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The Numeric Modelling and Simulation of the Mechano-hydraulic Servomechanisms

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Abstract

This paper presents the hacking systems of the servomechanisms that perform the automatic control of the position of a mechanism (platform, steer etc.) depending on the variation of the entrance size that can be of other natures than the mechanic nature.
The entrance size of the mechano-hydraulic of the servomechanism is represented by the command lever, and the exit size is represented by the position of the piston stick or the body of the hydraulic engine.

Key words: *Numeric modelling, Simulation, Mechano-hydraulic, Servomechanism.*

Introduction

Usually, the servomechanisms are supervision systems within the framework of which the automatic control of position for a mechanism is performed (platform, helm etc.), depending on the variations of an entrance measure that can also be of another nature than the mechanical one.

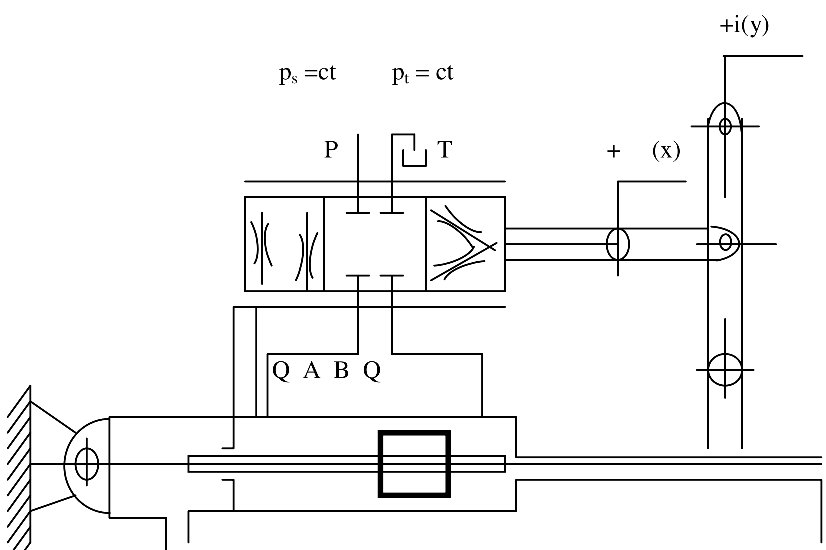

Fig. 1. Schematic diagram for a hydro-mechanical servomechanism

The schematic diagram for a hydro-mechanical servomechanism is presented in figure 1. In this paper the mathematical model of a servomechanism which has been installed in real operating conditions will be elaborated and its behaviour in dynamic regime by numerical simulation will be studied.

Mathematical Shaping

From the informational point of view, the hydro-mechanical servomechanisms have the following components:

- the comparison element (the mechanical comparator);
- the error amplification element (the hydraulic distributor);
- the execution element (the hydraulic motor).

The mathematical shaping of the hydro mechanical servomechanisms, whose scheme is presented in figure 2, consists of the writing of the following equations:

- the continuity equation;
- the equation of the mechanical comparator;
- the equation of the movement – debit – pressure converter;
- the movement equation for the hydraulic piston of the motor.

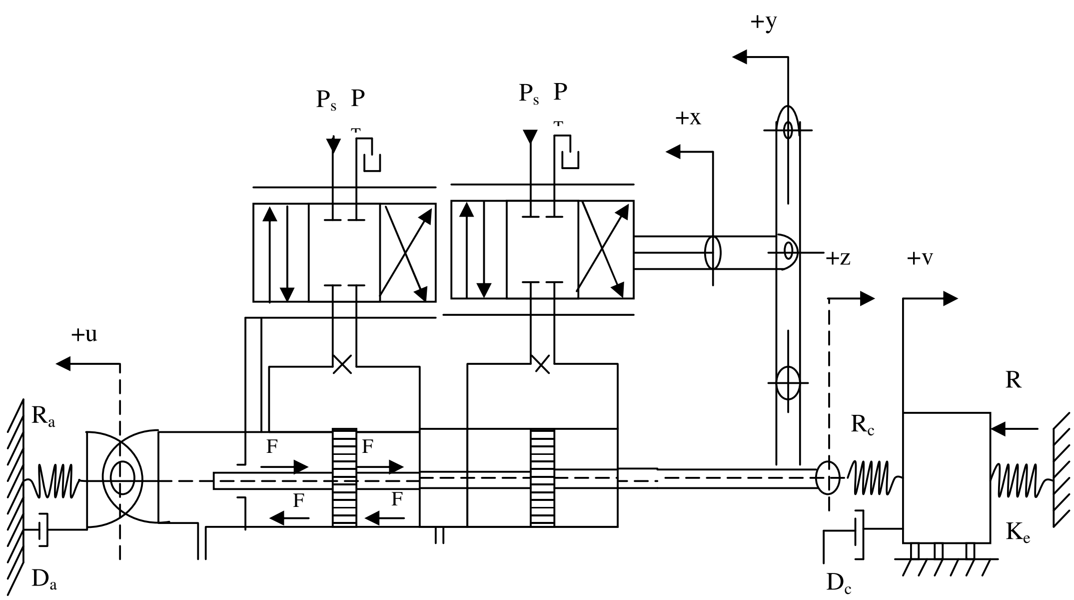

Fig. 2. Schematic diagram for the hydro mechanical servomechanism mounted in real operating conditions

The Continuity Equation

The Continuity Equation that Corresponds to the Movements from the Operating Hydraulic Systems

The continuity equation represents the mathematical expression for the weight conservation principle, applied to the fluid environment (continuous) in movement. In order to express this principle in a form which is specific for the operating hydraulic systems, a quantity of liquid, always made up of the same liquid particles, is defined as a liquid system.

If the liquid is homogenous, isotropic and if it has the isothermal elasticity mode *E*, then the continuity equation for a tube of elementary current (fig. 3) is:

$$\frac{\partial(\rho \cdot A)}{\partial t} + \frac{\partial(\rho \cdot A \cdot V)}{\partial s} = 0 \tag{1}$$

and the equation for the form of the liquid is:

$$\rho = \rho_0 \cdot e^{\frac{1}{E}\left(p - p_0\right)} \tag{2}$$

where A is the surface of the drain section,
V - the average speed of the liquid in the drain section,
ρ - the liquid density,
s - the space variable,
t - the time variable,
p - the liquid pressure.

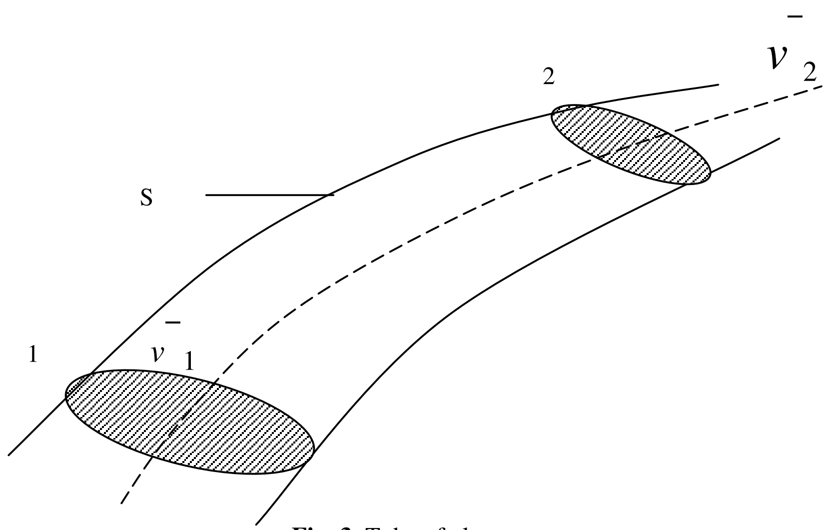

Fig. 3. Tube of elementary current

From the derivation of relations (1) and (2) we get the equations:

$$\frac{\partial(\rho \cdot A)}{\partial t} = \rho \cdot \frac{\partial A}{\partial t} + A \cdot \frac{\partial \rho}{\partial t} \tag{3}$$

$$\frac{\partial \rho}{\partial t} = \frac{\rho}{E} \cdot \frac{\partial p}{\partial t} \tag{4}$$

$$\frac{\partial p}{\partial t} = \frac{\partial p}{\partial t} + \frac{\partial p}{\partial s} \cdot \frac{\mathrm{d}s}{\mathrm{d}t} \tag{5}$$

From the equations (3), (4) and (5), it results:

$$\frac{\partial(\rho \cdot A)}{\partial t} = \rho \cdot \frac{\partial A}{\partial t} + \frac{A \cdot \rho}{E}\left(\frac{\mathrm{d}p}{\mathrm{d}t} - \frac{\partial p}{\partial s} \cdot \frac{\mathrm{d}s}{\mathrm{d}t}\right) \tag{6}$$

$$\frac{\partial(\rho \cdot V \cdot A)}{\partial s} = V \cdot A \cdot \frac{\partial \rho}{\partial t} + A \cdot \frac{\partial \rho}{\partial s} + \rho \cdot \frac{\partial(V \cdot A)}{\partial s} \tag{7}$$

$$\frac{\partial(\rho \cdot V \cdot A)}{\partial s} = V \cdot A \cdot \frac{\rho}{E} \cdot \frac{\partial \rho}{\partial s} + \rho \cdot \frac{\partial(V \cdot A)}{\partial s} \tag{8}$$

From the equations (1), (6) and (8), it results:

$$\rho \cdot \frac{\partial A}{\partial t} + \frac{A \cdot \rho}{E}\left(\frac{\mathrm{d}p}{\mathrm{d}t} - \frac{\partial p}{\partial s} \cdot \frac{\mathrm{d}s}{\mathrm{d}t}\right) + V \cdot A \cdot \frac{\rho}{E} \cdot \frac{\partial p}{\partial s} + \rho \cdot \frac{\partial(V \cdot A)}{\partial s} = 0 \tag{9}$$

$$\rho \cdot \frac{\partial A}{\partial t} + \frac{A \cdot \rho}{E} \cdot \frac{\mathrm{d}p}{\mathrm{d}t} + \frac{A \cdot \rho}{E}\left(V - \frac{\mathrm{d}s}{\mathrm{d}t}\right)\frac{\partial p}{\partial s} + \rho \cdot \frac{\partial(V \cdot A)}{\partial s} = 0 \tag{10}$$

Considering the walls of the tube of current as being rigid, the surface of the drain section does not modify while, as a consequence:

$$\frac{\partial A}{\partial t} = 0 \tag{11}$$

If we note with Q the liquid debit through the tube of current, then:

$$V \cdot A = Q \tag{12}$$

and also taking into consideration:

$$V = \frac{\mathrm{d}s}{\mathrm{d}t} \tag{13}$$

then the relation (10) becomes:

$$\frac{A \cdot \rho}{E} \cdot \frac{\mathrm{d}p}{\mathrm{d}t} + \rho \cdot \frac{\partial Q}{\partial s} = 0$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\frac{E}{A} \cdot \frac{\partial Q}{\partial s}$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{E}{\vartheta} \cdot (Q_1 - Q_2) \tag{14}$$

where ϑ is the liquid volume from the liquid system,

Q_1 -the liquid debit through the section 1,

Q_2 - the liquid debit through the section 2.

The elasticity mode E of the liquids used in the hydraulic operating systems varies in very large limits depending on their air content. In order to determine the performances of the hydro-mechanical servomechanisms, in the design phase or in the analyses phase, different values of the elasticity mode comprised in the interval 7000...14000 bars are used, the value of 14000 bars corresponding to the oil which does not contain dissolved gases.

The Equation of the Mechanical Comparator

For the servomechanism from the presented figure, the comparator is a mechanic lever with three articulations. The functional dependency between the characteristic signs of the servomechanism is expressed by the relation:

$$x(y,z) = y \cdot \mu y - z \cdot \mu z - \lambda \cdot u \tag{15}$$

the notes having the following meanings:
y - the entrance signal (lever movement at the level of the coupling articulation with the controlling rod of the servomechanism);
z - the exit signal (lever movement at the level of the coupling articulation with the exit rod of the servomechanism);
x - the control signal of the drawer (the lever movement at the level of the coupling articulation with the servo mechanism drawer);
$\mu y = \frac{b}{a+b}$ - cinematic amplification factor of the entrance signal;
$\mu y = \frac{a}{a+b}$ - cinematic amplification factor of the exit signal;
u – the movement of the servomechanism body because of the finite rigidity of the anchoring elements and of the structure;
λ - the coefficient that represents the movement influence of the servo mechanism body upon the operating cinematic chain of the drawer.

When the comparator is a symmetrical lever ($a = b$), then:

$$\mu y = \mu z = \mu \tag{16}$$

and the relation (15) becomes:

$$x(y,z) = \mu \cdot (z - y) = \mu \cdot \varepsilon \tag{17}$$

where $\varepsilon = y - z$.

The Condition of Stationary Regime of the Hydraulic Distributors

From the point of view of the automatic systems, the hydraulic distributors are movement - debit – pressure converters, whose static features are expressed through the relation:

$$F(P_s, x, Q_m, P) = 0 \tag{18}$$

where P_s represents the liquid pressure in the distributors' supply coupling,
x - the movement of the drawer,
Q_m - the debit spent by the operating hydraulic motor,
P - the pressure decline in the hydraulic motor.

For the hydraulic distributor, whose scheme is presented, we can write the following relations:

$$Q_1 = c_{d1} \cdot A_1(x) \cdot \sqrt{\frac{2(P_s - p_1)}{\rho}} \tag{19}$$

$$Q_2 = c_{d2} \cdot A_2(x) \cdot \sqrt{\frac{2(P_s - p_2)}{\rho}} \tag{20}$$

$$Q_3 = c_{d3} \cdot A_3(x) \cdot \sqrt{\frac{2(P_2 - p_r)}{\rho}} \tag{21}$$

$$Q_4 = c_{d4} \cdot A_4(x) \cdot \sqrt{\frac{2(p_1 - p_r)}{\rho}} \tag{22}$$

$$Q_m = Q_1 - Q_4 \tag{23}$$

$$Q_m = Q_1 - Q_2 \tag{24}$$

$$P = p_1 - p_2 \tag{25}$$

where Q_i is the debit through the "i" slot of the distributor,

Q_m - the debit spent by the motor,

$A_i(x)$ - the surface of the "i" slot of the distributor,

c_{di} - the debit coefficient of the "i" slot of the distributor,

p_r - the liquid pressure at the distributor's return coupling.

For a distributor with copulated symmetrical distribution slots and with equal debit coefficients, the debit supplied by the distributor for the motor can be calculated by the relation:

$$Q_m = Q_m(x, P) = c_d \cdot A(x) \cdot \sqrt{\frac{(P_s - P)}{\rho}} \tag{26}$$

The development in Taylor series of the relation 26 around a functioning point „o" leads to the following relation:

$$Q_m - Q_{mo} = \Delta Q = \left(\frac{\partial Q_m}{\partial x}\right)_o \cdot \Delta x + \left(\frac{\partial Q_m}{\partial P}\right)_o \cdot \Delta P + ... \tag{27}$$

The following coefficients are defined:

a) the debit amplification factor for the distributor:

$$K_{Qx} = \frac{\partial Q_m}{\partial x} \tag{28}$$

b) the debit – pressure coefficient;

$$K_{Qp} = \frac{\partial Q_m}{\partial P} \tag{29}$$

c) the pressure – movement coefficient:

$$K_{px} = \frac{\partial P}{\partial x} = \frac{K_{Qx}}{K_{Qp}} \tag{30}$$

With these notes the relation 22 becomes:

$$\Delta Q = K_{Qx} \cdot \Delta x - K_{Qp} \cdot \Delta P \tag{31}$$

In the paper, it is presented the mathematical model of the behaviour in a dynamic regime for the hydraulic distributor. The transfer function that describes this behaviour is:

$$H_A(s) = \frac{X(s)}{I(s)} = \frac{K_A}{\frac{s^2}{\varpi_n^2} + \frac{2\xi}{\varpi_n} + 1} \tag{32}$$

where $\varpi_n = \sqrt{\frac{K_s}{M_s}}$ is the pulsation of the free oscillations,

$\xi = \frac{1}{2} \cdot \frac{f_S}{\sqrt{K_S \cdot M_S}}$ is the amortisation factor,

K_A is the amplification factor,

M_s is the drawer weight,

K_s is the constant of the hydraulic main spring that operates upon the drawer

f is the coefficient of viscous friction.

The Movement Equations

The pressure force generated by the servomechanism is:

$$F = 2 \cdot P \cdot A_p \tag{33}$$

The pressure force generated on the covers of the servomechanism body stresses its body and implicitly the structure of the work machine. The movement equation of the servomechanism body is given by the relation:

$$F = R_a \cdot u + D_a \cdot u + m_{sv} \cdot u \tag{34}$$

where R_a is the anchoring rigidity of the servomechanism in the structure,

D_a - the amortisation coefficient of the structure,

m_{sv} - the weight of the servomechanism body,

u - the movement of the servomechanism body.

Because the weight m_{sv} of the servomechanism body is by far less than the weight of the operated machine, this can be neglected, in such a way that the relation (34) becomes:

$$F = R_a \cdot u + D_a \cdot u \tag{35}$$

The Movement Equation of the Servomechanism Piston

The pressure force that operates the servomechanism piston impresses to it a movement that can be determined by the relation:

$$F = m_p \cdot z + K_{fv} \cdot (z + u) + R_c \cdot (z\text{-}v) + D_c \cdot (z\text{-}v) \tag{36}$$

where R_c is the rigidity of the connection mechanism, connection between the servomechanism and the operated element (charge),

D_c - the amortisation coefficient of the connection mechanism,

z - the piston movement,

v - the weight movement for the operated element (charge),

K_{fv} - the coefficient of viscous friction,

m_p - the piston weight.

For the calculation of the friction force generated by the fittings with built – up barrel form, the following relation is recommended:

$$F_{fg} = 0{,}03\mu \cdot p \cdot \pi \cdot D_2 \cdot W \cdot d \tag{37}$$

where F_{fg} - the friction force caused by the fittings,

μ - the coefficient depending on the rubber's hardness and the pressure difference p,
p - the pressure difference between the fittings' tight rooms,
D_1 - the diameter of the canal bottom where the fitting is mounted,
D_2 - the diameter of the tight cylinder,
d - the diameter of the fitting's ring surface,

$$W = \frac{d-h}{d} 100\% \,,$$

$$h = \frac{D_2 - D_1}{2} \,.$$

Conclusions

The illustrated model shows that for the calculations practiced by the measurement, the use, is first generated by the hydraulic motor. This value of the friction force covers the definition for the servomechanism performances.

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Modelarea şi simularea numerică a servomecanismelor mecano-hidraulice

Rezumat

În lucrare sunt prezentate modelele sistemelor cu servomecanisme care realizează controlul automat al poziţiei unui mecanism (platformă, cârmă etc) în funcţie de variaţiile unei mărimi de intrare care poate fi şi de altă natură decât mecanică. Mărimea de intrare a servomecanismului mecanohidraulic o reprezintă poziţia levierului de comandă, iar mărimea de ieşire o reprezintă poziţia tijei pistonului sau a corpului motorului hidraulic.