

# The Modelling of the Electrohydraulic Servovalves

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## Abstract

*This paper presents an original model for the design of the debit servovalve with a force reaction which can determine, by numeric simulation, the dynamic performances of the servovalve. The couple motor is by definition an electromagnetic converter and therefore its behaviour in a dynamic regime is described using a set of differential equations with electromagnetic and movement sizes.*

**Key words:** *Servovalve, Electrohydraulic, Modelling, Mathematics shaping.*

## Introduction

The servovalves are electrohydraulic elements that realise the interface between the control electrical signals and the hydraulic motors, generators of mechanical work. The static features (linearity, precision, hysteresis) and the dynamic features (diminishing, phase difference, overshoot) of the servo valves directly influence the performances of the hydraulic operation system.

Currently, the servovalves are used in order to realise the automatic systems of supervision in position, speed and force. From the constructive point of view, the servovalves are manufactured with one, two or three hydraulic stages.

## The Scheme of the Servovalves with Force Reaction

The debit servovalves are made up of the following main assemblies:

- o the couple motor;
- o the differential hydraulic preamplifier (nozzle and palette);
- o the power hydraulic amplifier with drawer (distributor).

The functional scheme of the debit servovalve is presented in figure 1.

## The Mathematic Shaping of the Couple Motor

The couple motor represents the best known interface equipment in the field of the electrohydraulic systems and it is made under the form of a system with permanent magnet, directly coupled with the palette of the differential hydraulic preamplifier.

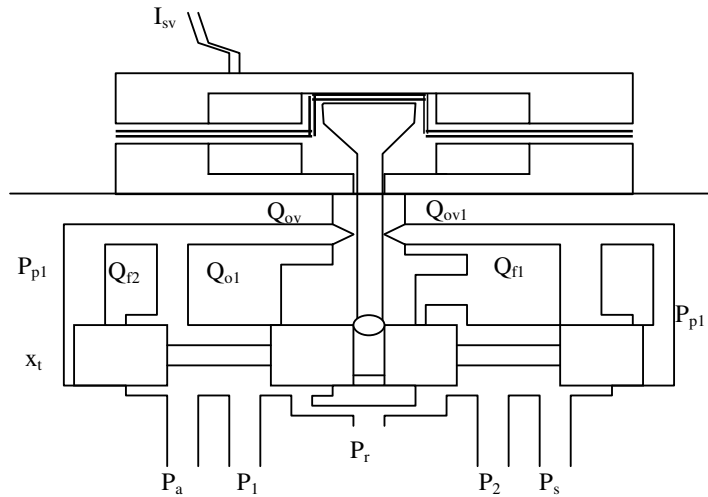


Fig. 1. The functional scheme of a debit servovalve

### The Feature of the Couple Motor

The feature of the couple motor represents the relation between the couple generated by the couple motor, the control current and the position of the mobile reinforcement of the couple motor, and it can be expressed by the relation:

$$M_{em}(\theta, i) = K_i \cdot i + K_m \cdot \theta + M_h \quad (1)$$

where  $M_{em}$  is the useful electromagnetic moment generated by the couple motor [Nm],

$K_i$  - the moment constant of the couple motor [Nm/A],

$i$  - the control current [A],

$K_m$  - the electromagnetic constant of the couple motor [Nm/rad],

$\theta$  - the inclination angle of the axle of the mobile reinforcement compared to its initial position, equal to the angle movement of the mobile reinforcement [rad unit],

$M_h$  - the hysteresis moment of the couple motor.

### The Movement Equation of the Mobile Reinforcement

The movement equation of the mobile reinforcement mobile is given by the relation:

$$M_{em}(t) = J \frac{d^2\theta}{dt^2} + \beta \frac{d\theta}{dt} + k_\theta \cdot \theta + M_s(t) \quad (2)$$

### The Electromagnetic Equation of the Couple Motor

The electromagnetic equation of the couple motor is given by the relation:

$$u(t) = \frac{1}{2} \cdot R \cdot i + L \cdot \frac{di}{dt} + 2 \cdot K_i \cdot \frac{d\theta}{dt} \quad (3)$$

where  $u(t)$  represents the tension at the terminals of the control bobbin,

$L$  represents the inductance of the bobbins of the couple motor,

$R$  is the bobbins' resistance.

After the application of the Laplace transformation for the equations (1), (2), (3) and considering  $M_h = 0$ , the following relationships are obtained:

$$U(s) - 2K_i \cdot s \cdot \theta = \frac{1}{2} \cdot R \cdot (T_e \cdot s + 1) \cdot I(s) \quad (4)$$

$$K_1 \cdot I(s) - M_s(s) = (k_\theta - K_m)[T_{em}^2 \cdot s^2 + 2\delta \cdot T_{em} \cdot s + 1] \cdot \theta(s) \quad (5)$$

where:

$$T_{em} = \frac{1}{\omega_n} = \sqrt{\frac{J}{k_\theta - K_m}} \quad (6)$$

$$\delta = \frac{\beta}{2\sqrt{J \cdot (k_\theta - K_m)}} \quad (7)$$

$$T_e = \frac{2 \cdot L}{R} \quad (8)$$

$$H(s) = \frac{\Theta(s)}{U(s)} \quad (9)$$

Considering the resistant moment  $M_s = 0$ , the transfer function of the couple motor between the exit size  $\theta$  and the control current  $i$  is:

$$\frac{\theta(s)}{I(s)} = \frac{\frac{K_i}{k_\theta - K_m}}{T_{em}^2 \cdot s^2 + 2\delta \cdot T_{em} \cdot s + 1} \quad (10)$$

The transfer function of the couple motor between the exit size  $\theta$  and the control tension  $u$  is:

$$\frac{\theta(s)}{U(s)} = \frac{\frac{K_i}{k_\theta - K_m}}{T_m \cdot T_{em}^2 \cdot s^2 + T_{em} \cdot (T_{em} + 2\delta \cdot T_e) \cdot s + \left( T_e \cdot \frac{K_\theta}{k_\theta - K_m} + 2\delta \cdot T_{em} \right) \cdot s + 1} \quad (11)$$

The stability condition of the couple motor is assured if  $K_m < k_\theta$ .

The time constants  $T_e$  and  $T_{em}$  are very little compared to the other constants of the servovalve ensemble, so as, in the current calculations the couple motor is considered as a strictly proportional element in the frequency field [0...1 000] Hz.

From the simplified expression of the transfer function:

$$\theta(s) = \frac{K_i}{k_\theta - K_m} I(s), \quad (12)$$

it results that for  $k_M - k_\theta$  the palette of the couple motor moves without having excited the couple motor. This phenomenon can be compared with a resonance phenomenon and must be avoided. Such thing is realised by the modification of the electrical gap between the fixed reinforcement and the mobile reinforcement of the couple motor for a flexible tube with the given rigidity  $k_\theta$ .

A smaller value of the nominator  $k_\theta - K_m$  leads to a bigger current sensibility of the couple motor.

## The Mathematical Shaping of the Mobile Ensemble of the Servovalve

The mobile ensemble of the servo valve is made up of the following elements:

- o the mobile reinforcement of the couple motor;
- o the palette;
- o the reaction rod;
- o the flexible tube.

The moments and the forces that operate the mobile ensemble are:

- o the electromagnetic couple generated by the couple motor;
- o the hydraulic forces generated by the fluid at the nozzles level;
- o the viscosity forces;
- o the reaction moment of the flexible supporting tube of the palette and of the mobile reinforcement;
- o the force of the distributor drawer on the reaction rod.

### The Hydraulic Force that Operates on the Palette

The hydraulic force that operates on the palette has two components: a hydrostatic component and a hydrodynamic component.

The hydrostatic force is generated by the differential pressure that operates on the palette:

$$F_{hsp} = (P_{p1} - P_{p2}) \cdot S_a \quad (13)$$

where:

$F_{hsp}$  - the hydrostatic force that operates on the palette [N];

$P_{p1}, P_{p2}$  - the pressure of the oil from the nozzle vents [N/m<sup>2</sup>];

$S_a$  - the surface of the fixed drain section of the nozzle [m<sup>2</sup>].

The hydrodynamic force that operates on the palette is given by the relation:

$$F_{hdp} = \frac{\rho}{2} \cdot \left( \frac{Q_{01}^2}{S_{01}} - \frac{Q_{02}^2}{S_{02}} \right) \quad (14)$$

where  $F_{hdp}$  is the hydrodynamic force that operates on the palette [N],

$\rho$  - the oil density [kg/m<sup>3</sup>],

$Q_{01}, Q_{02}$  - the debits through the variable apertures of the nozzle [m/s],

$S_{01}, S_{02}$  - the surfaces of the variable drain sections of the nozzle [m<sup>2</sup>].

The value of the hydraulic forces that operate on the palette is sensitively influenced by the form and the sizes of the drain aperture, by the drain regime through the apertures (laminar or turbulent), as well as by the position of the palette between the two nozzles.

The appearance of the sticking phenomenon of the palette to the nozzle sensitively modifies the general features of the servo valve and, due to this reason, different measures are foreseen to avoid this phenomenon even from the conception phase of the servo valve.

## The Equation of Static Stability for the Flexible Tube

The mobile ensemble of the servovalve is suspended on a flexible tube that assures the connection between the mobile reinforcement of the couple motor and the body of the servovalve, as well as the tightness of the couple motor.

The weight centre of the mobile part is considered to be situated on the axle of the mobile reinforcement of the couple motor. The mobile ensemble has a rotation movement and a translation movement.

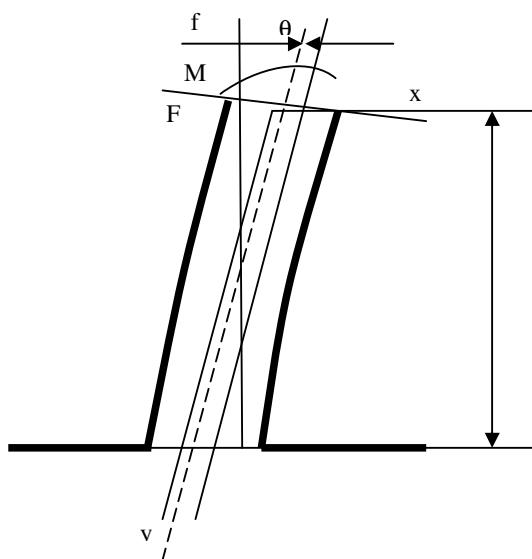


Fig. 2. The flexible tube of the servovalve

From the point of view of the material resistance, the flexible tube of the servo valve is a one end restraint bar, on which different forces and moments operate (fig. 2). These forces and moments have been reduced in the weight centre of the mobile ensemble that coincides to the free extremity of the tube. With the notations from the figure, the following relations can be written:

$$\begin{cases} f = \frac{M \cdot l^2}{2E \cdot I} + \frac{F \cdot l^3}{3E \cdot I} \\ \theta = \frac{M \cdot l}{E \cdot I} + \frac{F \cdot l^2}{2E \cdot I} \end{cases} \quad (15)$$

where  $f$  is the arrow of the free end of the flexible tube,

$\theta$  - the angle of the deformed middle fiber of the tube,

$E$  - the elasticity modulus of the material used to make the tube,

$I$  - the inertia moment of the transversal section of the tube,

$F$  - the resultant of the forces that operate on the tube, reduced in the weight centre of the mobile ensemble,

$M$  - the sum of the moments that operate on the flexible tube, reduced in the weight centre of the mobile ensemble,

$l$  - the length of the flexible tube.

If the system of equations (15) above is solved, it results:

$$\begin{aligned} F &= \frac{12E \cdot I}{l^3} \cdot f - \frac{6E \cdot I}{l^2} \cdot \theta \\ M &= \frac{6E \cdot I}{l^2} \cdot f - \frac{46E \cdot I}{l^2} \cdot \theta \end{aligned} \quad (16)$$

The relations (16) can also be written in the matrix form:

$$\begin{bmatrix} M \\ F \end{bmatrix} = \begin{bmatrix} K_1 & -K_2 \\ -K_2 & K_3 \end{bmatrix} \cdot \begin{bmatrix} \theta \\ f \end{bmatrix} \quad (17)$$

where:

$$K_1 = \frac{4 \cdot E \cdot I_p}{l}, \quad K_2 = \frac{6 \cdot E \cdot I_p}{l^2}, \quad K_3 = \frac{12 \cdot E \cdot I_p}{l^3}.$$

with the elements of the rigidity matrix expressed in [Nm/rad], [N], respectively [N/m].

The movement of the palette at the nozzle level is given by the relation:

$$X_p = f + l_1 \theta_p \quad (18)$$

where  $X_p$  is the palette movement,

$f$  - the movement of the weight centre of the reinforcement,

$l_1$  - the distance between the axle of the mobile reinforcement of the couple motor and the nozzles axle.

## The Movement Equation of the Distributor Drawer

The drawer of the distributor moves under the action of the following forces:

- o the control force;
- o the friction force;
- o the hydraulic forces;
- o the elastic force of reaction.

From the point of view of the dynamic behaviour of the drawer, only the longitudinal components of the forces mentioned above are interesting.

### The Control Force

The control force of the drawer is determined, based on the following calculation relation:

$$F_{cs} = A_s \cdot (P_{p1} - P_{p2}) \quad (19)$$

where  $F_{cs}$  is the control force of the drawer [N],

$A_s$  - the surface of the ends of the drawer [m<sup>2</sup>].

### The Friction Force

The dry friction forces that operate on the drawer are negligible, because between the socket and the drawer there is a quasi-uniform in oil.

The viscous friction forces are calculated by the relation:

$$F_{fvs} = K_{fv} \cdot X_s \quad (20)$$

$$K_{fv} = \frac{v \cdot \rho \cdot \pi \cdot D \cdot L}{J} \quad (21)$$

where  $F_{fvs}$  is the viscous friction force,

$K_{fv}$  - the coefficient of viscous friction [Ns/m],

$X_s$  - the speed of the drawer [m/s],

$v$  - the cinematic viscosity of the oil [m<sup>2</sup>/s],

$\rho$  - the oil density [kg/m<sup>3</sup>],

$D$  - the drawer diameter [m],

$L$  - the total contract length of the drawer with the socket [m],

$J$  - the radial free distance between the socket and the drawer [m].

## Hydraulic Forces

The hydraulic forces (radial and axial) that operate on the drawer appear because of the liquid drain through the distributor and they are called Bernoulli forces.

These forces operate both in drain stationary regime and also in transitory regime. In the design process, the analytical relations necessary for the determination of the values for these forces are obtained.

The stationary forces are calculated with the help of the impulse theorem, applied to a tube of current and they are given by the relation:

$$\rho \cdot Q \cdot (\bar{V}_1 - \bar{V}_2) = \Sigma F_{ext} \quad (22)$$

where  $\bar{V}_1$  and  $\bar{V}_2$  are the speeds of the fluid through the doors of the distributor,

$Q$  - the value of the debit that leads through the distributor,

$F_{ext}$  - the exterior forces that operate on the contour line of fluid volume.

With the notations from figure 3, the force used by the liquid jet operates on the drawer of the distributor and it is given by the relation:

$$F_0 = \rho \cdot Q_2 \cdot V_2 = \frac{\rho \cdot Q_2^2}{A_2} = \frac{\rho \cdot Q_2^2}{C_c \cdot A_0} \quad (23)$$

## The Linear Mathematical Model (transfer functions)

The transfer function of the servovalves represents the ratio between the Laplace transformation of the exit size (the debit at the exit coupling of the servovalve towards the hydraulic motor) and the Laplace transformation of its control current.

The calculation relation of the debit is:

$$Q = c_d \cdot X_s \cdot b \cdot \sqrt{\frac{P_s - P \cdot \text{sgn}(X_s)}{\rho}} \quad (24)$$

where  $b$  is the width of the distributor's window [m];

$P_s$  is the supply pressure of the servovalve [Pa];

$P$  represents the pressure decline between the couplings of the servovalve with the hydraulic motor [Pa].

## Conclusions

From the examination of the scheme we remark the fact that the value of the debit  $Q$  is proportional to the opening of the little drawer  $X_s$  and the amplification is constant.

In the same time we notice that the drain direction of the liquid through the servovalve depends on the direction of the current through the bobbins of the servovalve.

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## Modelarea servovalvelor electrohidraulice

### Rezumat

*Lucrarea prezintă un model original pentru proiectarea servovalvei de debit cu reacție de forță ce va putea determina, prin simulare numerică, performanțele dinamice ale unei servovalve. Motorul de cuplu fiind prin definiție un convertor electromecanic comportarea sa în regim dinamic este descrisă de un set de ecuații diferențiale, cu mărimi electromagnetice și de mișcare.*