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Construction of Spheres in Conditions of Tangency with the Vertical Plane

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Abstract

The present paper provides the construction of a sphere in some imposed conditions. The condition of tangency is one of the most required in the problems of descriptive geometry, as well as in applications like those from the technical drawing. The problem of the correct location of the important elements needed in the construction of a sphere, as the radius and the center, are solved in the present work for the case that the sphere must pass by three points and also must be tangent to the vertical plane of projection.

Key words: *sphere, tangency, vertical plane, projection.*

Introduction

The problem of the construction of a sphere in a condition of tangency is very interesting from a theoretical point of view and with an immediate practical applicability. The use of the methods of construction offered by the descriptive geometry helps solving this kind of problem. Also, these methods give the possibility to correctly locate the centre of the sphere and its radius, which are defined by the means of construction.

Building the Plane [P] Passing by the Three Points

In the present work, the author proposes to build a sphere which will go through three non co-linear points and must have tangency to the vertical plane of projection. To exemplify, there were imposed three points *A* (*a, a*\`), *B* (*b, b*\`), *C* (*c, c*\`) from the Euclidian space, of known co-ordinates, of which the horizontal projections a, b, c and the vertical projections a\`, b\`, c\` are shown in the figure 1. These three points determine the plane [P] (p_H, $p`_V$). It is built the straight line (*D*) (*d, d*\`) which contains the points *A* and *C*, as well as straight line (D_1) that goes through *A* and *B*. Also, we observe that (*D*) is a horizontal straight line, because the vertical projection *d*\` is parallel with the (*Ox*) axis. Through the vertical projections *v* and *v*\` found as the intersection between the straight lines (D) and (D_1) with the vertical plan of projection [V] we build $p`_V$, which is the intersection between [P] and the vertical principal plane of projection [V]. After the determination on the intersection p_X between the plane [P] and the (*Ox*) axis, p_H is built, which is the intersection between [P] and [H] ([H]- the horizontal plane of projection), being parallel with the horizontal projection d of the straight line (*D*).

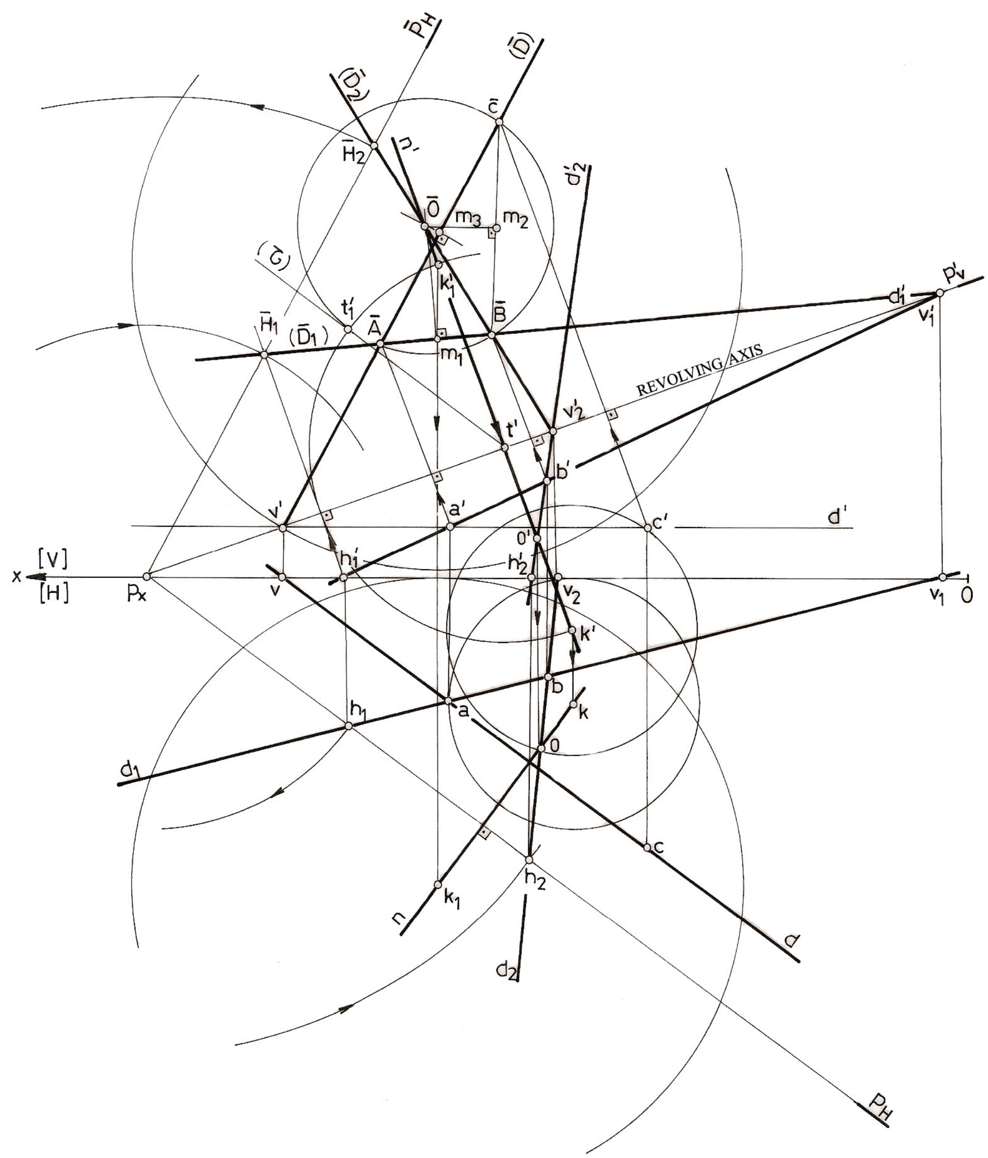

Fig. 1. The construction of spheres having tangency with the vertical plane

The Construction of the Revolved Plane with the Important Elements

To determine the center of the sphere, O, it is made a revolving of the plane [P] on the vertical plan. For this, from point h_1 (H_1 (h_1, h_1\`) – the intersection of $d_1$ with plane [H]) a circular arc is built in the trigonometric direction and at its intersection with the perpendicular built from $h`_1$ (the vertical projection of H_1) is found $\overline{H}_1$ - point H_1 revolved on the vertical plane. The $\overline{p_H}$, revolved projection of p_H is built by joining the points p_X and $\overline{H}_1$. In a similar way they build the revolved position in the vertical plane of the straight line (D), which is ($\overline{D}$), and of the straight line (D_1), which is ($\overline{D_1}$).

By adding perpendicular lines from the projections a\`, b\`, c\` on the revolving axis $p`_V$, at the intersections between these perpendicular lines and the straight afferent lines are found $\overline{A}$, $\overline{B}$, $\overline{C}$ - the revolved positions of points A, B, C. The $\overline{A}\ \overline{B}\ \overline{C}$ triangle is formed.

At the intersection of the perpendiculars on the middle of a segment m_1, m_2, m_3 of the triangle, the centre $\overline{O}$ of the circumscribed circle of the triangle is found, which is also the revolved position of the centre of the sphere. Point $\overline{O}$ is found on the straight line ($\overline{D_2}$).

The Solutions

It is obvious that the problem has two solutions. To determine the projections of the two spheres, there are brought from the revolved position the projections o and $o`$ by drawing the perpendicular line $n`$ on the revolving axis. At the intersection between $n`$ and $p`_V$, by correspondence, it is found o on d. It is marked the tangent (τ) from $t`$ at the centre $\overline{O}$ of the circle and it is found $t`_1$. The circle of the center $t`$ and the radius $t`t`_1$ cuts the projections $n`$ in $k`$ and $k`_1$. Through correspondence, on projection n, k and $k`$ are found. Points $K(k, k`)$ and $K_1(k_1, k_1`)$ are the centers of the spheres and it's obvious that the method of construction allows the precise determination of their co-ordinates it is.

The distance from k to (ox) axis represents the radius of the sphere of center K and the distance from k_1 to (ox) represents the radius of the center K_1 of the second sphere. The projections of the two spheres are represented in the figure 1.

Conclusion

As a conclusion, the graphical construction of a tangent sphere to a vertical plane of projection passing through three known points was made by using the revolving method on a vertical plane of the plane determined by the three points, offering a quick solution, precise and simple enough to obtain the center and the radius of the sphere.

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Construcția sferelor în condiții de tangență cu planul vertical de proiecție

Rezumat

În prezenta lucrare, autoarea a propus construirea unei sfere în condiții de tangență la planul vertical de proiecție, trecând, de asemenea, prin trei puncte necoliniare impuse din spaţiu .Problema a fost rezolvată prin mijloacele geometriei descriptive. Problema cea mai importantă a constituit-o identificarea precisă a coordonatelor centrului sferei şi a razei acesteia. Folosirea metodei rabaterii pe un plan principal de proiecție (planul vertical) a fost de un real folos pentru poziționarea corectă a centrului sferei. Aşa cum s-a arătat în lucrare, problema a avut două soluții constructive.