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The Calculus of the Proper Circular Frequencies for the Rod Pumping Units

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Abstract

In the paper, it is presented a calculus methodology for the proper circular frequencies of a rod pumping unit, using the finite elements method. It is established the minimum number of elements in order to minimize the error between the finite element results and those obtained from the general theory of axial vibrations of beams with continuous mass. The results obtained are analysed in a calculus example.

Key words: *proper circular frequencies, finite element method*

Theoretical Considerations

The fundamental circular frequency is an important characteristics of the structure that allow the selecting of the optimal technological parameters in order to have a goodl function of the entire equipment.

The proper circular frequencies of the free vibrations of a uniform beam, p_1, can be expressed by the relation (1):

$$p_1 = \frac{\pi}{2} \cdot \sqrt{E / \rho \cdot l^2} \tag{1}$$

where: E – is the longitudinal elasticity modulus ; ρ - the density of the material of the beam;.
l- the length of the beam.

The calculus methodology using the finite elements method is overpassing the following steps [3]:

a) the calculation of the first proper circular frequencies meshing the entire structure into only one finite element;
b) the calculation of the first proper circular frequencies meshing the entire structure in two finite elements;
c) the calculation of the first proper circular frequencies meshing the entire structure in three finite elements;
d) establishing of the minimum number of elements that assure a very small percentage of error (less than 0.1%);

By meshing the entire structure in different numbers of elements, the following conclusions have been emphasised.

When the meshing has been made into one finite element (fig. 1), considering the starting point (having zero displacements) embedded, the following equation have been obtained:

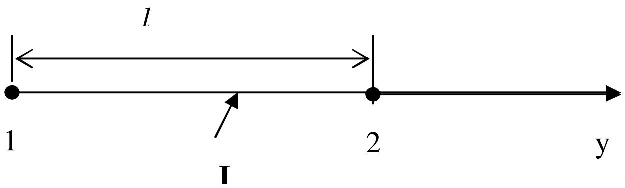

Fig.1 Structure meshed in one finite element

$$\frac{E \cdot A}{l} - \frac{\rho \cdot A \cdot l}{3} \cdot p^2 = 0 \tag{2}$$

Solving the above equation the proper circular frequency has the form :

$$p = 1{,}732\sqrt{E/_{\rho \cdot l^2}} \tag{3}$$

Meshing the entire structure in two finite elements (fig. 2), the following equation is obtained:

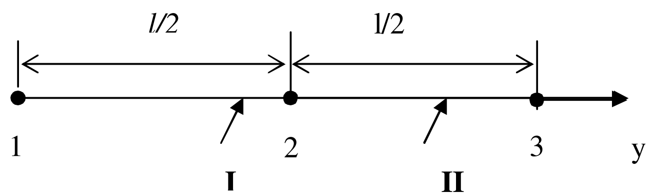

Fig.2 Structure meshed in two finite elements

$$0{,}0486\rho^2 l^2 p^4 - 1{,}67\rho E p^2 + 4\frac{E^2}{l^2} = 0 \tag{4}$$

The above equation has two positive roots, p and p*, that represent the first two circular frequencies of the structure:

$$p = 1{,}61\sqrt{E/_{\rho \cdot l^2}} \qquad \text{and} \qquad p^* = 5.63\sqrt{E/_{\rho \cdot l^2}} \tag{5}$$

By meshing the entire structure in three finite elements (fig. 3), the following equation is obtained:

$$0{,}00446p^6 - 1{,}5278\frac{Ep^4}{\rho \cdot l^2} + 12\frac{E^2 \cdot p^2}{\rho^2 \cdot l^4} - 27\frac{E^3}{\rho^3 \cdot l^6} = 0 \tag{6}$$

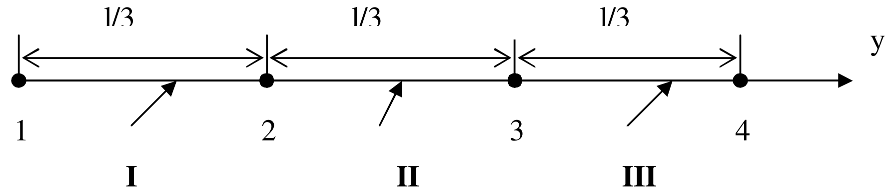

Fig. 3. Structure meshed in three finite elements

The above equation has as solutions the first three proper circular frequencies :

$$p = 1{,}588\sqrt{E/_{\rho \cdot l^2}}\ ; \quad p^* = 5{,}196\sqrt{E/_{\rho \cdot l^2}}\ ; \quad p^{**} = 9.462\sqrt{E/_{\rho \cdot l^2}} \tag{7}$$

The cases analysed above suggest that the finite element method may reach some errors if the number of finite elements is small and the number of proper frequencies obtained is equal with the number of the elements.

This is the reason why, when it is necessary to study a dynamical response (in displacements of stresses) it is necessary to mesh the structure into a big number of elements.

If the calculus error is defined by the relation:

$$Er = \frac{p - p_1}{p_1} \cdot 100[\%] \quad , \tag{8}$$

the following values are obtained:

Table 1

Number of finite elements	**1**	**2**	**3**
***Error* [%]**	10.26	2.495	1.09

From the data presented in table 1 it can be noticed that:

- the dynamical analysis of the structure can reach unacceptable results if the entire structure is meshed only in one finite element;
- if the structure is meshed in two finite elements the errors are less than 2.5% and become acceptable.

The theoretical study has been continued with the dynamical analysis of the structure using an specialised programme that allows such type of analysis. The structure has been meshed in up to twenty finite elements and the errors obtained in every case are presented in table 2. The beam has the following geometrical and material data:

$$E = 2{,}1 \cdot 10^{11}\, N/m^2 ; \rho = 7850 kg/m^3 ; l = 2100 m .$$

For this case the first circular frequency can be expressed by the following relation:

$$p_1 = \frac{\pi}{2} \cdot \sqrt{E/_{\rho \cdot l^2}} \Rightarrow p_1 = 3{,}868 rad/s$$

Table 2

Number of finite elements	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**20**
Error [%]	10.03	2.53	1.24	0.62	0.465	0.258	0.207	0.155	0.103	0.077	0

Analysing the data presented in table 2 it can be observed that if the number of finite elements is higher than 10, the error is less than 0.1%. When the structure is meshed in twenty finite elements the results obtained using the finite elements method is the same with the theoretical one (error is zero). Using the same method a dynamic analysis for a pumping equipment has been performed. The analysed rod line has the entire length of 900 m and is build by two segments with the lengths l_1 and l_2 and have the external diameters: 7/8 in, respective 3/4 in.

The results obtained are presented in table 3.

Table 3

Nr. crt.	l_1[m]	l_2[m]	p[s^{-1}]
1	100	800	9.28
2	200	700	9.54
3	300	600	9.72
4	400	500	9.84
5	450	450	9.86
6	500	400	9.84
7	600	300	9.72
8	700	200	9.54
9	800	100	9.28

It can be noticed that the circular frequencies remain inside the domain [9.28 – 9.84] rad/s. It can be concluded that the relation (1) reaches a 9.25% maximum error.

A similar study has been made for a structure with three different cross sectional area and lengths: 504 m, 567 m, 1029 m and the diameters respectively 1 in, 7/8 in, 3/4 in.

The same type of finite elements have been used and every part of the structure has been meshed in twenty finite elements.

The values of the first ten axial circular frequencies are presented in table 4.

Table 4

The proper mode	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Circular frequencies [s^{-1}]	0.7079	1.83	3.098	4.204	5.607	6.73	7.99	9.05	10.42	11.52

The circular frequency calculated with the relation (1) is $p_1 = 3.868 s^{-1}$ and using the finite elements method it is $p = 2 \cdot \pi \cdot n = 2 \cdot \pi \cdot 0.7079 = 4.447 s^{-1}$. It can be noticed that the error reach the maximum value of 15%.

Conclusions

From the analysis presented above it can be noticed that :

- the proper frequency of the combined structure is about 15% away from the proper frequency of the structure with a single cross sectional area; ignoring this aspect can contribute to a mall-function of the entire structure;
- it has been established that the minimum number of finite elements in which a beam structure have to be meshed in order to obtain a minimum error is 10 elements;
- it may be observed that the finite element method allows the calculation of a number of proper frequencies that is equal with the number of finite elements in which the structure has been meshed. This is the reason why, when a dynamical response is required, the structure has to be meshed in a number of finite elements that is higher than the number of necessary proper modes.

The above methodology has the advantage that it is very simple and it can be easily programmed in order to obtain some specialized results.

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Calculul frecvenţelor proprii ale unităţilor de pompare cu balansier

Rezumat

În lucrare se prezintă o metodologie de determinare a frecvenţelor proprii ale unităţilor de pompare cu balansier. Este utilizata metoda elementelor finite şi este stabilit numărul minim de elemente necesar pentru a nu exista erori intre rezultatele numerice şi cele rezultate din teoria vibraţiilor barelor cu masă continuă. Rezultatele obţinute sunt analizate pe exemple de calcul.