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The Torsion of Thin Wall U-Profile Section Poles

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Abstract

The paper presents a method to calculate displacement and stresses that appear in thin wall U-profile sections poles. Displacements of the section is considered to be restrained, due to links and torsion momentum variation along the pole, which produces both torsion and bending alongside the pole. The results obtained using this method differs from those using the elementary torsion stress theory that considers section movement to be free. When projecting this pole types the stress verification must be done using this calculation method.

Key words: *torsion, bending-torsion moment, bending-torsion bimoment, stress.*

Sectorial Characteristics for the U-Profile Section

The center of gravity O and the center of bending-torsion C are both found on the symmetry axis Oz (fig. 1, a).

We choose as the arbitrary pole the P point obtained from the intersection of the Oz axis and the median line of the profile heart, and as the origin radius PO (fig. 1, a).

In regard to the P pole, the gravity center O is situated at the following distance:

$$e = PO = \frac{\left(s_1 + \frac{t_2}{2}\right)^2 \cdot t_1}{2 \cdot \left(s_1 + \frac{t_2}{2}\right) \cdot t_1 + \left(s_2 - t_1\right) \cdot t_2} \tag{1}$$

The sectorial coordinates chart

$$\omega_P = \int_0^s h_P \cdot ds$$

is given in fig. $1,b$. On the B_1B_2 wing, it is positive because it is obtained by rotating the origin axis Pz in the positive way from the Ox axis (anti-clockwise for an observer watching the graph) while on the B_3B_4 wing, it is negative.

The value on the Oy axis in B_1 is double the area of the PB_1B_2 triangle,

$$\omega_P(B_1) = \frac{1}{2} \cdot s_1 \cdot s_2 \tag{2}$$

The y coordinates of each point on the middle contour of the section are represented as the diagram in fig. $1,c$.

Using Veresceaghin's rule, we calculate the integral:

$$\int_{(A)} y \cdot \omega_P \cdot dA = -\frac{1}{4} \cdot s_1^2 \cdot s_2^2 \cdot t_1 \tag{3}$$

The moment of inertia reported to the Oz axis:

$$I_z = \frac{1}{12} \cdot \left[\left(s_1 + \frac{t_2}{2} \right) \cdot h^3 - \left(s_1 - \frac{t_2}{2} \right) (s_2 - t_1)^3 \right]. \tag{4}$$

The position of C on the Oz axis is determined from the paper [2],

$$a_z = PC = -\frac{\int_{(A)} y \cdot \omega_P \cdot dA}{I_z} = \frac{s_1^2 \cdot s_2^2 \cdot t_1}{4 \cdot I_z} \tag{5}$$

The chart $\omega = \omega_C$ with the pole in C is represented in fig. $1,d$.

The values of the sectorial coordinates in report to the C pole are:

$$\omega(B_1) = \frac{1}{2} \cdot s_2 \cdot (s_1 - a_z) \; ; \; \omega(B_2) = -\frac{1}{2} \cdot s_2 \cdot a_z \tag{6}$$

The static sectorial moment S_ω is calculated, according to [2], with the formula

$$S_\omega = \int_0^s \omega \cdot t \cdot ds, \tag{7}$$

where s is measured on the middle contour from the B_4 end of the lower leg to the current point; the S_ω chart is drawn in fig. $1,e$.

On the lower and upper leg S_ω varies on a parabola with the maximum value in E, and F respectively, so that $B_3F = B_2E = a_z$,

$$S_{\omega\max} = S_\omega(F) = S_\omega(E) = -\frac{s_2 \cdot (s_1 - a_z)^2 \cdot t_1}{4}. \tag{8}$$

The static sectorial moments in B_2 and B_3 are equal:

$$S_\omega(B_2) = S_\omega(B_3) = S_\omega(F) + \frac{1}{4} \cdot a_z^2 \cdot s_2 \cdot t_1 \ . \tag{9}$$

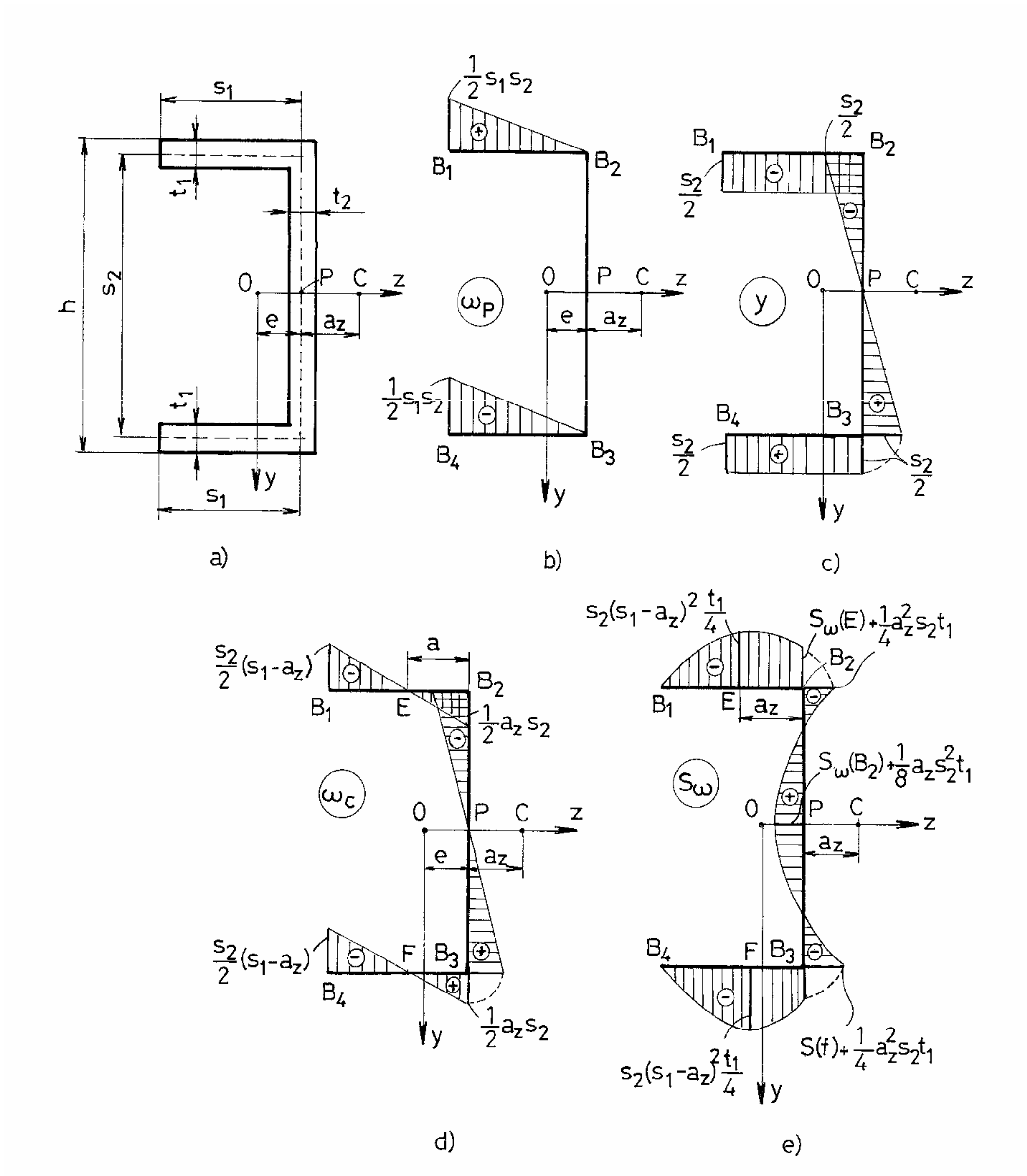

Fig. 1

On the heart S_ω it varies according to a parabola with the maximum value in P,

$$S_{\omega \max} = S_\omega(P) = S_\omega(B_2) + \frac{1}{8} \cdot a_z \cdot s_2^2 \cdot t_2 \ . \tag{10}$$

The sectorial moment of inertia is obtained, according to [2],

$$I_{\omega} = \int_{(A)} \omega^2 \cdot dA \tag{11}$$

and is solved using Verescеaghin's rule.

The torsion moment of inertia with the free section displacement:

$$I_t = \frac{1}{3} \cdot \left[\left(s_1 + \frac{t_2}{2} \right) \cdot t_1^3 + (s_2 - t_1) \cdot t_2^3 \right] \tag{12}$$

Application

Considering that the AB beam (fig. $2, a$) – simply supported for bending at the ends A and B and encased for torsion at the same ends – is pressed in $C(x = a)$ with a moment M_C, on the Ox axis (fig. $2, a$); the particular solution is given by [1],

$$\overline{\varphi} = -\frac{M_C}{k \cdot G \cdot I_t} [k \cdot (x - a) - shk(x - a)]_{x>a} \tag{13}$$

The limit conditions are:

$$\varphi_0 = 0 \,;\; B_{\omega,0} = 0 \text{ for } x = 0$$

and

$$\varphi(2a) \,;\; B(2a) = 0 \text{ for } x = 2a \,.$$

Considering the momentum equations in respect to the Ox axis and the anti-symmetrical nature of the moment of torsion we get $M_{x,0} = -\frac{M_C}{2}$.

With these values, the expression of the φ section rotation, of x distance from the Cx axis, as given by [1], is

$$\varphi = \varphi_0' \frac{shkx}{k} + \frac{M_C}{2 \cdot k \cdot G \cdot I_t} \cdot (k \cdot x - shkx) - \frac{M_C}{k \cdot G \cdot I_t} \cdot [k \cdot (x - a) - shk(x - a)]_{x>a} \tag{14}$$

From the limit condition, for $x = 2a$ in B,

$$\varphi(2a) = \varphi_0' \frac{sh2ka}{k} - \frac{M_C \cdot sh2ka}{2 \cdot k \cdot G \cdot I_t} + \frac{M_C}{k \cdot G \cdot I_t} \cdot shka = 0 \tag{15}$$

and therefore:

$$\varphi_0' = \frac{M_C}{2 \cdot G \cdot I_t} \cdot \left[1 - 2 \cdot \frac{shka}{sh2ka} \right] \tag{16}$$

Using this value in (14) we obtain the expression of the torsion rotation φ, (fig. $2, c$):

$$\varphi=\frac{M_C}{2\cdot k\cdot G\cdot I_t}\cdot\left\{kx-2\cdot\frac{shka}{sh2ka}\cdot shkx-2\cdot\left[k\cdot(x-a)-shk(x-a)\right]_{x>a}\right\} \quad (17)$$

and the derivates φ' (fig. $2,d$),

$$\varphi'=\frac{M_C}{2\cdot G\cdot I_t}\cdot\left\{1-2\cdot\frac{shka}{sh2ka}\cdot chkx-2\cdot\left[1-chk(x-a)\right]_{x>a}\right\}; \quad (18)$$

$$\varphi''=\frac{M_C\cdot k}{G\cdot I_t}\cdot\left[-\frac{shka}{sh2ka}\cdot shkx+shk(x-a)\Big|_{x>a}\right]; \quad (19)$$

$$\varphi'''=\frac{M_C\cdot k^2}{G\cdot I_t}\cdot\left[-\frac{shka}{sh2ka}\cdot chkx+chk(x-a)\Big|_{x>a}\right]; \quad (20)$$

With the expressions in [1] we determine:

- the bending-torsion bimomentum

$$B_\omega=-E\cdot I_\omega\cdot\varphi''=\frac{M_C}{k}\cdot\left[\frac{shka}{sh2ka}\cdot shkx-shk(x-a)\Big|_{x>a}\right]; \quad (21)$$

  the chart is illustrated in fig. $2,c$;
- the bending-torsion momentum

$$M_\omega=E\cdot I_\omega\cdot\varphi'''=M_C\cdot\left[-\frac{shka}{sh2ka}\cdot chkx+chk(x-a)\Big|_{x>a}\right]; \quad (22)$$

  as shown in fig. $2,f$;
- the moment of torsion corresponding to the free movements

$$M_t=M_x-M_\omega=-G\cdot I_t\cdot\varphi'=M_C\left\{\frac{shka}{sh2ka}\cdot chkx-0{,}5+\left[1-chk(x-a)\right]_{x>a}\right\}; \quad (23)$$

  the chart is in fig. $2,g$.

The chart values are calculated for the section in fig. $2,b$, which has:

$$I_z=87.4426\cdot10^6 mm^4;\quad a_z=\frac{s_1^2\cdot s_2^2\cdot t_1}{4\cdot I_z}=44.5mm;$$

$$e=PO=31.9mm;\quad I_t=23.824\cdot10^4 mm^4;$$

$$I_\omega=137.884\cdot10^9 mm^6;$$

$$\frac{G}{E}=\frac{1}{2\cdot(1+\mu)}=0.385;$$

$$k=\sqrt{\frac{G\cdot I_t}{E\cdot I_\omega}}=0.81561\cdot10^{-3}\frac{1}{mm};$$

$$a=1.5m=1.5\cdot10^3 mm;\quad k\cdot a=1.2234;\quad 2\cdot k\cdot a=2.4468$$

The stresses are calculated from [1], with the formulas:

$$\sigma_\omega = \frac{B_\omega}{I_\omega} \cdot \omega \; ; \; \tau_\omega = \frac{M_\omega \cdot S_\omega}{t \cdot I_\omega} \cdot \omega \; ; \; \tau_t = \frac{M_t}{I_t} \cdot t \tag{24}$$

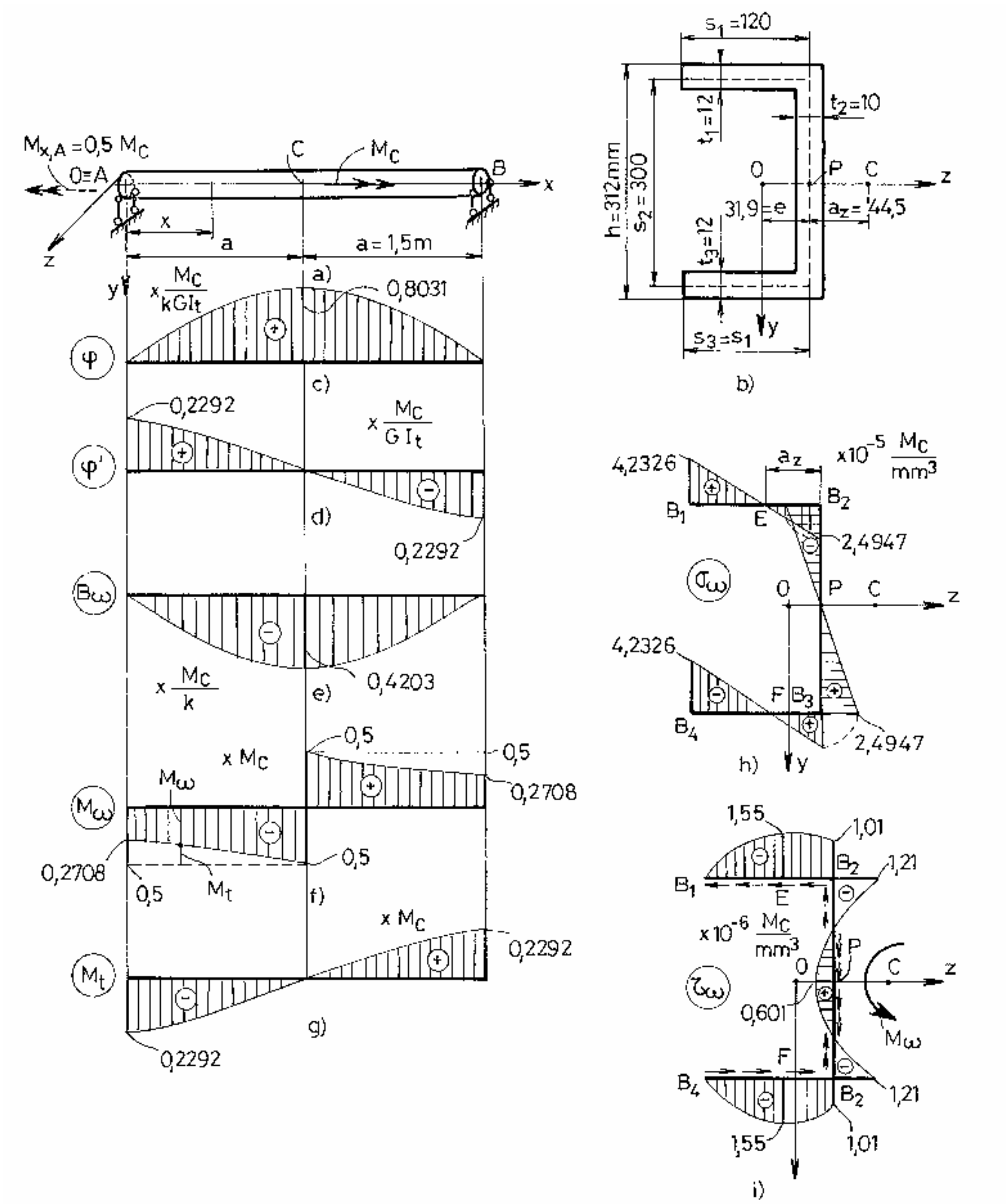

Fig. 2

The chart for the normal stresses σ_ω is given in fig. $2, h$.

The bending-torsion bimomentum has the maximum value in the $C(x = a)$ section,

$$B_\omega(C) = 0.4203 \frac{M_C}{k}$$

and considering

$$\omega(B_2) = -66.75 \cdot 10^2 mm^2 ,\ \omega(B_1) = 113.25 \cdot 10^2 mm^2 ,$$

we get:

$$\sigma_\omega(B_1) = 4.232 \cdot 10^{-5} \frac{M_C}{mm^3} ;\ \sigma_\omega(B_2) = -2.4947 \cdot 10^{-5} \frac{M_C}{mm^3} .$$

$$\sigma_\omega(B_4) = -\sigma_\omega(B_1) \ ;\ \sigma_\omega(B_3) = -\sigma_\omega(B_2) .$$

The tangential stress chart τ_ω is shown in fig. $2, i$.

In section C -right, $C(a+\varepsilon)$, $M_\omega = 0.5 \cdot M_C$ and while :

$$S_\omega(E) = S_\omega(F) = -513 \cdot 10^4 mm^4 ;$$

$$S_\omega(B_2) = S_\omega(B_3) = -334.78 \cdot 10^4 mm^4 ;$$

$$S_\omega(P) = S_\omega(B_3) + \frac{1}{8} \cdot a_z \cdot s_2^2 \cdot t_2 = 165.845 \cdot 10^4 mm^4 ,$$

the values for τ_ω are:

$$\tau_\omega(E) = \tau_\omega(F) = -1.55 \cdot 10^{-6} \frac{M_C}{mm^3} ;$$

$$\tau_\omega(B_2 - talpă) = -1.01 \cdot 10^{-6} \frac{M_C}{mm^3} ;$$

$$\tau_\omega(B_2 - inimă) = -1.21 \cdot 10^{-6} \frac{M_C}{mm^3} ;$$

$$\tau_\omega(P) = 0.601 \cdot 10^{-6} \frac{M_C}{mm^3} .$$

From the σ_ω (fig. $2, h$) and τ_ω (fig. $2, i$) charts, in the B_1 point of the $C(a+\varepsilon)$ section,

$$\sigma_\omega(B_1) = 4.2326 \cdot 10^{-5} \frac{M_C}{mm^3}$$

and

$$\tau_\omega(B_1) = 0 ,\ \tau_t = 0$$

The resistance condition, in accordance with T_τ ,

$$\sigma_{ech} = \sigma_\omega(B_1) = 4.2326 \cdot 10^{-5} \frac{M_C}{mm^3} \le \sigma_a = 150 \frac{N}{mm^2} .$$

Therefore: $M_{C\max} = 35.44 \cdot 10^5 N \cdot mm = 3.544 kN \cdot m$.

In all the other beam points the resistance condition is met.

So, in E :

$$\sigma_\omega(E) = 0\,;$$

$$\tau_\omega(E) = \tau_{max} = 1.55 \cdot 10^{-6} \frac{M_C}{mm^3} = 1.55 \cdot 10^{-6} \cdot 3.544 \cdot 10^6 = 5.5 \frac{N}{mm^2}\,;$$

$$\sigma_{ech} = 2 \cdot \tau_{\omega max} = 11 \frac{N}{mm^2} < \sigma_a = 150 \frac{N}{mm^2}\,.$$

Conclusions

The paper presents a method to calculate displacement and stresses that appear in thin wall U-profile sections poles.

Displacements of the section are considered to be blocked, due to links and torsion momentum variation along the pole, which produces both torsion and bending alongside the pole. Therefore, normal and tangential stresses appear in this type of poles. For thin sections, this stresses become very high thus breaking the poles. So a resistance check is needed, using the above-presented method.

References

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Torsiunea barelor drepte cu pereţi subţiri, având secţiunea în formă de profil U

Rezumat

În lucrare este prezentată o metodă exactă pentru determinarea stării de tensiuni în secţiunea unei bare cu pereţi subţiri de forma unui profil U, considerând deplanările secţiunilor împiedicate, supusă la torsiune. Ca urmare a modului cum sunt considerate deplanările, în secţiunea barei apar şi tensiuni normale, pe lângă cele tangenţiale, astfel încât pentru verificare este nevoie să se aplice o teorie de rezistenţă.