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New Calculation Method for Biphasic Gas-Water Flows through Pipes

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Abstract

Generally speaking, when transporting natural gas through pipelines high quantities of water may occur, this giving the mixture a biphasic character. There are several theories on biphasic transport, but they have certain limits or are applied only using empirical coefficients whose valability is unnsure. The method proposed by the paper, even if using one of Baker's ideas, is an exact one and does not require experimental numerical coefficients. The method uses hydraulic resistance coefficients for monofhasic flow in pipelines, no matter the type of movement (laminar or turbulent) or the hydraulic nature of the pipeline (flat, semi-flat or completely rugose).

Key words: *biphasic flow, pressure falls*, *Reynolds number, laminar, turbulent*

Introduction

Over 50 years ago (in 1949) Lockhart and Martinelli presented, for the first time, a calculation relation for the pressure dropping of a biphasic flow in horizontal pipes. Although the method is empirical, it has a theoretical support. The method admits that the pressure is equal in the two biphasic pressure falls. Nevertheless, the results of the calculation never led to the equality of pressures. The method proposed makes possible the realization of this equality with a high precision.

The Lockhart-Martinelli Method

The pressure fall on each phase $\left(\frac{\Delta p}{l}\right)_i$, i is an index that is valid for the gasous phase ă ($i = g$) and for the liquid one ($i = a$), is calculated with the classical formula:

$$\left(\frac{\Delta p}{l}\right)_i = \rho_i \frac{v_i^2}{2} \frac{\lambda_i}{d_i}. \quad (1)$$

The average speed v_i corresponding to phase i , is calculated according to the formula

$$v_i = 4\frac{M_i}{\pi d_i^2 \rho_i} \ , \tag{2}$$

where d_i is the equivalent diameter through which the phase indicated by i flows. We understand that $d_i \leq d$, d being the interior diameter of the transportation pipe. The average speed of the phase, if it would flow by itself through the pipe, is of

$$v_{si} = 4\frac{M_i}{\pi d^2 \rho_i} \ . \tag{3}$$

Between the biphasic pressure gradient $\left(\frac{\Delta p}{l}\right)_b$ and any of the two gradients of the transporting phases $\left(\frac{\Delta p}{l}\right)_{si}$ there are the relations

$$\left(\frac{\Delta p}{l}\right)_b = \Phi_i^2\left(\frac{\Delta p}{l}\right)_{si} = \alpha_i^{\frac{n-2}{2}}\left(\frac{d}{d_i}\right)^{\frac{5-m}{2}} , \tag{4}$$

In which α_i has two values that refer to the slide between the phases. The m exponent may vary according to the two values of i.

In the situation in which for the two phases the hydraulic strength coefficients are calculated according to Blasius formula, than

$$\left(\frac{\Delta p}{l}\right)_b = \left(\frac{\Delta p}{l}\right)_{si} \alpha_i^{\frac{m-2}{2}}\left(\frac{d}{d_i}\right)^{\frac{5-m}{2}} . \tag{5}$$

The authors have admitted, without specifying the rightness of the present method, that the flowing rhythms of the two phases are established according to Reynolds number, Re_i and its critical value is $(Re_{si})_{cr} \approx 1000$. The Reynolds number, Re_{si}, is calculated according to the classical formula

$$Re_{si} = \frac{\rho_i v_{si} d}{\mu_i} , \tag{6}$$

μ_i being the dynamic viscid of i phase from the mixture

In the situation in which during the movement of the biphasic fluid $Re_{si} < 1000$, the movement of the respective phase is flaky, and if $Re_{si} \geq 1000$, then the movement becomes turbulent. The conclusion is, consequently, that, without a theoretical or experimental part, the authors have admitted that the transition from flaky to turbulent movement takes place at this value of the Reynols number, equal to 1000.

The pressure gradient $\left(\frac{\Delta p}{l}\right)_b$ can be calculated by the means of formula (4), and the two gradients, respectively i =1 for the liquid and i =2 for the gaseous phase, satisfy the relation

$$X = \sqrt{\frac{\left(\frac{\Delta p}{l}\right)_{sa}}{\left(\frac{\Delta p}{l}\right)_{sg}}} = \frac{v_{sa}}{v_{sg}}\sqrt{\frac{\rho_a \lambda_{sa}}{\rho_g \lambda_{sg}}} . \tag{7}$$

In the situation in which λ_a şi λ_g is calculated according to Blasius, than X parameter has the value of

$$X = \left(\frac{v_{sa}}{v_{sg}}\right)^{7/8}\left(\frac{\rho_a}{\rho_g}\right)^{3/8}\left(\frac{\mu_a}{\mu_g}\right)^{1/8} . \tag{8}$$

The Silviu Stan Method

In order to ground the Lockhart – Martinelli correlations, for the two phases, we will write the obvious relations:

$$A_a\left(\frac{\Delta p}{l}\right)_b - \tau_{0a}P_a + S = 0 \tag{9}$$

$$A_g\left(\frac{\Delta p}{l}\right)_b - \tau_{0g}P_g - S = 0 , \tag{10}$$

for which τ_0 represent the effort to the pipe's wall for each phase, and P_i is the pipe's perimeter in contact with the respective phase. A_a and A_g are the transversal sections through the pipe with section $A = \pi\frac{d^2}{4}$, occupied by phases and obviously $A = A_a + A_g$.

By adding relations (9) and (10) we get

$$A\left(\frac{\Delta p}{l}\right)_b = \tau_a P_a + \tau_g P_g \tag{11}$$

where, to simplify, for the efforts τ_{0a} and τ_{0g} the index representing the significance of the fact that they are used for the interior wall of the pipe has been omitted. These may be calculated with the help of

$$\tau_a = \rho_a \frac{v_a^2}{8}\lambda_a,\ \tau_g = \rho_g \frac{v_g^2}{8}\lambda_g \tag{12}$$

where λ_a and λ_g are the hydraulic strength coefficients that are calculated according to the Reynolds numbers Re_a and, respectively, Re_g. Each of the Reynolds numbers is defined with the average speed v_a, or v_g, and the interior diameter of the pipe d.

The average speeds v_a and v_g are calculated according to the mass flaws M_a and M_g

$$v_a = \frac{4M_a}{\rho_a \pi d_a^2};\ v_g = \frac{4M_g}{\rho_g \pi d_g^2}. \tag{13}$$

With the help of these measures, it results that the water fraction a has the following expression

$$a = \frac{\rho_{gN}}{\rho_g} \frac{v_a}{v_g} \left(\frac{d_a}{d_g} \right)^2 \qquad (14)$$

where ρ_{gN} represents the density of the gases in normal conditions.

Formula (9) can also be written under the form of

$$A_a \rho_b \frac{v_b^2}{2} \frac{\lambda_b}{d} = \rho_a \frac{v_a^2}{8} \lambda_a \left(P_a - \sqrt{H_a d} \right) \qquad (15)$$

where H_a is an equivalent length between aria A_a and A_g. In a similar way, formula (10) can be written under the form of

$$A_g \rho_b \frac{v_b^2}{2} \frac{\lambda_b}{d} = \rho_g \frac{v_g^2}{8} \lambda_g \left(P_g + \sqrt{H_a d} \right). \qquad (16)$$

The ration of the two members of the last equations leads to the relation

$$\frac{A_a}{A_g} = \frac{\rho_a}{\rho_g} \frac{\lambda_a}{\lambda_g} \frac{v_a^2}{v_g^2} \frac{P_a - \sqrt{H_a d}}{P_g + \sqrt{H_a d}} \approx \frac{\rho_a}{\rho_g} \frac{\lambda_a}{\lambda_g} \frac{v_a^2}{v_g^2} \frac{P_a}{P_g}, \qquad (17)$$

That can be also written under the form of

$$\frac{A_a}{A_g} = a^2 \frac{\rho_a \rho_g}{\rho_{gN}^2} \frac{\lambda_a}{\lambda_g} \frac{P_a}{P_g} \left(\frac{d_g}{d_a} \right)^4. \qquad (18)$$

The P_a and P_g perimeters are written according to the d_a and d_g diameters, which renders

$$\left(\frac{d_a}{d_g} \right)^{(1)} = \left[a^2 \frac{\rho_a \rho_g}{\rho_{gN}^2} \frac{\lambda_a}{\lambda_g} \right]^{1/5} \qquad (19)$$

The index 1 attached above the ratio $\frac{d_a}{d_g}$ indicated that this is the first iteration obtained as a consequence of the approximation in (15). We can introduce the symbols

$$d_a = d_g C_g, \; d_g = d_a C_a, \qquad (20)$$

where

$$C_g = \left(a^2 \frac{\rho_a \rho_g}{\rho_{gN}^2} \frac{\lambda_a}{\lambda_g} \right)^{1/5}, \; C_a = C_g^{-1} = \left(\frac{1}{a^2} \frac{\rho_{gN}^2}{\rho_a \rho_g} \frac{\lambda_g}{\lambda_a} \right)^{1/5}. \qquad (21)$$

On the other side we have:

$$\frac{A_a}{A_g} = C_g^2, \; \frac{A_g}{A_a} = C_a^2 \qquad (22)$$

That allow the expression of A_a and A_g areas according to the total area A

$$A_a = \frac{A}{1 + C_a^2}, \; A_g = \frac{A}{1 + C_g^2}. \qquad (23)$$

The ratio λ_a / λ_g from the expression of C_g and C_a is for the moment unknown, which determines also that C_g and C_a to be undetermined.

The equation (15) in which $\sqrt{H_a d}$ is neglected compared to P_a can be written under the form of

$$\rho_b \cdot \frac{v_b^2 \cdot \lambda_b}{2d} = \sqrt{1 + C_a^2} \cdot \frac{\rho_a \cdot v_a^2 \cdot \lambda_a}{2d} \tag{24}$$

because

$$\frac{P_a}{A_a} = \frac{4}{d_a} = \frac{4\sqrt{1+C_a^2}}{d}.$$

The equation (16) can be similarly written under the form of

$$\rho_b \frac{v_b^2 \lambda_b}{2d} = \sqrt{1 + C_g^2} \frac{\rho_g v_g^2 \lambda_g}{2d} \tag{25}$$

because now

$$\frac{P_g}{A_g} = 4\frac{\sqrt{1+C_g^2}}{d}.$$

From the previous relations we can understand that

$$\frac{A_a}{A_g} = \left(\frac{d_a}{d_g}\right)^2, \ \frac{P_a}{P_g} = \frac{d_a}{d_g}. \tag{26}$$

The equation (19) is written under the equivalent form of

$$\left(\frac{d_a}{d}\right)^{(1)} \lambda_g^{\frac{1}{5}} = \left(a^2 \frac{\rho_a \rho_g}{\rho_{gN}^2}\right)^{\frac{1}{5}} \left(\frac{d_g}{d}\right)^{(1)} \lambda_a^{\frac{1}{5}}. \tag{27}$$

In the situation in which the two phases would flow by themselves through the pipe, the average speeds are considered to be v_{sa} and v_{sg}, and the strength coefficients are to be λ_{sa} and, correspondingly λ_{sg}. Between these two speeds we can establish the relations

$$v_{sa} = v_a\left(\frac{d_a}{d}\right), \ v_{sg} = v_g\left(\frac{d_g}{d}\right). \tag{28}$$

In the case in which we introduce, just like Lockhart – Mertinelli, the defined parameter X in the relation

$$X = \sqrt{\frac{\left(\frac{\Delta p}{l}\right)_{sa}}{\left(\frac{\Delta p}{l}\right)_{sg}}} = \frac{v_{sa}}{v_{sg}} \sqrt{\frac{\rho_a \lambda_{sa}}{\rho_g \lambda_{sg}}}, \tag{29}$$

The double indexes *sa* and *sg* attached to the pressure gradient $\left(\frac{\Delta p}{l}\right)$ indicate that the latter is calculated for the situation in which just the liquid phase flows through the pipe (*sa*) and respectively, just the gaseous phase (*sg*).

If we note down the X_s parameter equivalent to the X, namely

$$X_s = \sqrt{\frac{\left(\frac{\Delta p}{l}\right)_a}{\left(\frac{\Delta p}{l}\right)_g}} = \frac{v_a}{v_g}\sqrt{\frac{\rho_a \lambda_a}{\rho_g \lambda_g}}, \tag{30}$$

Between *X* and X_s the following relation exists

$$\frac{X}{X_s} = \left(\frac{d_a}{d_g}\right)^2 \sqrt{\frac{\lambda_{sa}}{\lambda_{sg}} \frac{\lambda_g}{\lambda_a}}. \tag{31}$$

The immediate result of the (28) is

$$\left(\frac{d_a}{d}\right)^2 = \frac{v_{sa}}{v_a}, \left(\frac{d_g}{d}\right)^2 = \frac{v_{sg}}{v_g}, \tag{32}$$

And in order to obtain the ration $\frac{\lambda_a}{\lambda_g}$ we must fist do certain hypothesis related to the flowing rhythms, defined by the numbers Re_a, Re_{sa}, Re_g and Re_{sg}. At the Reynolds number, *Re*, the indexes *a, sa, g* and *sg* have been added, just as in the case of the expressions of *X* and X_s.

Next, we take into consideration just two cases, namely that both numbers Re_a and Re_g are specific to flaky rhythm (Re_a and Re_g < 3000) and the flowing turbulences (Re_a and $Re_g \geq$ 3000). For the flaky movement, correspondingly Re_a and Re_g < 3000, it is surely simultaneously made and the in equations Re_{sa} and Re_{sg} < 3000. For the turbulent flowing (Re_a and $Re_g \geq 3000$) it is possible that one of the two numbers Re_{sa} and Re_{sg}, or even both, to be inferior to the critical value of 3000.

In order to simplify the thinking, we admitted that both pairs of Reynolds numbers indexed with *a, sa, g* and *sg* are simultaneously either smaller than 3000 or superior to that specific value. In the first case, because λ_a, λ_{sa} şi λ_g, λ_{sg} are calculated according to Stokes relation ($\lambda \cdot Re$ = 64), we have the fallowing result

$$\frac{d_a}{d} = \frac{Re_a}{Re_{sa}} = \frac{\lambda_{sa}}{\lambda_a};\ \frac{d_g}{d} = \frac{\lambda_{sg}}{\lambda_g}. \tag{33}$$

For the second situation when the four flowing rhythms are turbulent, the strength coefficients are calculated according to Blasius formula $\lambda = \frac{0,3164}{Re^{0,25}}$, and we immediately have

$$\frac{d_a}{d} = \frac{Re_a}{Re_{sa}} = \left(\frac{\lambda_{sa}}{\lambda_a}\right)^4;\ \frac{d_g}{d} = \frac{Re_{sg}}{Re_g} = \left(\frac{\lambda_{sg}}{\lambda_g}\right)^4. \tag{34}$$

Consequently, we have for the flaky rhythms

$$\frac{d_a}{d_g} = \left(\frac{\lambda_g}{\lambda_a}\frac{\lambda_{sa}}{\lambda_{sg}}\right)^4, \tag{35}$$

And for the turbulent rhythms

$$\frac{d_a}{d_g} = \frac{\lambda_g}{\lambda_a}\frac{\lambda_{sa}}{\lambda_{sg}}. \tag{36}$$

Each d_a/d_g ratio can be introduced in (14), which would lead to the equalities

$$\frac{\lambda_a}{\lambda_g} = \left(\frac{\lambda_{sa}}{\lambda_{sg}}\right)^{5/6}\left(a^2\frac{\rho_a\rho_g}{\rho_{gN}^2}\right)^{-1/6}, \tag{37}$$

for the flaky rhythms, and

$$\frac{\lambda_a}{\lambda_g} = \left(\frac{\lambda_{sa}}{\lambda_{sg}}\right)^{20/21}\left(a^2\frac{\rho_a\rho_g}{\rho_{gN}^2}\right)^{-1/21} \tag{38}$$

for the turbulent rhythms.

The two values of the λ_a/λ_g ratio are written under the unique form

$$\frac{\lambda_a}{\lambda_g} = \left(\frac{\lambda_{sa}}{\lambda_{sg}}\right)^{a}\left(a^2\frac{\rho_a\rho_g}{\rho_{gN}^2}\right)^{b} = \left(\frac{\lambda_{sa}}{\lambda_{sg}}\right)^{a} F^b, \tag{39}$$

Where the *F* parameter is calculated according to the formula

$$F = a^2\frac{\rho_a\rho_g}{\rho_{gN}^2}$$

and it depends mainly by the water ratio *a*. The values of the coefficients are $a = 5/6$ and $b = -1/6$ for the flaky movements and, respectively $a = 20/21$ and $b = -1/21$ for turbulent movements.

The pressure gradient for the biphasic flow $\left(\frac{\Delta p}{l}\right)_b$ is expressed by one of the relations (4). In the case in which we admit one of it, than,

$$\left(\frac{\Delta p}{l}\right)_b = \sqrt{1+C_a^2}\,\frac{\rho_a v_a^2\lambda_a}{2d}. \tag{40}$$

If the gradient $\left(\frac{\Delta p}{l}\right)_b$ is expressed in a similar way as in the case of Lockhart – Martinelli method, namely

$$\left(\frac{\Delta p}{l}\right)_b = \Phi_a^2\cdot\left(\frac{\Delta p}{l}\right)_{sa} \tag{41}$$

or

$$\left(\frac{\Delta p}{l}\right)_b = \Phi_g^2 \cdot \left(\frac{\Delta p}{l}\right)_{sg} \tag{42}$$

where Φ_a^2 and Φ_g^2 are number with the values given by

$$\Phi_a^2 = \frac{\sqrt{1+C_g^2}}{C_g}\left(\frac{d}{d_a}\right)^{2+m} ; \Phi_g^2 = \frac{\sqrt{1+C_a^2}}{C_a}\left(\frac{d}{d_g}\right)^{2+m} . \tag{43}$$

The m = 1 coefficient for the flaky movement and m =1/4 for the turbulent movement. We notice the ratios between the pipe's diameter and the ones equivalent for the a and g phases are

$$\frac{d}{d_a} = \sqrt{\frac{A}{A_a}} = \sqrt{1+C_a^2} \tag{44}$$

and

$$\frac{d}{d_g} = \sqrt{\frac{A}{A_g}} = \sqrt{1+C_g^2} . \tag{45}$$

Consequently, it results that Φ_a^2 and Φ_g^2 are now under the form of

$$\Phi_a^2 = \frac{\sqrt{1+C_g^2}}{C_g} \cdot \left(1+C_a^2\right)^{\frac{2+m}{2}} , \Phi_g^2 = \frac{\sqrt{1+C_a^2}}{C_a} \cdot \left(1+C_g^2\right)^{\frac{2+m}{2}} , \tag{46}$$

Where m has the values previously notified (m = 1 for the flaky flows where Φ_a^2 will be written down as Φ_{all}^2 and Φ_g^2 through Φ_{gll}^2 and, for the turbulent flows, m = -1/4, where the parameters will be written down as Φ_{att}^2 and, respectively, Φ_{gtt}^2).

Because $C_g C_a = 1$ and $C_g^2 C_a^2 = 1$, it consequently results that the determination of the parameters Φ_a^2 and Φ_g^2 is not difficult to reach if at least one of it is given (C_a or C_g) . If we consider for instance , C_g given by the first relation (22) we can consequently write

$$C_g = F^{1/5}\left(\frac{\lambda_a}{\lambda_g}\right)^{1/5} . \tag{47}$$

On the other side, the ratio $\frac{\lambda_a}{\lambda_g}$ is given by the formula (39) which renders

$$C_g = \left(\frac{\lambda_{sa}}{\lambda_{sg}}\right)^{\frac{a}{5}} F^{b+\frac{1}{5}} = \left(\frac{\lambda_{sa}}{\lambda_{sg}}\right)^{\frac{a}{5}} F^{\frac{5b+1}{5}} . \tag{48}$$

The C_a value can be easily triggered. The two parameters C_a and C_g can be consequently easily triggered, because F can be calculated if the water ratio a is known.

$$C_a = \left(\frac{\lambda_{sa}}{\lambda_{sg}} \right)^{-\frac{a}{5}} F^{-\frac{5b+1}{5}} . \tag{49}$$

The equivalent forms of the parameters Φ_a^2 and Φ_g^2 can be traced down, namely

$$\Phi_a^2 = \left(1 + C_a^2\right)^{\frac{3+m}{2}} , \ \Phi_g^2 = \left(1 + C_g^2\right)^{\frac{3+m}{2}} , \tag{59}$$

with the same notification that for the flaky movements $m = 1$, and for the turbulent movements $m = 1/4$.

We can appreciate that the equivalent diameters d_a and d_g obtained as a result of the calculations are

$$d_a = \frac{d}{\sqrt{1 + C_g^2}} , \ d_g = \frac{d}{\sqrt{1 + C_a^2}} \tag{60}$$

which represent the first iterative values of the diameters.

The C_a and C_g measures can be easily established taking into consideration the fact that the pressure gradients $\left(\frac{\Delta p}{l} \right)_b$ expressed according to Φ_a^2 or Φ_g^2 must be identical. Which renders

$$\frac{\Phi_a^2}{\Phi_g^2} = \frac{\rho_g v_{sg}^2 \lambda_{sg}}{\rho_a v_{sa}^2 \lambda_{sa}} = \left(\frac{1 + C_g^2}{1 + C_a^2} \right)^{\frac{3+m}{2}} . \tag{61}$$

The right member can be written down under the fallowing forms

$$\left(\frac{1 + C_g^2}{1 + C_a^2} \right)^{\frac{3+m}{2}} = \left(C_g^2 \right)^{\frac{3+m}{2}} = \left(C_a^{-2} \right)^{\frac{3+m}{2}} ,$$

from where we can immediately understand that

$$C_g = \left(\frac{\rho_g \cdot v_{sg}^2 \cdot \lambda_{sg}}{\rho_a \cdot v_{sa}^2 \cdot \lambda_{sa}} \right)^{\frac{1}{3+m}} ; \ C_a = C_g^{-1} = \left(\frac{\rho_g \cdot v_{sg}^2 \cdot \lambda_{sg}}{\rho_a \cdot v_{sa}^2 \cdot \lambda_{sa}} \right)^{-\frac{1}{3+m}} . \tag{62}$$

We consequently notice that the d_a and d_g parameters can be easily obtained, values that are written down by means of $d_a^{(1)}$ and,, correspondingly, $d_g^{(1)}$, which represents the number one order iteration of these diameters.

If we write down by (i+1) the iteration of this order of the d_a and d_g diameters, meaning $d_a^{(i+1)}$ and, respectively, $d_g^{(i+1)}$ than

$$d_a^{(i+1)} = d_a^{(i)} + \varepsilon_a d \, ; \ d_g^{(i+1)} = d_g^{(i)} + \varepsilon_g d \tag{63}$$

ε_a and ε_g are two small parameters introduced for the d_a diameter and, respectively, d_g. Between ε_a and ε_g we can obtain a connection taking into account the existence of the relation

$$C_a^{(i+1)} C_g^{(i+1)} = 1 \tag{64}$$

where $C_a^{(i+1)}$ and $C_g^{(i+1)}$ are iteration of i+1 order of the C_a and, respectively, of C_g, which signifies that, by replacing it in the ratio

$$C_a^{(i+1)} = \frac{d_g^{(i+1)}}{d_a^{(i+1)}} = \frac{d_g^{(i)} - \varepsilon_g \cdot d}{d_a^{(i)} - \varepsilon_a \cdot d} = \frac{C_a^{(i)} - \varepsilon_g \cdot \sqrt{1+\left(C_a^{(i)}\right)^2}}{C_g^{(i)} - \varepsilon_a \cdot \sqrt{1+\left(C_g^{(i)}\right)^2}} \cdot \frac{\sqrt{1+\left(C_g^{(i)}\right)^2}}{\sqrt{1+\left(C_a^{(i)}\right)^2}} \tag{65}$$

and in a similar manner in

$$C_g^{(i+1)} = \frac{d_a^{(i+1)}}{d_g^{(i+1)}} = \frac{d_a^{(i)} + \varepsilon_a \cdot d}{d_g^{(i)} - \varepsilon_g \cdot d} = \left(C_a^{(i+1)}\right)^{-1}, \tag{66}$$

which allow to obtain

$$\varepsilon_a^{(i)} = \frac{1}{4\sqrt{1+\left(C_a^{(i)}\right)^2}} \cdot \left[1-2\left(C_g^{(i)}\right)\right]; \; \varepsilon_g^{(i)} = \frac{1}{4\sqrt{1+\left(C_g^{(i)}\right)^2}} \cdot \left(2C_a^{(i)} - 1\right) \tag{67}$$

When reaching these relations we took into account

$$\left(d_a^{(i)}\right)^2 + \left(d_g^{(i)}\right)^2 = d^2 . \tag{68}$$

The interrelation between ε_a and ε_g is given by the relations (67). We can obtain a similar expression starting from the obvious equality

$$\left(d_a^{(i)} + \varepsilon_a \cdot d\right)^2 + \left(d_g^{(i)} - \varepsilon_g \cdot d\right)^2 = d$$

This results in the physical sense solution for

$$\varepsilon_g = \frac{d_g^{(i)}}{d} - \sqrt{\left(\frac{d_g^{(i)}}{d}\right)^2 - \varepsilon_a\left(2\frac{d_a}{d} + \varepsilon_a\right)} \tag{69}$$

In the case in which $\varepsilon_a\left(2\frac{d_a}{d} + \varepsilon_a\right)$ present reduce values, for a reduce x argument, the approximate equality $\sqrt{1+x} \approx 1 + \frac{x}{2}$, than ε_g given by the last formula is written in the following way

$$\varepsilon_g = \frac{\varepsilon_a}{2} \cdot \frac{d_g^{(i)}}{d}\left(2\frac{d_a^{(i)}}{d} + \varepsilon_a\right) \tag{70}$$

The SYLVY Calculation Programme

On the basis of the theories presented above, in order to check the method we proposed, the Sylvy calculation programme has been elaborated and with its help we could calculate the pressure gradients for gases, respectively water phases, for different values of the gases flows and their humidity, for a 5 pipe.

For the calculation programme, we considered ε_a = 0.0000015, which means a very high precision of the calculations, because, at a diameter of d = 0.125 m the result was d_a = 0.11919 and d_g = 0.03766 m and the ε_g value for the last iteration was $\varepsilon_g = 4{,}3091 \cdot 10^{-7}$.

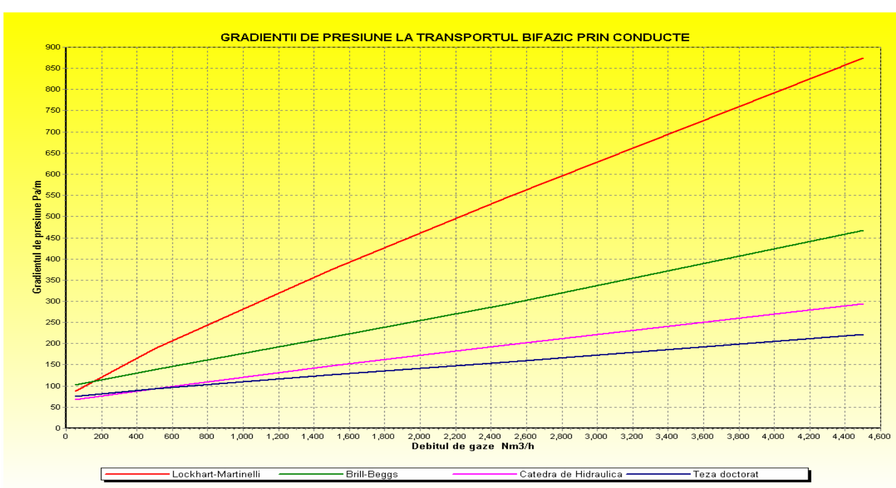
Conclusions

The main reason that leads us towards the conclusion that the method we propose is correct is the fact that the pressure gradients for the two phases have approximately the same values, which corresponds to the reality of the biphasic transportation pipes.

In order to have a proper comparison of the three studying methods, Lockhart-Martinelli's, Brill-Beggs's and the method belonging to the Thermo technical Hydraulics and Deposits Engineering Departments from UPG Ploiesti, the calculation program offers, in the same conditions, the values of the pressure gradients calculated on the base of corresponding algorithms for the respective methods.

The calculation program has been tried on a 5 pipe that transports biphasic mixture with a flow of gases of 50 m^3_N/h, 500 m^3_N/h, 1500 m^3_N/h, 2500 m^3_N/h and 5000 m^3_N/h, with different humidity. Synthetically, the results are presented in the following chart and diagram:

Calculation method	S. Stan	Hydraulic Department	Lockhart-Martinelli	Bell-Brigs
Specific transportation conditions	$\left(\frac{\Delta p}{l}\right)_a / \left(\frac{\Delta p}{l}\right)_g$	$\left(\frac{\Delta p}{l}\right)_b$	$\left(\frac{\Delta p}{l}\right)_b$	$\left(\frac{\Delta p}{l}\right)_b$
Q_{gN}= 50 m^3/h a = 0,75467	76.71/75.92	67.77	87.85	103.17
Q_{gN}= 500 m^3/h a = 0,07547	94.96/94.16	93.79	187.72	138.46
Q_{gN}= 1.500 m^3/h a = 0,02516	127.21/126.41	147.25	375.03	216.58
Q_{gN}= 2.500 m^3/h a = 0,01509	157.96/157.16	197.59	547.99	294.41
Q_{gN}= 5.000 m^3/h a = 0,00839	222.44/221.64	294.49	875.10	467.57



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Metodă nouă de calcul pentru curgerea bifazică gaze-apă prin conducte

Rezumat

În general, la transportul gazelor naturale prin conducte apar şi cantități mari de apă, ceea ce conferă acestui amestec un caracter bifazic. Există mai multe teorii privind transportul bifazic, dar, acestea au anumite limite sau sunt aplicabile utilizând coeficienți empirici ai căror valabilitate este incertă. Metoda propusă în lucrare, deşi utilizează o idee a lui Baker, este exactă şi nu necesită utilizarea unor coeficienți numerici de natură experimentală. În cazul metodei propuse sunt utilizați coeficienți de rezistență hidraulică pentru curgerea monofazică prin conducte indiferent de caracterul mişcării (laminar, turbulent) sau natura hidraulică a conductei (netedă, seminetedă sau complet rugoasă).