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A Method of Calculation of the Axial Beam Reaction

Şerban Vasilescu

Universitatea Petrol – Gaze din Ploieşti, Bd. Bucureşti 39, Ploieşti
e-mail: vserban @mail.upg-ploiesti.ro

Abstract

In the paper it is presented a methodology that allows the calculation of the axial reaction of the beams transversally loaded.
The solution is put under the form of origin parameters and may be applied to any type of external loads and limit conditions. The results obtained are analysed in a calculus example.

Key words: *axial reaction, loads, cantilever.*

General Equations

A straight beam subjected to bending is considered. The origin parameters are: v_o; φ_o; M_o; T_o (fig. 1).

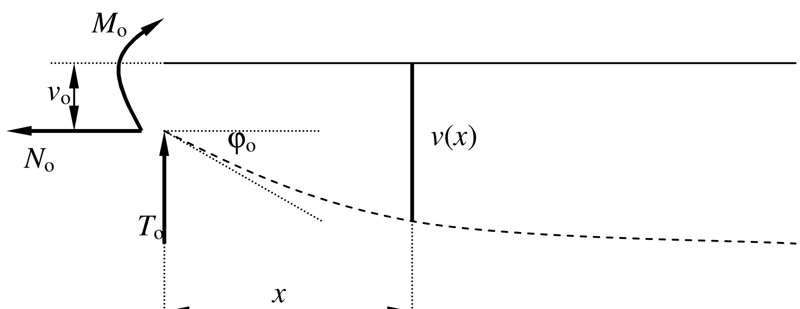


Fig. 1. The origin parameters of the beam

The differential equation of the elastic curve of the beam is [2] :

$$\frac{d^2 v}{dx^2} = -\frac{M(x)}{EI} \tag{1}$$

where $M(x)$ is the bending moment in the current section and the product EI represents the bending rigidity of the beam.

The bending moment can be expressed by reducing from the origin :

$$M(x) = M_o + T_o x - N_o(v - v_o) \tag{2}$$

Replacing (2) into (1), it results :

$$\frac{\mathrm{d}^2 v}{\mathrm{d}x^2} - \frac{N_o}{EI} v = -\frac{1}{EI}(M_o + T_o x + N_o \cdot v_o) \tag{3}$$

If the following notation is accepted :

$$k^2 = \frac{N_o}{EI} \tag{4}$$

the solution of the differential equation (3) can be put under the form :

$$v(x) = C_1 \cdot e^{kx} + C_2 \cdot e^{-kx} + \frac{M_o + T_o \cdot x}{N_o} + v_o \tag{5}$$

The C_1 and C_2 constants can be calculated from the limit conditions :

$$x = 0 \Rightarrow v = v_o, \varphi = \varphi_o \tag{6}$$

Using the limit conditions (6) the constants C_1 and C_2 can be written under the form :

$$\begin{aligned} C_1 &= \frac{1}{2}\left(\frac{\varphi_o}{k} - \frac{T_o}{kN_o} - \frac{M_o}{N_o}\right) \\ C_2 &= -\frac{1}{2}\left(\frac{\varphi_o}{k} - \frac{T_o}{kN_o} + \frac{M_o}{N_o}\right) \end{aligned} \tag{7}$$

Replacing (7) in (5) it results the expression of the deflection of the beam :

$$v(x) = v_o + \frac{\varphi_o}{2k}\left(e^{kx} - e^{-kx}\right) - \frac{M_o}{2N_o}\left(e^{kx} + e^{-kx} - 2\right) - \frac{T_o}{2kN_o}\left(e^{kx} - e^{-kx} - 2kx\right) + \bar{v} \tag{8}$$

The particular solution $\bar{v}$ is a function of the external loads of the beam and can be expressed using a translation of the starting point in the corresponding terms of the (8) relation.

Taking into consideration the differential relations between shear force and bending moment and the external loads [3], it can be calculated the expressions of the slope, bending moment and shear force :

$$\varphi(x) = \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{\varphi_o}{2}\left(e^{kx} + e^{-kx}\right) - \frac{M_o k}{2N_o}\left(e^{kx} - e^{-kx}\right) - \frac{T_o}{2N_o}\left(e^{kx} + e^{-kx} - 2\right) + \bar{\varphi} \tag{9}$$

$$M(x) = -EI\frac{\mathrm{d}\varphi}{\mathrm{d}x} = -\frac{\varphi_o N_o}{2k}\left(e^{kx} - e^{-kx}\right) + \frac{M_o}{2}\left(e^{kx} + e^{-kx}\right) + \frac{T_o}{2k}\left(e^{kx} - e^{-kx}\right) + \overline{M} \quad (10)$$

$$T(x) = \frac{\mathrm{d}M}{\mathrm{d}x} = -\frac{N_o \varphi_o}{2}\left(e^{kx} + e^{-kx}\right) + \frac{M_o k}{2}\left(e^{kx} - e^{-kx}\right) + \frac{T_o}{2}\left(e^{kx} + e^{-kx}\right) + \overline{T} \quad (11)$$

In the mentioned relations the $\overline{\varphi}, \overline{M}, \overline{T}$ terms represent the particular solutions of the respective functions.

The relation (11) can be corrected by expressing the shear force on the deformed shape of the beam, using the differential relation [3] :

$$T^d = \frac{\mathrm{d}M}{\mathrm{d}x} = T + P\frac{\mathrm{d}v}{\mathrm{d}x} \quad (12)$$

The relations (8) and (9) can be used for any types of external loads and limit conditions.

The General Algorithm

The calculation of the axial reactions from the bounds of the beams that are transversally loaded is very important because these reactions are usually neglected and the effect of such forces may increase the values of stresses.

In this respect, it is important to establish the values of the axial forces as exactly possible.

The relations (8) and (9) can solve this problem by using the limit conditions that correspond to the external bounds of the beam.

In order to reach some numerical results two cases are considered : a double supported beam at the extremities loaded with a concentrated force (fig. 2a) and a double embedded beam loaded with the same force (fig. 2b).

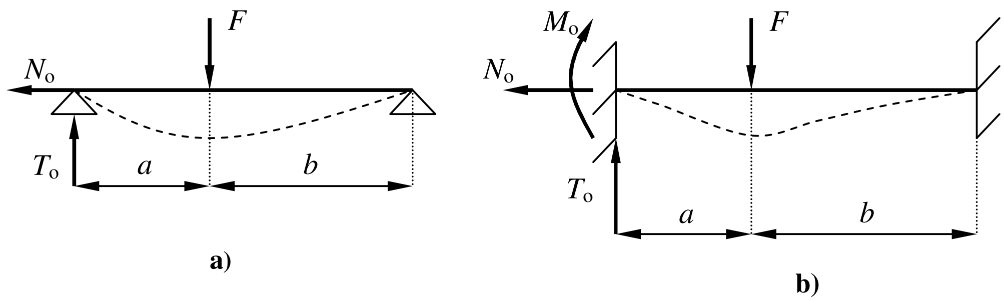


Fig. 2. A beam subjected to an external load

For the case presented in figure 2a, the known origin parameters are $v_o = 0$ and $M_o = 0$. From the limit conditions ($x = l \Rightarrow v = 0, M = 0$) the constants φ_o and T_o are determined, the function of the axial reaction N_o which is still unknown and after that the length of the elastic curve of the beam is calculated with the relation:

$$l^{'} = \int_{0}^{a} \sqrt{1+\varphi_1^2}\,dx + \int_{a}^{l} \sqrt{1+\varphi_2^2}\,dx \tag{13}$$

where φ_1 and φ_2 are the slopes of the beam in the [0, a] and respective [a, l] intervals. The difference between the length of the elastic curve and the initial length of the beam has to be the same with those produced by the axial reaction :

$$\Delta l = l' - l = \frac{N \cdot l}{EA} \tag{14}$$

The relation (14) can be corrected, to improvement the precision with the influence of the axial component of the shear force in the deformed shape of the beam :

$$\Delta l' = l' - l = \frac{N_o l}{EA} + \int_{o}^{a} \frac{T_1 dv}{EA} + \int_{a}^{l} \frac{(-T_2)\mathrm{d}v}{EA} \quad , \tag{15}$$

where T_1 and T_2 are the shear forces on the two intervals of the beam.

If the shear forces are supposed to be nearly constant, the relation (15) can be written in the simplified form :

$$\Delta l' = \frac{N_o l}{EA} + \frac{F \cdot v(a)}{EA} \tag{16}$$

The N_o unknown can be determined by solving the (14) and (16) equations.

In the case presented in figure 2b, the limit conditions in origin are $v_o=0$ and $\varphi_o=0$ and in the end of the beam $v(l)=0$ and $\varphi(l)=0$. Using the above conditions, the M_o and T_o parameters can be determined.

Calculus Example

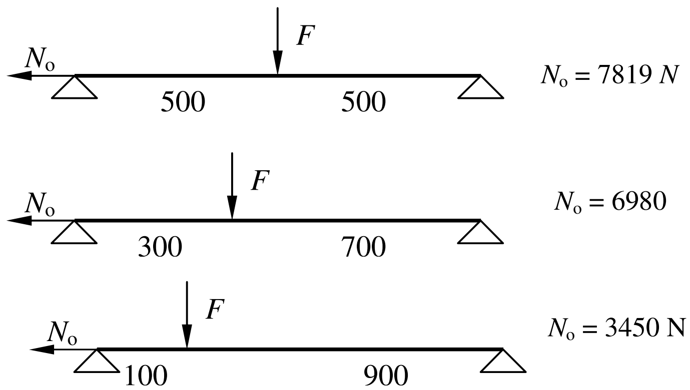
In order to exemplify the calculus methodology presented above it is considered a beam identically with those presented in [2]. The geometrical characterises of the beam are : the length l = 1m, the external force F= 260 N, the longitudinal elasticity modulus E = 210000 N/mm^2. The cross sectional area of the beam is a rectangle with the dimensions b=40 mm and h = 6 mm.

For the two cases presented in figure 2, it is necessary to calculate the axial force in origin in the hypothesis that the external force F acts at 500 mm, 300 mm and 100 mm from the starting point.

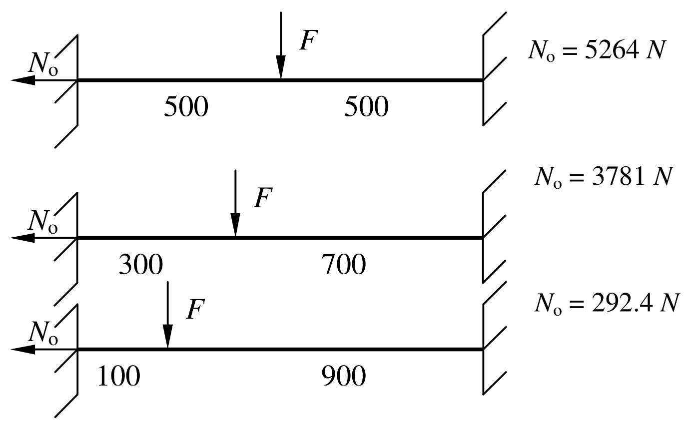
In order to solve numerically the equations(14) or (16), a specialised calculus programme has been realised to calculate the first root of the above equations .

The following results have been obtained :

- for the case presented in figure 2a :



- for the case presented in figure 2b :



It can be noticed that in comparison with the results presented in [2] the differences are 5% for the first case (fig. 2a) and 8% for the second case (fig. 2b).

Conclusions

The methodology presented in this paper allows the calculation of the axial force for any other external loads and limit conditions.

The formulation of the origin parameters in the deflection of the beam simplify the using of the limit conditions

References

1. Pandrea M., Model neliniar pentru calculul eforturilor axiale din bara dublu articulată acționată transversal de o forță concentrată, *Buletinul UPG seria tehnică*, volum LVII, nr. 4/2005.
2. Pandrea M., Calculul aproximativ al forței de întindere din bara dublu încastrată acționată transversal la mijloc de o forță concentrată, *Buletinul UPG seria tehnică*, volum LVII, nr. 4/2005.
3. Posea N., *Rezistența materialelor* , Editura Didactică şi Pedagogică, Bucureşti, 1979.
4. Posea N., Vasilescu Ş., General Solution of Symmetrically Loaded Thin Cylindrical Shells, Applying the Laplace Transform, *Revue Roumaine des Sciences Techniques, Serie de Mecanique Appliquee*, nr. 2 (32), Editura Academiei, Bucureşti, 1987.

5. Vasilescu Ş., Talle V., Badoiu D., *Rezistenţa materialelor- tehnici de calcul şi proiectare*, Editura Universitatii din Ploieşti, 2002.
6. Vasilescu S., Talle V, Manea C., Transversal Vibrations in Fluid Crrying Pipelines, *International Scientific Conference "microCAD 2002"*, Miskolc, Ungaria

O metodă de calcul a reacțiunilor axiale pe bare drepte

Rezumat

În lucrare se prezintă o metodologie de determinare a reacţiunii axiale pe bare drepte static nedeterminate. Metodologia este importantă deoarece acest efect era neglijat până în prezent şi evidențierea sa poate avea efecte în special în lagărele arborilor supralegaţi solicitaţi transversal. Metodologia prezentată este aplicabilă oricăror tipuri de legături şi oricăror sarcini transversale.

Soluția ecuației diferențiale obținute este pusă sub forma parametrilor în origine, ceea ce facilitează determinarea constantelor din condiții la limită. Rezultatele obținute sunt analizate pe un exemplu de calcul şi sunt comparate cu rezultate obținute în alte lucrări în care se utilizează metode aproximative.