BULETINUL Universității Petrol – Gaze din Ploiești	Vol. LVIII No. 2/2006	77 - 84	Seria Tehnică

Aspects of the Non-linear Buckling Analysis of the MU200 Mast

Gheorghe Gherghe*, Nicolae Posea**

* SC Timken SA, Str Dr. Gh. Petrescu 25, Ploiești
e-mail: george.gherghe@timken.com

** Universitatea Petrol – Gaze din Ploiești, Bd. București 39, Ploiești
e-mail: nposea@upg-ploiesti.ro

Abstract

This article presents the most important aspects of the post-buckling analysis of the MU200 mast, such as the critical load at which the bifurcation point of the equilibrium appears, the critical load at which the limitation point of the equilibrium appears and the post-buckling behaviour of the MU200 mast. After the presentation of the theoretical aspects and the principal iterative methods used in the non-linear buckling analysis, the post-buckling behaviour of the MU200 mast is analysed. At the end a new method for the study of the buckling of masts is presented.

Key words: *non-linear buckling, instability zone, arc-length method, post-buckling analysis.*

Theoretical Aspects of the Stability Analysis in the Non-linear Field

A major characteristic of the study of the buckling in the non-linear field as opposed to the study of the buckling in the elastic linear field is represented by the presence of instability areas in the post-buckling area. The buckling in the elastic linear field displays only a linear behaviour in the pre-buckling area up to the embranchment of the balance position where P_{crit} does appear.

These defining elements of the two types of buckling study are represented in figure 1.

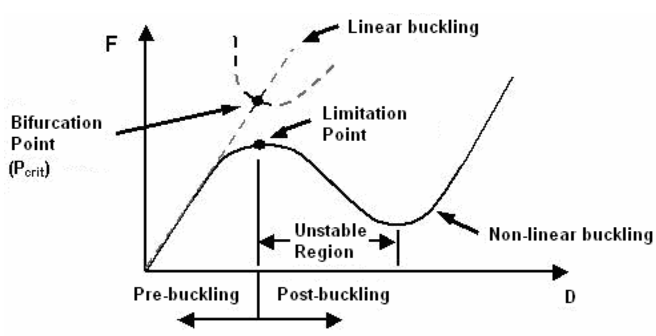

Fig. 1. Buckling analysis characteristics

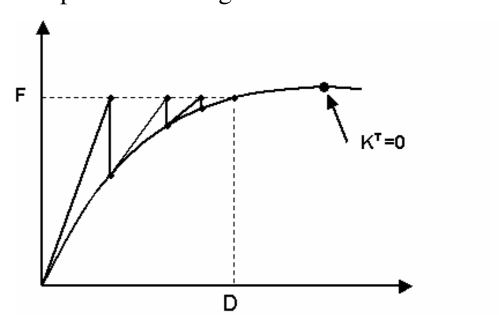

Fig. 2. Newton-Raphson method

The structure of the equations of equilibrium in the finite element method for a structure which displays a non-linear behaviour is:

$$\underline{K}^T(\underline{D})\ \underline{D} = \underline{F}\,, \tag{1}$$

in which $\underline{K}^T(\underline{D})$ is the matrix of the tangential structural rigidity, $\underline{D}$ is the nodal displacement vector and $\underline{F}$ is the vector of the nodal forces. Relation (1) can be written in the following incremental structure [3]:

$$\underline{K}^T(\underline{D}^{i-1})\ \Delta\underline{D}^i = \Delta\underline{F}^i\,, \tag{2}$$

where $\underline{K}^T(\underline{D}^{i-1})$ is the tangential matrix to the computation step *i* (which is the function of nodal displacement at the previous step *i-l*), $\Delta\underline{D}^i$ are the displacements of the nodes to step *i* and $\Delta\underline{F}^i$ are the nodal forces at step *i*.

Out of equation (2) $\Delta\underline{D}^i$ displacements are obtained [3]:

$$\Delta\underline{D}^i = \left[\underline{K}^T(\underline{D}^{i-1})\right]^{-1}\ \Delta\underline{F}^i\,. \tag{3}$$

On the basis of relation (3) the total displacements to step *i* turn into:

$$\underline{D}^i = \underline{D}^{i-1} + \Delta\underline{D}^i \tag{4}$$

At each and every computational step, therefore at step *i* as well, follows the nodal residual force which is evaluated through the expression [3]:

$$\Delta\underline{\Im}^i = \underline{F}^i - \underline{K}^T(\underline{D}^{i-1})\ \underline{D}^{i-1}\,, \tag{5}$$

and the computation continues until a value of the effaceable residual force is obtained within two consecutive steps.

Iterative Solving Techniques Used in the Stability Study of the Non-linear Field

The best known solving method is the Newton-Raphson method. Therefore, the Newton-Raphson method iterates equation (2) until the remnant of both external and internal forces is smaller than an imposed approximation. The Newton-Raphson method increases the value of external forces applied with an increment and keeps constant this value of external forces applied during the afferent iterations of each and every step until the convergence is reached. The method cannot reach the desired convergence if the tangent stiffness matrix is zero as it is depicted in figure 2. In order to solve such structures at which the tangent stiffness matrix is zero or negative the arc-length method is used which is also an iterative method. This method emerged as a consequence of the need to develop algorithms that would allow describing the structural behaviour over the limit point (see figure 1). The arc-length method was developed by Ricks [1] and Wempner [2].

This method multiplies the external forces with a loading parameter λ, and the equation of equilibrium takes the form of:

$$\underline{K}^T(\underline{D})\ \underline{D} = \lambda\,\underline{F}\,. \tag{6}$$

The main limitation of iterative methods when trying to solve the system of equation (6) appears around the limit point, i.e. for a loading level specified by $\lambda =$ const. There may not exist an intersection point between the structural behaviour curve specified in (6) and the loading level. In order to overpass this shortcoming, the arc-length method looks for the intersection of the curve specified by (6) and the curve s=const., where s is a circular arc with its centre in the equilibrium point determined at the previous step and of the length:

$$s = \int ds\,, \tag{7}$$

where ds is expressed as [3]

$$ds = \sqrt{d\,\underline{D}^T\, d\,\underline{D} + d\lambda^2\, \underline{F}^T\, \underline{F}}\,. \tag{8}$$

In relation (8), $\underline{D}$ represents the vector which contains the displacements of the structural nodes, $\underline{F}$ is the vector of the external forces, and λ is the parameter of the loading.

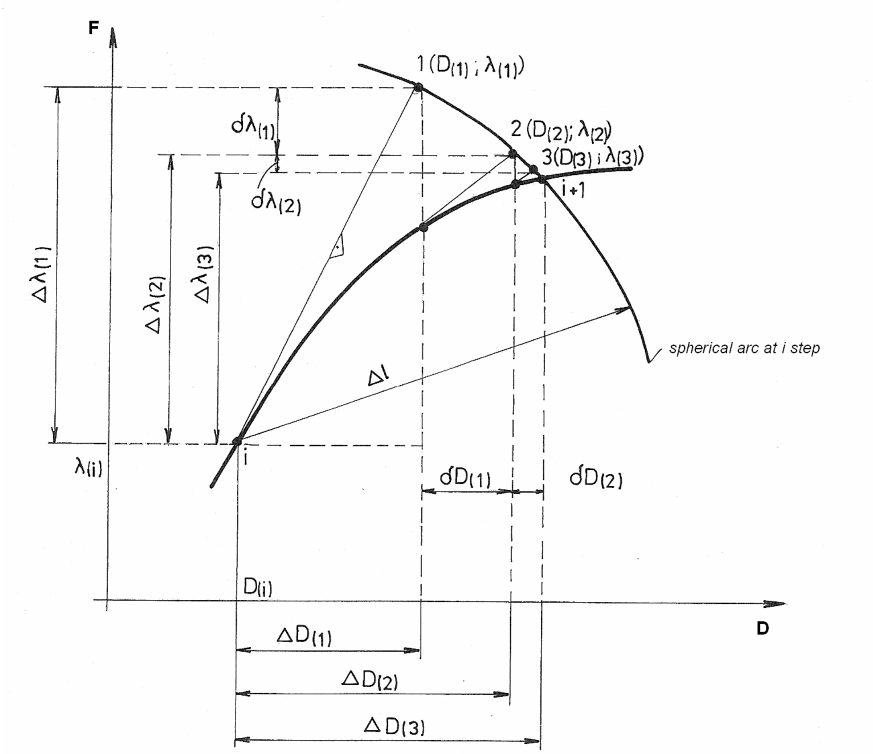

Fig. 3. The arc-length method

Relation (8) can be expressed in the following incremental form [3]:

$$\Delta\,\underset{\sim}{D}^T\;\Delta\,\underset{\sim}{D} + \Delta\lambda^2\;\underset{\sim}{F}^T\;\underset{\sim}{F} - \Delta l^2 = 0\,, \tag{9}$$

where Δl is the constant radius of the arc.

Observation: $\Delta\,\underline{D}$ and $\Delta\lambda$ are incremental not iterative quantities. For iterative quantities the symbols $\delta\,\underline{D}$ and $\delta\lambda$ will be used.

The essence of the arc-length method is that the parameter of loading λ becomes a variable. It is thus obtained a system of n equations with n+l unknown quantities. In order to solve this situation, the system of equation (6) has to be completed with the constraint (9) and the resulting system will be solved using the Newton-Raphson iterative method.

The Post-Buckling Analysis of the Mast MU200

In the following pages, the post-buckling behaviour of the structure of the mast MU200 will be studied. The mast MU200 is a slender iron construction made of straight bars (rolled sections) solded at the ends. The mast is positioned above the drillhole and has the following main functions:

- the propping of drilling pipes and tubular goods using the system of control, and also the control of the mast during entering/extraction operations;
- the arrangement and propping of the piles of poles and tubings.

The MU 200 mast is a spatial iron construction made of straight bars rigidly tied at the nodes. For the post-buckling study of the mast the method of the finite element whose unknown quantities are the displacements of the structural nodes shall be used. For the discretisation of the structure the uni-dimensional finite element of bar type with 6 degrees of freedom on the node is used, as it is shown in figure 4.

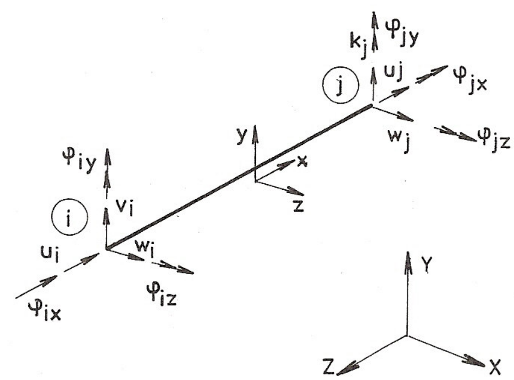

Fig. 4. One-dimensional finite element

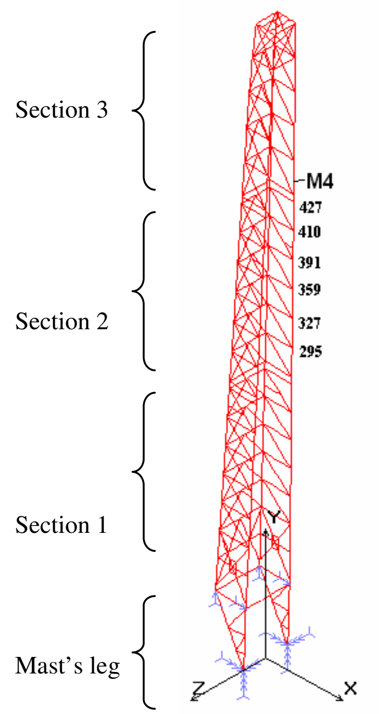

Fig. 5. The finite elements model

The discretization of the structure of the MU200 mast is specified in figure 5, along with the leans-on presented by the structure. Analysing figure 5, one can observe that the mast is made of the mast's leg and three assembled sections. The need for analysis in the non-linear field of the mast's buckling arises from the fact that this structure presents a non-linear behaviour of the same type with the one represented in figure 1 and the analysis made in the elastic linear field accentuated a dangerous behaviour of the sections 2 and 3 of the mast especially because of the local phenomena of loss of stability. Thus the sole specification of the value of the loading at which the mast loses its position of equilibrium through the embranchment of the equilibrium (i.e. a general buckling) is not sufficient and it can even be dangerous and the structure can present the areas with a local buckling which propagate in the entire structure at reduced loading values, values that are specified by the limitation point of the equilibrium. In the following lines the results of the analysis made in the non-linear field of the buckling are presented. This analysis shows the behaviour of the structure on both sides of the limitation point of the equilibrium. As it has already been specified, on the basis of the analysis of the buckling in the linear field the sections 2 and 3 of the mast have been proved to be the most dangerous areas that require an analysis in the non-linear field. Thus, the post-buckling behaviour of the MU200 mast will be specified through the specification of the curve of equilibrium for those nodes afferent to the gantry M4 on section 2 using the arc-length method if the mast is loaded with a unit force at the hook. The curve of equilibrium in the arc-length method is expressed using the following co-ordinates: the loading parameter (λ) and the displacements of the nodes (D), as it has been specified previously.

Using a specialised computational program which has the arc-length method implemented, the behaviour of the structure around the equilibrium point is determined. The study of section 2 of the MU200 mast will allow the attainment of the equilibrium curve for the nodes placed on a gantry of section 2.

The nodes 295, 327, 359, 391, 410 and 427 belonging to the gantry M4 have been chosen and they are arranged in an ascending way on the height of section 2 (see figure 5). Thus in figure 6 it is represented the variation graphic of the multiplication factor of loading λ as related to its displacement on the x axis for the nodes that are placed on the M4 gantry of section 2.

The variation graphic of the multiplication factor of loading as related to its displacement on the y axis for the nodes that are placed on the M4 gantry of section 2 is not represented because the displacements on this axis are very small. Also in figure 7 it is represented the variation graphic of the multiplication factor of loading as related to its displacement on the z axis for the nodes that are placed on the M4 gantry of section 2.

Conclusions

Analysing the figures 6 and 7, the following conclusions can be drawn:

- the critical point of limitation of the equilibrium corresponds to a loading that has the following value $P_{crit}^{\lim it} = 3627\ kN$;
- the mast MU200 exhibits a post-critic unstable behaviour, instability which is obtained through the propagation of the local buckling throughout the entire structure which leads to the general collapse of the structure.
- the MU200 mast is a structure which is liable to drawbacks, a thing that must be kept in mind during its execution phase as well as during its exploitation phase.

Thus the critical load at which the limitation of the equilibrium appears has been obtained:

$$P_{crit}^{\lim it} = 3627\ kN\ . \tag{10}$$

From an analysis previously performed in the elastic linear field the critical load at which the embranchment of the equilibrium appears has been determined [4]:

$$P_{crit}^{bif} = 4666\ kN\ . \tag{11}$$

The critical load at which the lost of the equilibrium appears must be determined through the contrasting of the two values. Thus, for the MU200 mast the critical load is:

$$P_{cr} = \min\left[P_{crit}^{bif},\ P_{crit}^{\lim it}\right] = \min\left[4666\ kN,\ 3627\ kN\right] = 3627\ kN\ . \tag{12}$$

It can be observed that the value of the critical load at which the limitation of the equilibrium appears as specified by relation (10) is with 22.3% smaller than the value of the critical load at which the embranchment of the equilibrium appears – value represented in relation (11).

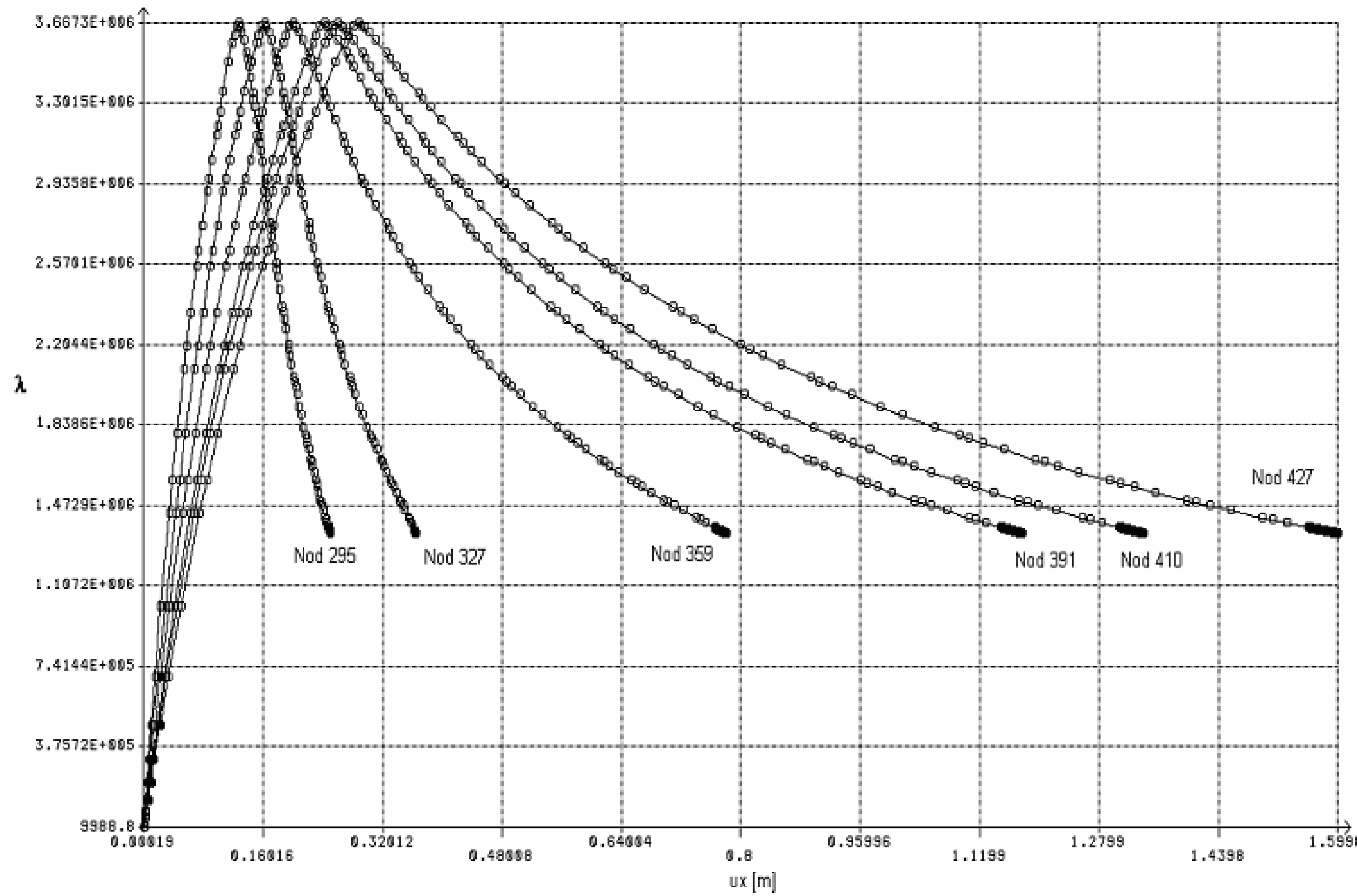

Fig. 6 – Multiplication loading factor as related to its displacement on the *x* axis

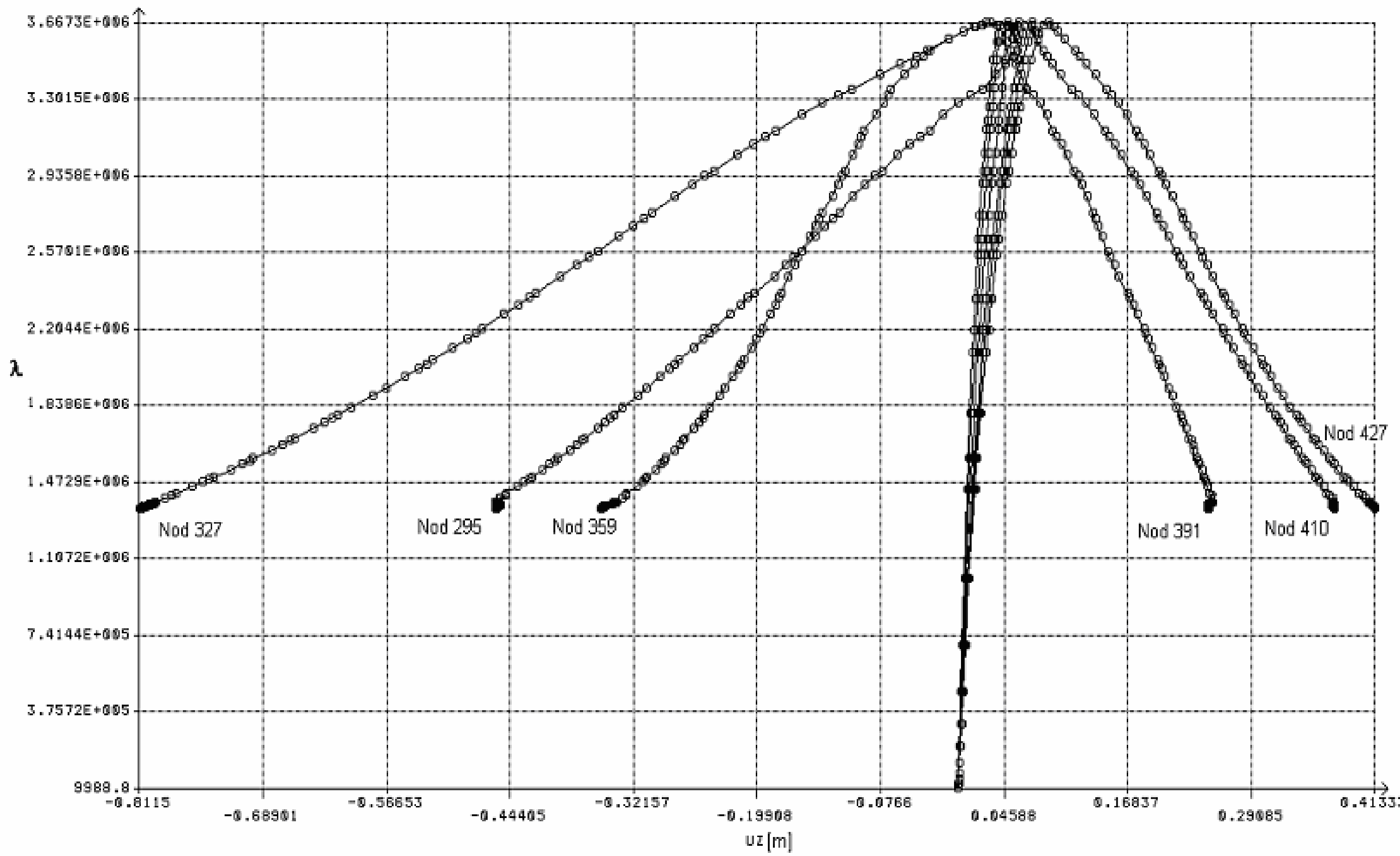

Fig. 7 – Multiplication loading factor as related to its displacement on the *z* axis

The above mentioned observation lets one draw the conclusion that the sole analysis of the buckling in the elastic linear field is not sufficient.

On the basis of this observation, it is suggested that the computational manner of the masts at buckling should be realised following the three steps mentioned bellow:

- step 1- the study of the buckling in the elastic linear field which gives us the value of the critical loads at which the embranchment of the equilibrium and the forms of loss of stability appear;
- step 2 - the study of the buckling in the elastic non-linear field accentuating the behaviour of the dangerous areas specified at step 1. This analysis indicates the value of the critical loads at which the limitation of the equilibrium appears;
- step 3- the determination of the critical loads at which the mast loses its stability through the contrasting of the values afferent to the embranchment of the equilibrium to the values afferent to the limitation of the equilibrium.

References

1. Riks, E., An incremental approach to the solution of snapping and buckling problems, *International Journal of Solids and Structures*, 1979.
2. Wempner, G.A., Discrete approximations related to nonlinear theories of solids, *International Journal of Solids and Structures*, 1971.
3. Crisfield, M.A., *Non-linear finite element analysis of solids and structures*, 1991.
4. Gherghe, Gh., *Contributii privind stabilitatea dinamica a masturilor*, Teza de doctorat, Universitatea Petrol-Gaze Ploiesti, 2006.
5. Zienkiewicz, O.C., *The Finite Element Method in Engineering Science*, McGraw-Hill, London, 1971.
6. Timoshenko, S., Gere, J., *Teoria stabilităţii elastice*, Editura Tehnică, Bucureşti, 1967.

Aspecte privind flambajul mastului MU200 în domeniul neliniar

Rezumat

In cadrul acestui articol sunt prezentate cele mai importante aspecte ale comportării post-flambaj a mastului MU200, cum ar fi: sarcina critică de pierdere a stabilităţii prin bifurcarea echilibrului, sarcina critică de pierdere a stabilităţii prin limitarea echilibrului şi răspunsul post-flambaj al mastului MU200.

După prezentarea principalelor aspecte teoretice şi a principalelor metode iterative de rezolvare a ecuaţiilor de mişcare utilizate în studiul flambajului în domeniul neliniar, este studiată comportarea post-flambaj a mastului MU200. La sfârşitul acestui articol este propusă o nouă metodă care permite studiul complet al flambajului (flambajul în domeniul liniar şi flambajul în domeniul neliniar) masturilor de foraj.