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Regarding a Special Class of Shells

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Abstract

The purpose of this paper is to determine a special class of shells (using the Laplace equation) in the hypothesis that a relation between the meridional (N_φ) and circumferential (N_θ) forces is accepted from the beginning. Such a condition is important in order to minimise the efforts in shells and to determine the optimum shape of the shell. The algorithm reaches a general differential equation that is integrated for some different tips of limit conditions and some special kind of shells are obtained. It is also provided an approximate method that reaches easier the form of the shell.

Key words: *shell, meridional, circumferential.*

General Aspects

A vertical cylindrical shell embedded with an unknown shell is considered (fig. 1).

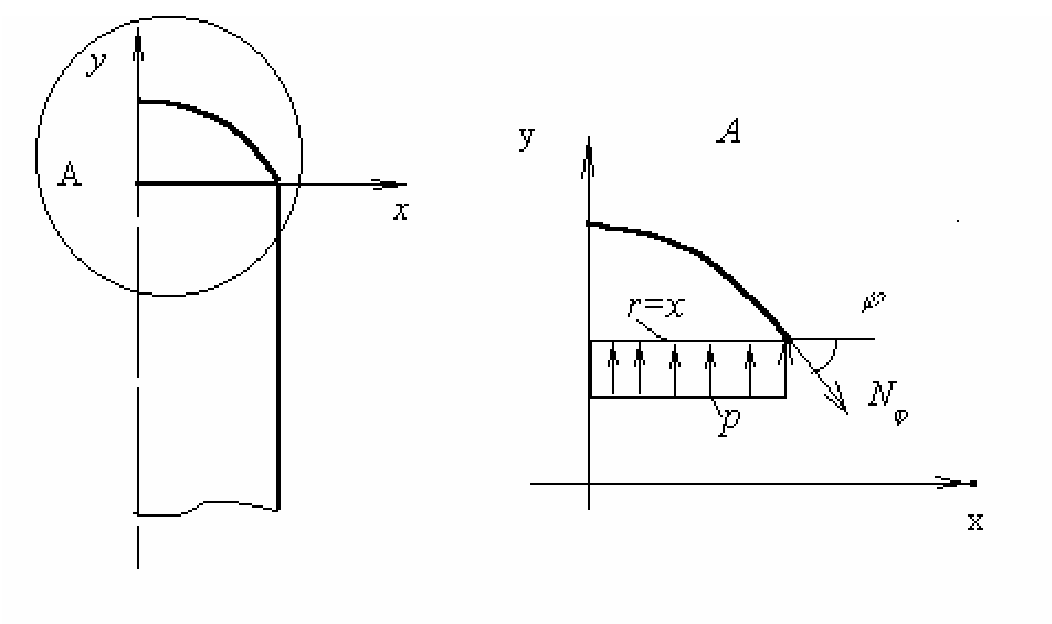

Fig. 1. The meridional curve of the shell

The cylindrical shell has the a radius and is loaded with an uniform pressure p. In order to determine the efforts in the head, a complete section of the shell is realised (at the current r radius) and the equation of vertical forces is used:

$$-N_{\varphi} \cdot 2 \cdot \pi \cdot r \cdot \sin\varphi + p \cdot \pi \cdot r^2 = 0 \tag{1}$$

From equation (1) the N_{φ} meridional effort (in units of force by unit of length) is obtained :

$$N_{\varphi} = \frac{p \cdot r}{2 \cdot \sin\varphi} \tag{2}$$

The connection between the meridional (N_{φ}) and circumferential (N_{θ}) efforts is realised by the Laplace equation [1]:

$$\frac{N_{\varphi}}{r_1} + \frac{N_{\theta}}{r_2} = p \tag{3}$$

where the r_1 and r_2 are the main radius of curvature of the surface.

Usually in the membrane shell theory, after the calculation of the meridional effort (N_{φ}) with the (2) relation, the Laplace equation (3) is used in order to determine the circumferential effort (N_{θ}). After this, the thickness of the shell is calculated using a strength theory.

The Equations of the Shells

In this paper a special kind of hypothesis is used:

$$N_{\theta} = k \cdot N_{\varphi} \tag{4}$$

where k is a numerical coefficient that characterise the shell.

The (4) hypothesis establishes from the beginning a relation between the efforts of the shell. This relation is a general one so that it does not restrict the generality of the problem.

Replacing the (4) relation into (3) the meridional effort (N_{φ}) is obtained:

$$N_{\varphi} = \frac{p \cdot r_1 \cdot r_2}{r_2 + k \cdot r_1} \tag{5}$$

Because r_1 is the main radius of curvature of the meridian curve, from figure 1 it can be observed that this radius can be expressed by an analytical relation [2]:

$$r_1 = \frac{\left(1 + y'^2\right)^{3/2}}{y''} \tag{6}$$

Because the main curves are normal in the current point of intersection, the r_2 radius of curvature can be expressed by relation (fig. 1):

$$r_2 = \frac{r}{\sin\varphi} = \frac{x}{\sin\varphi} \tag{7}$$

In the current point (fig. 1) the trigonometric function ($\sin\varphi$) can be expressed taking into consideration the co-ordinates of the points (x, y):

$$\sin\varphi = \frac{tg\varphi}{\sqrt{1+tg^2\varphi}} = \frac{y'}{\sqrt{1+y'^2}} \tag{8}$$

Taking into consideration relation (8), the radius of curvature r_2 takes the form:

$$r_2 = \frac{x}{\sin\varphi} = \frac{x\cdot\sqrt{1+y'^2}}{y'} \tag{9}$$

Replacing the (6) and (9) relations in (5) it results the following form for the N_φ effort:

$$N_\varphi = \frac{\dfrac{p\cdot\left(1+y'^2\right)^{3/2}}{y''}\cdot\dfrac{x\cdot\sqrt{1+y'^2}}{y'}}{\dfrac{x\cdot\sqrt{1+y'^2}}{y'}+k\dfrac{\left(1+y'^2\right)^{3/2}}{y''}} = \frac{p\cdot x\cdot\left(1+y'^2\right)^{3/2}}{x\cdot y''+k\cdot y'\cdot\left(1+y'^2\right)} \tag{10}$$

Taking into consideration (fig. 1) that at the current point the r radius is equal with the x abscissa, the (2) relation can be put under the form:

$$N_\varphi = \frac{p\cdot r}{2\cdot\sin\varphi} = \frac{p\cdot x\cdot\sqrt{1+y'^2}}{2\cdot y'} \tag{11}$$

Because (10) and (11) express the same effort, by equalling the above relations the following differential equation is obtained:

$$(2-k)\cdot\left(\frac{dy}{dx}\right)^3 + (2-k)\cdot\frac{dy}{dx} - x\cdot\frac{d^2y}{dx^2} = 0 \tag{12}$$

The (12) differential equation describes the shape of some different kind of shells that can be considered the head of the cylindrical shell presented in figure 1.

In order to realise a correct junction between the cylinder and his head, some limit conditions have to be accepted. For example, if the radius of the cylinder is considered to be equal with a, the limit conditions are:

$$x = a \Rightarrow y = 0;\ \ y' = -\infty \tag{13}$$

If in the junction the two shells do not necessarily have a continuity of derivatives, the (13) limit conditions can be put under the general form:

$$x = a \Rightarrow y = 0;\ \ y' = -b \tag{14}$$

where b is a real number in the interval $(0,-\infty)$.

In the case that the approximation $\left(\frac{dy}{dx}\right)^2 << 1$ can be accepted, the differential equation (12) can be put under the form:

$$x\frac{d^2y}{dx^2} - (2-k)\frac{dy}{dx} = 0 \tag{15}$$

The advantage of the (15) differential equation is that is easier to be integrated in comparison with (12).

Numerical Example

In order to exemplify the analytical results, a cylinder with radius a = 1m is considered that is subjected to an uniform pressure p. The problem asks to determine the form of his head shells accepting for k values in the interval $[0,3]$.

The (12) differential equation has been numerically integrated, accepting the limit conditions under (14) form (b = -1). The shapes of some particular shells are presented in the following figures.

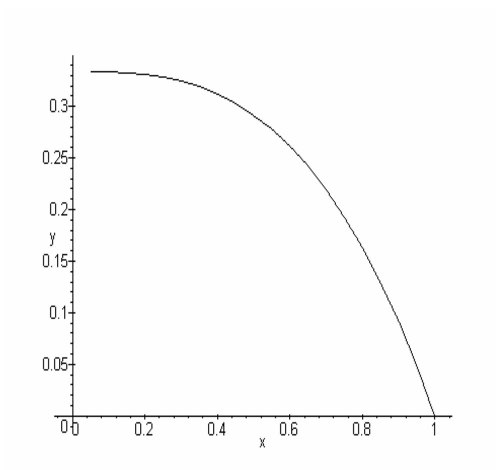

Fig. 2. Meridian curve for $k = 0$

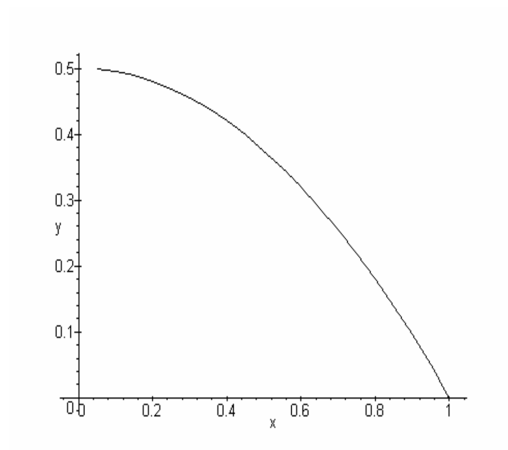

Fig. 3. Meridian curve for $k = 1$

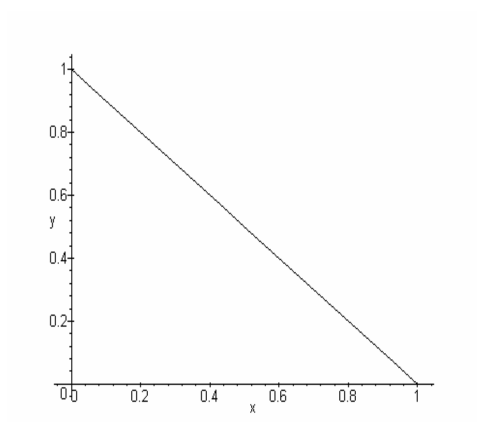

Fig. 4. Meridian curve for $k = 2$

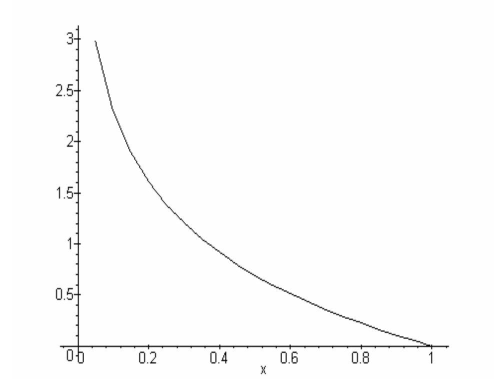

Fig. 5. Meridian curve for $k = 3$

In figures 2, 3, 4 and 5 are presented the meridional curves of the respective shells. It can be noticed that the first case (that corresponds to $N_\theta = 0$) has the minimum ratio between the semi-axes of the shell. The third case corresponds to a conical head (fig. 4) and the last one (fig. 5) to a shell that has an inconvenient form.

If instead of (12) the (15) differential equation is used with the same limit conditions, the form of the shells can be integrated and can be written under the forms:

$$k = 0 \Rightarrow y(x) = \frac{a}{3}\left(1 - \frac{x^2}{a^2}\right)$$

$$k = 1 \Rightarrow y(x) = \frac{a}{2}\left(1 - \frac{x^2}{a^2}\right) \tag{16}$$

$$k = 2 \Rightarrow y(x) = a - x$$

$$k = 3 \Rightarrow y(x) = a \ln \frac{a}{x}$$

If the analytical equations (16) are graphically represented nearly the same curves as those represented in figures 2, 3, 4 and 5 are obtained. This is a proof that the approximation accepted is efficient and can be used if the slope of the shell is smaller than the unit.

The curves (16) can be studied also in the plastic range, where the most efficient method of analysis is to use the equations of the shell.

The methodology can be used in order to establish the shape of the shells for any other values of *k* and limit conditions.

Conclusions

The paper presents a methodology that establishes the form of a head shell that is connected with a cylindrical shell subjected to an internal uniform pressure *p*.

The algorithm accepts the hypothesis that between the circumferential and meridional effort it exists a relation ($N_\theta = k \cdot N_\varphi$) and determine the differential equation of the shell. Solving numerically the differential equation some special kinds of shells have been obtained.

It is also provided an approximate method that facilitates the integration of the differential equation and reaches the analytical equation of the curves. The results can be applied for any values of *k* coefficient and any other kind of limit conditions.

The analytical equations of the meridian curves of the shells allow the study of the elasto-plastic behaviour in order to establish the plastic bending moment and to increase the capacity of the shell.

Using the same algorithm there can be established any other meridian curves that correspond to different limit conditions.

It can be also noticed that, for the classic limit conditions the cylindrical, spherical and conical shells are obtained. These results correspond to the expectations and make this methodology to be an important one.

The shells obtained for any limit conditions can be easily manufactured using the same technology as the existent one.

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Asupra unei clase speciale de învelişuri

Rezumat

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