pulses in the interval 0 – T/4.

Spectral Analysis of the Synthetic Voltage

The effective values of voltage harmonics and the distortion coefficients are given by the following formulas [1, 3, 6, 7]:

° the effective values of the n^{th} harmonic:

$$U_{efn} = \frac{b_n}{\sqrt{2}},\ n = 1,\ 3,\ 5, \ldots; \tag{12}$$

° the total effective value:

$$U_{ef.t} = \left[\frac{1}{T}\int_0^T u^2(t)\,\mathrm{d}t\right]^{\frac{1}{2}} = 2E\left[\frac{1}{T}\sum_{k=1}^{m}\left(t_{2k+1} - t_{2k}\right)\right]^{\frac{1}{2}}; \tag{13}$$

° the total effective value of the high harmonics:

$$U_{ef.t.a} = \left[\frac{1}{T}\int_0^T\left(\sum_{n=3,5,..}^{\infty} b_n \sin n\omega t\right)^2 \mathrm{d}t\right]^{\frac{1}{2}} = \left[\frac{1}{2}\sum_{n=3,5,..}^{\infty} b_n^2\right]^{\frac{1}{2}} = \left[U_{ef.t}^2 - U_{ef.1}^2\right]^{\frac{1}{2}}; \tag{14}$$

° the distortion coefficient k_{d1}, defined as the square root of the ratio between the conducting power in harmonics and the conducting power in the fundamental:

$$k_{d1} = \frac{U_{ef.t.a}}{U_{ef.1}} = \left[\frac{U_{ef.t}^2}{U_{ef.1}^2} - 1\right]^{\frac{1}{2}}; \tag{15}$$

° the distortion coefficient k_{d2}, defined as the square root of the ratio between the conducting power in harmonics and the conducting power in the synthetic voltage:

$$k_{d2} = \frac{U_{ef.t.a}}{U_{ef.t}} = \frac{k_{d1}}{\left(1 + k_{d1}^2\right)^{\frac{1}{2}}}. \tag{16}$$

Summary of Part A

The paper was split in two parts. Part A contains a brief introduction, the principle electric scheme of a mono-phase inverter, the commutation program of the mono-phase inverter for resistive-inductive charge and the mathematical design of proposed PWM control algorithm. The proposed mathematical model is based on the equality between the effective value of the fundamental of the synthetic voltage to the effective value of the proposed sinusoidal voltage at the terminals of the charge and nullifying or diminishing of the weight of low frequency harmonics up to order $4m - 2$.

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O abordare îmbunătățită a algoritmului de comandă PWM al invertorului monofazat

- Partea A -

Rezumat

Articolul prezintă un suport matematic al algoritmului de comandă PWM pentru sinteza îmbunătățită a tensiunii de ieşire a invertorului monofazat, având ca obiective aproximarea valorii efective a fundamentalei cu valoarea efectivă a tensiunii sinusoidale propuse la bornele sarcinii şi diminuarea ponderii armonicelor de joasa frecvența în conținutul armonic al tensiunii.

Momentele de comutație ale pulsurilor sunt calculate în condițiile în care fundamentala tensiunii sintetice este egală cu tensiunea sinusoidală propusă şi armonicele superioare până la ordinul 4m – 2 sunt nule. Pentru simularea numerică a modelului s-a folosit pachetul Matlab. Rezultatele simulării sunt prezentate numeric şi grafic; se confirmă validitatea suportului matematic al algoritmului de comandă PWM propus. Scopul acestui articol este să se obțină un invertor care să asigure o sursă de alimentare apropiată de forma sinusoidală. De asemenea sunt prezentate câteva considerații practice.

Articolul este împărțit în două părți. Partea A conține o scurtă introducere, schema electrică de principiu a invertorului monofazat, programul de comutație al invertorului monofazat cu sarcină rezistiv – inductivă, modelul matematic al algoritmului de comandă PWM propus şi analiza spectrală a tensiunii sintetice.