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Properties of the Measure of a Physical Size

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Abstract

This paper is a sequel of the work [1] where the notions of "physical size" and its "measure" are cleared up, and the use of the term "measure" in metrology, not only to appoint the measure standard, but also for the quantitative expression of a physical size, obtained experimentally or by calculation, is proposed. In the present work, as a result, the properties of the measure of the physical sizes on the basis of its general expression are determined. These properties will be used in the framework of future papers.

Key words: *physical size, measure, properties of a measure.*

Introduction

The present paper is a sequel of ideas regarding the physical sizes and their measures, expressed in the work [1] by the author (see also [2]).

The physical sizes represent the measurable common properties of the different classes of objects or processes, occurring in nature or created by man.

The properties of the objects and processes are phenomena [1].

Phenomena are external manifestations of objects and processes that are the manifestations of their essence [1]. They may be empirically find out and experimentally emphasized.

Therefore, the properties of the objects and processes can be experimentally determined by measuring.

The result of the measuring operation of a physical size in certain conditions and accuracy is the measure of the respective physical size at the measuring moment [1].

In the mentioned paper [1], the use of the term "measure" in metrology,both to appoint the result of the measuring action/operation, the materialized conventional unit (the measure standard), indication (of a measuring means) – see [3] and [4] –, and also for the quantitative expression of a physical size, generally, determined even by calculation as a product of a number (called numerical value of the measure) and the measure unit, is suggested. This is argued by the existence of these metrological meanings of the term "measure" in world-wide languages, as English, French and German. The last above defined meaning of the term "measure" may be adopted by extension even if it does not appear well-defined in the analysed languages [1].

In this way, as it was shown in [1], the measure of a physical size M , noted with $\mu(M)$, determined either by measuring or by calculation, represents the product of the measure numerical value $w(M)$, and the considered unit of measure $[M]$, in accordance with the relation:

$$\mu(M) = w(M) \cdot [M], \tag{1}$$

where μ is *the operator of measure*, and $w(M) \in \mathbf{R}$.
If $w(M) = 1$, then

$$\mu(M) = [M] \equiv \mu_1(M), \tag{2}$$

where $\mu_1(M)$ is *the unitary measure/the unit of measure of the physical size M*, and μ_1 is *the unitary measure operator.*

The Properties of the Measure

Further, the measure properties of a physical size on the basis of the general measure expression having the shape (1) are determined (see also [2]).

● *The relation of biunique correspondence between the multitude of the measures, having the same unit of measure, of a physical size, and the multitude of the real numbers.* Let be $\mathbf{M}_{[M]}$ the multitude of the measures $\mu(M)$ of a physical size M, with the same unit of measure $[M]$, and $\mathbf{R}$ the multitude of the real numbers. As $w(M) \in \mathbf{R}$, to any real number will correspond a measure of a physical size M, and to any measure having the same unit of measure of the physical size M, will correspond a real number.

Observation 1. Generally, the sign of the measure of a physical size is depending on the scale chosen for the measure of that size, on the considered reference point for measure zero, respectively, that is it depends on the reference physical size, or it results in accordance with the definition of the physical size. In this way, it is known that temperature has positive and negative measures on the Celsius scale (therefore, the measure Celsius, or the relative measure of the temperature has positive or negative values), but the Kelvin measure or the thermodynamic/absolute measure of the temperature has only positive values. Also, the altitude has both positive and negative measures, if it is taken into in consideration a scale where the measure zero will correspond to sea level. Instead, the elongation represents a size which may have only positive measures, while the contraction has measures whose numerical values are only negative if both of the sizes are defined by means of the following relation:

$$\Delta l = l - l_0, \tag{3}$$

where l_0 is the initial length and l the final length. A strain gauge connected to a strain-measuring instrument indicates positive values of specific deformation in the case where the part on which it is applied will support on elongation and negative values if the respective part contracts. Also, the acceleration, according to its definition, is a size which may have both positive measures (in accelerated motion) and negative measures (in slowing motion).

● *The theorem of order relation of the measures of a physical size.* Two measures of a physical size are equal if and only if they have the same unit of measure and their numerical values are equal:

$$\left\{\mu^{(1)}(M) = \mu^{(2)}(M)\right\} \Leftrightarrow \left\{[M]^{(1)} = [M]^{(2)} \text{ and } w^{(1)}(M) = w^{(2)}(M)\right\}, \tag{4}$$

and they are unequal if and only if, being expressed dependent on the same unit of measure, their numerical values are unequal in the same sense:

$$\left\{\mu^{(1)}(M) < \mu^{(2)}(M)\right\} \Leftrightarrow \left\{w^{(1)}(M) < w^{2}(M) \mid [M]^{(1)} = [M]^{(2)}\right\} \tag{5}$$

and

$$\left\{\mu^{(1)}(M) > \mu^{(2)}(M)\right\} \Leftrightarrow \left\{w^{(1)}(M) > w^{2}(M) \mid [M]^{(1)} = [M]^{(2)}\right\} \tag{6}$$

or if, having the same numerical value, the units of measure are unequal in the same sense as the measures themselves:

$$\left\{\mu^{(1)}(M) < \mu^{(2)}(M)\right\} \Leftrightarrow \left\{[M]^{(1)} < [M]^{(2)} \mid w^{(1)}(M) = w^{(2)}(M)\right\} \tag{7}$$

and

$$\left\{\mu^{(1)}(M) > \mu^{(2)}(M)\right\} \Leftrightarrow \left\{[M]^{(1)} > [M]^{(2)} \;\middle|\; w^{(1)}(M) = w^{(2)}(M)\right\}. \tag{8}$$

● *The theorem of completeness.* Any row of intervals of the numerical values of a measure (of a physical size M) $\left(w_i, w_i'\right)$, with $\left(w_{i+1}, w_{i+1}'\right) \subset \left(w_i, w_i'\right)$, defines uniquely a measure $\mu(M)$, whose numerical value $w(M) \in \mathbf{R}$ has the property

$$w_i(M) \le w(M) \le w_i'(M),\ \forall i \in \mathbf{N}, \tag{9}$$

$\left(w_i, w_i'\right)$ being the accuracy interval which depends on the measuring instrument (and implicitly on the adopted unit of measure), so that

$$\mu_i(M) \le \mu(M) \le \mu_i'(M),\ \forall i \in \mathbf{N}. \tag{10}$$

It is proved on the basis of property/theorem of completeness of the real number multitude and on the basis of biunique correspondence relation among the measure multitude, with the same unit of measure of a physical size, and the multitude of real numbers.

● *The inverse proportionality between the numerical value of the measure and the unit of measure.* For a certain accuracy of measuring carried out at a given moment, that is, for a certain measure of a physical size, $\mu(M)$, *the numerical value of the measure is inversely proportional to the used measure unit:*

$$w(M) = \frac{\mu(M)}{[M]}. \tag{11}$$

● *The theorem of comparison of two measures of a physical size.* If two unequal measures of a physical size are compared, the smaller the small measure is, the more it divides the great measure. In this way, if $\mu^{(1)}(M)$ and $\mu^{(2)}(M)$ are two unequal measures of M, and if $\mu^{(1)}(M)$ is of λ times smaller than $\mu^{(2)}(M)$, that is

$$\mu^{(1)}(M) = \frac{\mu^{(2)}(M)}{\lambda},\ \lambda \in \mathbf{R}_+, \tag{12}$$

then, by using the expression of measure for $\mu^{(1)}(M)$, for which the unit of measure $[M]$ is taken into consideration, will result:

$$\mu^{(2)}(M) = w^{(1)}(M) \cdot \lambda \cdot [M],$$

that is

$$\frac{\mu^{(2)}(M)}{w^{(1)}} = \lambda \cdot [M]. \tag{13}$$

● *The consequence of the theorem of comparison of two measures of a physical size (of making a unit of measure from another unit).* Any unit of measure may result either by multiplying, or dividing by another unit of measure. This assertion is expressed in the following way: any unit of measure may be a measure with another unit of measure (which will divide the respective measure). In this way, in accordance with the measure definition, the left-hand member in the equality (13) is a unit of measure noted by $[M]^{(2)}$ and the relation (13) becomes

$$[M]^{(2)} = \lambda \cdot [M],\ \lambda \in \mathbf{R}_+. \tag{14}$$

Generally, if $\mu_1 \equiv [M]$ is a (basic) unit of measure for any physical size, then there is $\lambda_i \in \mathbf{R}_+$, $i = \overline{1, n}$, so that, other units of measure for the respective size in the shape

$$\mu_1^{(i)} = \lambda_i \cdot \mu_1 \tag{15}$$

or

$$[M]^{(i)} = \lambda_i \cdot [M] \tag{16}$$

are obtained.

For example, "Near by stood six stone jars, the kind used by the Jews for ceremonial mashing, each holding by two or three measures" (John's Gospel, 2, Wedding in Cana, 6), "a measure having about 39 litres" (Idem). Therefore, a measure represents the capacity of a stone jar (see also [1]), which may be a unit of measure, representing other two or three measures/units of measure, each of them having 39 litres.

Observation 2. Usually, a new unit of measure is chosen, which is multiple or submultiple of another unit of measure (of a type-unit or a basis one). It is again mentioned that the multiple/submultiple of a type-unit of measure represents a unit of measure, which is a whole number times greater/smaller than the type-unit.

Observation 3. The division of the unit of measure, in fact is a method used for measurements, which suggested a construction mode of real numbers (as decimal fractions having an infinity of decimals) [8]. In this way, in order to measure any size as possible more accurately, the units of measure are used in turn

$$\mu_1^{(i)} = \frac{\mu_1^{(1)}}{10^{i-1}},\ i=1,\ 2,\ ..., \tag{17}$$

theoretically represented by an infinite row, in the way that each of the successive measurement provides in turn a new decimal of the numerical value of the respective size measure. Theoretically, the row of these decimals is infinite. In practice, the numerical value of measure is expressed/approximated by a fraction with a finite number of decimals, according to the wanted accuracy; for example, a third from the length of a segment of 16 mm represented by micron accuracy (as a unit of measure) or valuable, in the decimal system, with an accuracy of 10^{-3} is

$$\frac{16 \cdot \text{mm}}{3} = 5\frac{1}{3} \cdot \text{mm} \approx 5{,}333 \cdot \text{mm}.$$

Observation 4. Other examples may be even the definitions of some units of measure, adopted in the course of time (see [5]÷[7] and [9]), as: the meter definition adopted at the 11^{th} General International Conference for Measures and Weights (GICMW) from October 1960, for which as a base standard of length was considered the wave length in vacuum, of 0,605 780 21 μm, of the orange spectral radiation of the Krypton atom 86 emitted by transition between the energy levels $2p_{10}$ and $5d_5$ ("meter is the length equal to 1 650 763,73 length in vacuum of radiation corresponding to transition between the energy levels $2p_{10}$ and $5d_5$ of the Krypton atom 86"); the definitions of second (the initial one: "the fraction 1/86 400 in a medium solar day"; the second, decided by the International Committee of Measures and Weights in 1956 and ratified in 1960 by the 11^{th} GICMW: "the fraction 1/31 556 925,974 7 in the tropical year for 1900 January 0 to 12 hours of the ephemerid time"; the third was decided by 13^{th} GICMW in 1967: "duration of 9 192 631 770 periods of the radiation corresponding to transition between the two hyperfine levels of energy of the fundamental state of the Caesium atom 133").

● *Invariance of measure in case of changing the unit of measure*. The measure of a physical size is invariably in case of changing the unit of measure that is written as

$$w^{(i)}(M) \cdot [M]^{(i)} = const.,\ \ \forall i \in \mathbf{N}, \tag{18}$$

i being the number of order of the measure.
For example,

$$\mu(F) = 1\,281 \cdot \text{kgf} = 12\,566{,}61 \cdot \text{N} = 12{,}56661 \cdot \text{kN}.$$

Demonstration. Let be

$$\mu(M) = w^{(1)}(M) \cdot [M]^{(1)}$$

and let be $[M]^{(2)}$ an other unit of measure for M, so that

$$[M]^{(2)} < [M]^{(1)}.$$

Then, as unequal measures, there is $\lambda > 1$, so that

$$[M]^{(2)} = \frac{[M]^{(1)}}{\lambda}.$$

Because $[M]^{(2)}$ is inversely proportional to $w^{(2)}(M)$, will result that $[M]^{(2)}$ divides λ times more $\mu(M)$ than $[M]^{(1)}$ does, that is

$$w^{(2)}(M) = \lambda \cdot w^{(1)}(M).$$

Therefore

$$w^{(2)}(M) \cdot [M]^{(2)} = \lambda \cdot w^{(1)}(M) \cdot \frac{[M]^{(1)}}{\lambda} = w^{(1)}(M) \cdot [M]^{(1)} = \mu(M).$$

q. e. d.

Demonstration may be simpler made on the basis of the following reasoning: as the numerical value of measure will result by dividing it with the unit of measure, also the unit of measure, as a measure itself, can be divided by another unit of measure, and so on, that is

$$\mu(M) = w^{(1)}(M) \cdot [M]^{(1)} = w^{(1)}(M) \cdot \lambda_1 \cdot [M]^{(2)} = w^{(2)}(M) \cdot [M]^{(2)} = \ldots$$
$$\ldots = w^{(i)}(M) \cdot [M]^{(i)} = w^{(i)}(M) \cdot \lambda_i \cdot [M]^{(i+1)} = w^{(i+1)}(M) \cdot [M]^{(i+1)},$$

where

$$w^{(i+1)}(M) = w^{(i)}(M) \cdot \lambda_i \text{ and } \lambda_i = \frac{[M]^{(i+1)}}{[M]^{(i)}},\ i = \overline{1, n}.$$

● *Consequence 1 (of the invariance of measure in case of changing the unit of measure), of the expression of a measure depending on any unit of measure of the physical size.* Any measure of a physical size determined by a certain unit of measure may be written depending on any other unit of measure of the respective physical size.

Really, if $\mu(M)$ is a measure of the physical size M, determined with any unit of measure $[M]^{(i)}$, $i = \overline{1, n}$, that is

$$\mu(M) = w^{(i)}(M) \cdot [M]^{(i)},$$

then another unit of measure $[M]$ may be chosen, in the way that (according to the property of invariance)

$$[M]^{(i)} = \lambda_i \cdot [M],$$

and the respective measure becomes

$$\mu(M) = \lambda_i \cdot w^{(i)}(M) \cdot [M],\ i = \overline{1, n}. \tag{19}$$

● *Consequence 2 (of the invariance of measure in case of changing the unit of measure), of the proportion of the numerical value ratio and of the inverse one of the units of measure of the same measure.* For one and the same measure, the ratio of the numerical values is inversely proportional to the ratio of the units of measure or the ratio of numerical values and the inverse one of the units of measure build up a proportion:

$$\frac{w^{(i)}}{w^{(1)}} = \frac{\mu_1^{(1)}}{\mu_1^{(i)}}. \tag{20}$$

● *Consequence 3 (of the invariance of measure in case of changing the unit of measure), of increasing the measurement accuracy by decreasing the unit of measure.* By choice of a smaller and smaller unit of measure,

$$\mu_1^{(i)} < \mu_1^{(1)},\ i = \overline{2, n},$$

the numerical values of the measure become greater and greater,

$$w^{(i)} > w^{(1)},\ i = \overline{2,n},$$

according to the relation:

$$w^{(i)} = w^{(1)} \cdot \frac{\mu_1^{(1)}}{\mu_1^{(i)}},$$

which increases the measurement accuracy (see observation 3). For example, if $\mu_1^{(i)},\ i = \overline{2,n}$, is chosen, according to relation (17), then

$$w^{(i)} = 10^{i-1} \cdot w^{(1)},\ i = \overline{2,n}.$$

● *The principle of algebraic summation of the physical sizes.* Only the physical sizes having the same nature/being the same kind are algebraically summated.

● *The property of value additivity.* The algebraic sum measure of the physical sizes having the same nature, for which the same unit of measure is used, is equal to the product of algebraic sum of the numerical values of the respective measures and the adopted unit of measure. Therefore

$$\mu\left(\sum_{i=1}^{n} M_i\right) = \sum_{i=1}^{n} w(M_i) \cdot [M], \tag{21}$$

if $M_i,\ i = \overline{1,n}$, they are physical sizes having the same nature and $[M] = [M_i],\ i = \overline{1,n}$.

Demonstration. As $M_i,\ i = \overline{1,n}$, they are measures of the same nature, they can be separately measured and their measures summated, that is

$$\mu\left(\sum_{i=1}^{n} M_i\right) = \sum_{i=1}^{n} \mu(M_i).$$

But, in accordance with the measure definition:

$$\mu(M_i) = w(M_i) \cdot [M_i],$$

is obtained

$$\sum_{i=1}^{n} \mu(M_i) = \sum_{i=1}^{n} w(M_i) \cdot [M_i].$$

$M_i,\ i = \overline{1,n}$, being sizes of the same nature, the same unit of measure may be used. Let be $[M]$ this unit of measure:

$$[M_1] = [M_2] = \ldots = [M_i] = \ldots = [M_n] = [M].$$

Further, will result

$$\sum_{i=1}^{n} w(M_i) \cdot [M_i] = \sum_{i=1}^{n} w(M_i) \cdot [M].$$

Taking into account the previous relations,

$$\mu\left(\sum_{i=1}^{n} M_i\right) = \sum_{i=1}^{n} \mu(M_i) = \sum_{i=1}^{n} w(M_i) \cdot [M_i] = \sum_{i=1}^{n} w(M_i) \cdot [M]$$

is written.

q. e. d.

Observation 5. The property of the value additivity will express the fact that it makes no sense the application of the unit measure operator to the algebraic sum of the physical sizes having the same nature, for which the same unit of measure is adopted.

Observation 6. Because, according to the measure definition,

$$\mu\left(\sum_{i=1}^{n} M_i\right) = w\left(\sum_{i=1}^{n} M_i\right)\cdot[M]$$

is written, taking into account the equality (21), will result

$$\sum_{i=1}^{n} w(M_i) = w\left(\sum_{i=1}^{n} M_i\right). \tag{22}$$

● *Consequence 1 (of the property regarding the value additivity), in case of variation measure of a physical size.* From point of view of measure, a variation of a physical size takes place only by variation of the numerical value of its measure. This thing is expressed by the following relation

$$\mu(\Delta M) = \Delta w(M)\cdot[M]. \tag{23}$$

But, according to (22), the equality

$$\Delta w(M) = w(\Delta M) \tag{24}$$

is true.

● *Consequence 2 concerning the property of the value additivity in case of infinitesimal measure.* As a result of the consequence 1, *only the numerical value of a measure can tend to zero*; in this way:

$$\{\mu(\Delta M)\to 0\} \Leftrightarrow \{\Delta\mu(M)\to 0\} \Leftrightarrow \{w(\Delta M)\to 0\}, \tag{25}$$

that is the variation measure of a physical size will tend to zero, only if the measure variation of that size tends to zero, the numerical value of that variation measure of the physical size tends to zero, respectively.

Therefore, the affirmation "Variation of a physical size tends to zero" is not correct. Instead, it is correct to say: "The physical size is not changed", "The variation measure of a physical size tends to zero", or "The measure variation of a physical size tends to zero", respectively.

● *The property of the extended value additivity.* (*The algebraic summation of the physical sizes having the same nature but different units of measure*.) Also the physical sizes having the same nature/being the same kind, but different units, are algebraically summated.

Really, if $\mu^{(i)}(M_i)$, $i=\overline{1,n}$, are the measures of those n physical sizes having the same nature, which are determined with different units of measure $[M_i]^{(i)}$, that is

$$\mu^{(i)}(M_i) = w(M_i)\cdot[M]^{(i)},$$

then, passing to the same unit of measure $[M]$ for all the measures, by means of the relation having the shape (19), which becomes

$$\mu^{(i)}(M_i) = \lambda_i \cdot w^{(i)}(M_i)\cdot[M],\ \lambda_i\in \mathbf{R}_+,$$

and by using the property of value additivity, will result:

$$\sum_{i=1}^{n} \mu^{(i)}(M_i) = \left(\sum_{i=1}^{n} \lambda_i \cdot w^{(i)}(M_i)\right)\cdot[M]. \tag{26}$$

Conclusions

In this paper the discussion from [1], regarding the physical size and its measure, is continued. It is reminded that the term of "measure" is used by the author – despite of distinction made in [3] and [4] – both in order to designate the result of measuring action/operation, the materialized conventional unit (measure standard), indication (a measuring instrument) and for the quantitative expression of a physical size, generally, determined even by calculation, as a product of a number (called numerical value of the measure) and the unit of measure.

In this way, in the present work, the properties of measure of a physical size on the basis of its general expression having the shape (1) are determined: the biunique correspondence relation among the multitude of the measures, with the same unit of measure, of a physical size and the multitude of real numbers; the theorem of the relation of order in case of measure of a physical size; the theorem of completeness; the inversely proportionality between the numerical value of measure and the unit of measure; the theorem of comparison of two measures of a physical size and its consequence; the invariance of measure in case of changing of the measure unit and its three consequence; the principle of algebraic summation of the physical sizes; the property of the value additivity and its consequences; the property of extended value additivity.

These properties will be used, in the future in the frame of other works, in order to express the measure of a physical size determined by a relation among other sizes and to emphasize a series of other aspects concerning the physical size and its measure.

References

1. Parepa, S., On the physical size and its measure. *Buletinul Universităţii Petrol-Gaze din Ploieşti. Seria tehnică*. Vol. LIX, No. 2/2007.
2. Parepa, S., *Studiul sistemului de rotire al instalaţiilor pentru forajul sondelor de diametre mari*. Teză de doctorat, Universitatea din Petroşani, 2007.
3. ⁂*SR 13251. Vocabular internaţional de termeni fundamentali şi generali în metrologie*. Institutul Român de standardizare (IRS), Aprilie 1996.
4. ⁂ *SR 13434. Ghid pentru evaluarea şi exprimarea incertitudinii de măsurare*. Ediţia 1. Asociaţia de Standardizare din România (ASRO), Iulie 1999.
5. ⁂ *Sistemul internaţional de unităţi.* Ediţia a III-a în limba română. (Traducere din limba franceză după ediţia a cincea a lucrării elaborate de Biroul Internaţional de Măsuri şi Greutăţi). Editura Academiei Republicii Socialiste România, Bucureşti, 1989.
6. ⁂ *Directiva 80/181/CEE a Comunităţii Economice Europene din 20 decembrie 1979 privind apropierea legislaţiilor statelor membre referitoare la unităţile de măsură şi de abrogare a Directivei 71/354/CEE.*
7. ⁂ *Guvernul României. Hotărâre privind aprobarea unităţilor de măsură legale*. Nr. 755, Bucureşti, 14 mai 2004. Monitorul Oficial al României, Partea I, Nr. 475/27.V.2004.
8. ⁂ *Mică enciclopedie matematică.* Editura tehnică, Bucureşti, 1980.
9. ⁂ *Sistemul internaţional de unităţi.* (Traducere din limba franceză după ediţia a treia a lucrării elaborate de Biroul Internaţional de Măsuri şi Greutăţi). Editura didactică şi pedagogică, Bucureşti, 1979.

Proprietăţile măsurii unei mărimi fizice

Rezumat

Acest articol este continuarea lucrării [1], în care se clarifică noţiunile de „mărime fizică" şi de „măsură" a acesteia şi se propune utilizarea termenului „măsură" în metrologie nu numai pentru a desemna etalonul de măsură, dar şi pentru expresia cantitativă a unei mărimi fizice, în general, determinată experimental sau prin calcul. Ca urmare, în lucrarea de faţă se determină proprietăţile măsurii unei mărimi fizice pe baza expresiei sale generale. Aceste proprietăţi vor fi utilizate în cadrul unor lucrări viitoare.