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# The Study of the Dynamic Stability Curves of the MU200 Mast

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## Abstract

*This article presents the most important aspects of the dynamic stability curves of the MU200 mast. These dynamic stability curves represent a great instrument which is used in order to determine the area in which the MU200 mast can operate in safe conditions. After the presentation of the theoretical aspects, the stability chart of the MU200 mast is analysed. Also, a new method for the study of the stability chart of MU200 mast is presented. The way this study is conducted can easily be applied to every mast used in the petroleum industry.*

**Key words:** *dynamic stability, dynamic stability curves, instability zone, stability chart.*

## Theoretical Aspects of the Dynamic Stability Analysis

For the study of the dynamic stability curves, the structure is considered under the action of periodic axial forces of form:

$$P(t) = P_s + P_d \cos(\omega t) \ , \tag{1}$$

where $P_s$ is the static component and $P_d$ is the dynamic component of the axial forces.
The relation (1) can be written depending on the value of critical charge of miss the stability, as follows:

$$P(t) = \alpha P_{cr} + \beta P_{cr} \cos(\omega t) \tag{2}$$

with $\alpha$ and $\beta$ coefficients in the (0…1) domain.

For the case of the act of axial periodic forces of form (2), the equations of motion system take the form [1]:

$$\underset{\sim}{M}\, \ddot{\underset{\sim}{\eta}} + \left[\underset{\sim}{R}^E - \alpha P_{cr}\, \underset{\sim}{R}^G - \beta P_{cr}\, \underset{\sim}{R}^G \cos(\omega t)\right] \underset{\sim}{\eta} = 0 \ . \tag{3}$$

The system of equations (3) represents a system of differential equations of the order two with periodic coefficients of Mathieu type.

In accordance to the theory of differential equations with periodic coefficients, the limit among the stable regions and the one unstable can be built with the help of periodic solutions of period $2T$ and $T$ [2, 3], where $T$ is the period of the periodic axial forces.
Because the solutions of period $2T$ represent a big practical importance [2, 3], the solution of the system (3) is chosen as a solution of period $2T$.

Consequently, the system (3) has the following solution [2, 3]:

$$\underset{\sim}{\eta} = \underset{\sim}{A}\sin\left(\frac{\omega t}{2}\right) + \underset{\sim}{B}\cos\left(\frac{\omega t}{2}\right) \tag{4}$$

By entering the relation (4) in the system (3) and grouping terms in $\sin\left(\frac{\omega t}{2}\right)$ and $\cos\left(\frac{\omega t}{2}\right)$ a set of algebraic homogeneous equations in $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is obtained.

The condition for this set of algebraic homogeneous equations to have solutions is:

$$\begin{vmatrix} \underset{\sim}{R}^E - \alpha P_{cr}\underset{\sim}{R}^G - \frac{\beta}{2}P_{cr}\underset{\sim}{R}^G - \frac{\omega^2}{4}\underset{\sim}{M} & 0 \\ 0 & \underset{\sim}{R}^E - \alpha P_{cr}\underset{\sim}{R}^G + \frac{\beta}{2}P_{cr}\underset{\sim}{R}^G - \frac{\omega^2}{4}\underset{\sim}{M} \end{vmatrix} = 0. \tag{5}$$

The equation (5) is best known as the equation of the dynamic stability zones and will be used in order to obtain the dynamic stability curves for the MU200 mast.

Through particularisation of the equation (5), the particular cases of dynamic stability analysis is obtained: the free vibration and the static loss of stability.

In the case of free vibrations no axial forces are present, so $\alpha = \beta = 0$. Also, the following notation takes place: $p = \frac{\omega}{2}$.

Consequently, the equation (5) takes the form:

$$\left|\underset{\sim}{R}^E - p^2\underset{\sim}{M}\right| = 0, \tag{6}$$

which represents the natural frequency equation of free vibration case.

In the case of the static loss of stability, no vibrations are present, so $\alpha = 1$ and $\beta = \omega = 0$. With these notations, the equation (5) becomes:

$$\left|\underset{\sim}{R}^E - P_{cr}\underset{\sim}{R}^G\right| = 0, \tag{7}$$

which represents the critical value equation of the static loss of stability.

So, the relation (5) contains the correct cases of free vibrations and the static loss of stability. This thing confers certainty as the relation (5) is correctly based and permits the easily determination of the zones in which the structure can operate in complete safety.

## Dynamic Stability Analysis of the MU200 Mast

For the MU200 mast, the BOLOTIN method will be applied [2] with the aim to obtain the dynamic stability curves. In accordance to this method, the curves of dynamic stability are obtained through the solution of the equation (5).

For the BOLOTIN method to be applied, the MU200 mast is considered to be schematised just as arisen from figure 1.

Regarding the schema presented in figure 1, the following elements are known:

- o the mast mass is concentrated at the ends and the middle of the sections: $m_I/2$=4550 kg; $m_I/4+m_{II}/4$=4880 kg; $m_{II}/2$=5210 kg and $m_{II}/4$=2605 kg;
- o the spans length is: $l_I/2$=7 m; $l_{II}/2$=13.5 m.

The geometric features which were considered for the sections 1, 2 and 3 of the MU200 mast are represented in figure 2, as follow:

- o for the section 1, it was considered the cross-section represented in figure 2 a;

- o for the sections 2 and 3, the cross-section represented in figure 2 b was considered.

Using the finite element method, the natural frequencies and the critical buckling parameter for the MU200 mast, as represented in figure 1, are obtained:

- o the first 4 frequencies are: $p_1$=6,2694 rad/s; $p_2$=28,659; $p_3$=69.9 rad/s; $p_4$=107,587 rad/s;
- o the axial force at which the buckling appears: $P_{cr}$=3251,69 kN.

The simple schematisation of the MU200 mast permitted to obtain the solution of the equation (5) and the critical values ($\beta$, $\frac{\omega}{2p}$) which determine the limits of the dynamic stability zones.

Because in the equation of the stability curves (5) there are, besides the ($\beta$, $\frac{\omega}{2p}$) parameters, another two influences (influence of the $\alpha$ parameter and the influence of the natural frequencies), for the study of the curves of stability the following approach is adopted:

- o the variation of the stability zones depending on the natural frequencies (the parameter $\alpha$ is maintained constantly) is represented;
- o the variation of the stability zones depending on the $\alpha$ parameter is represented.

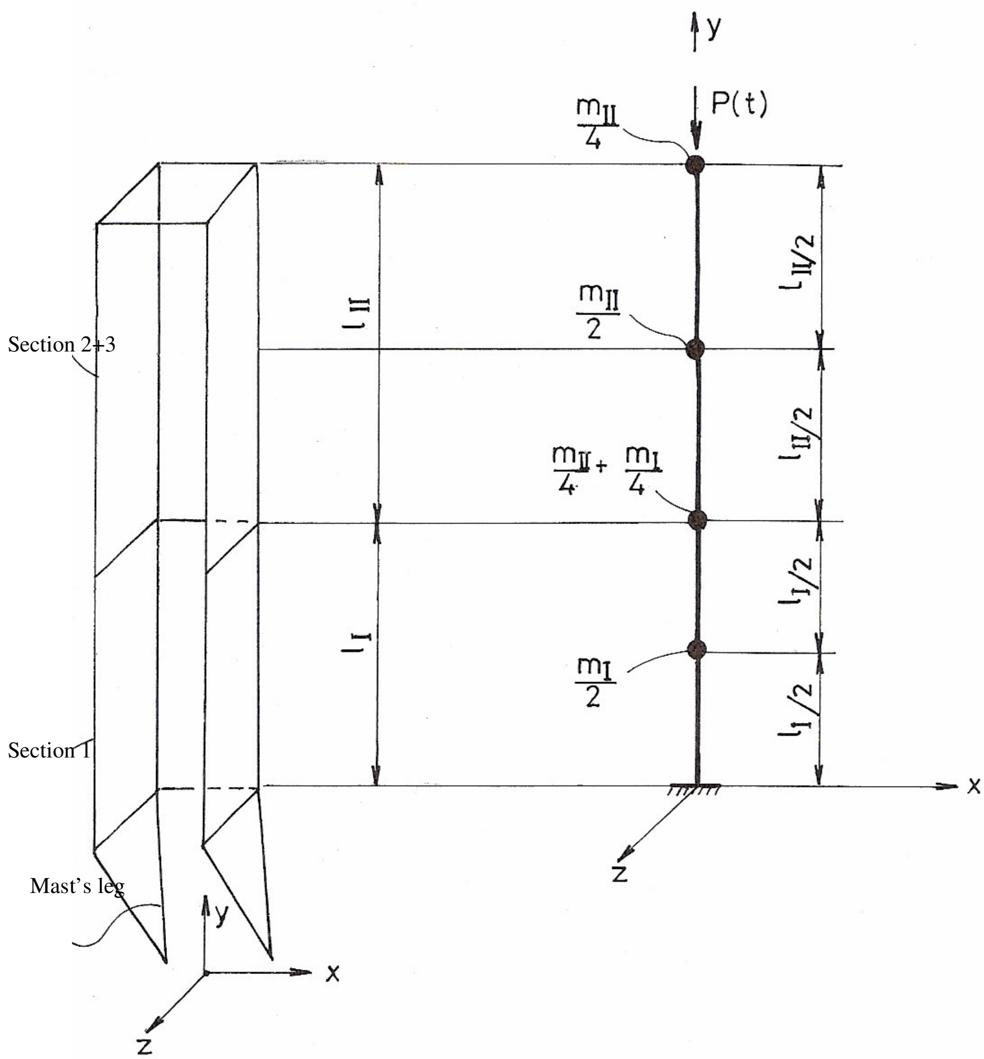
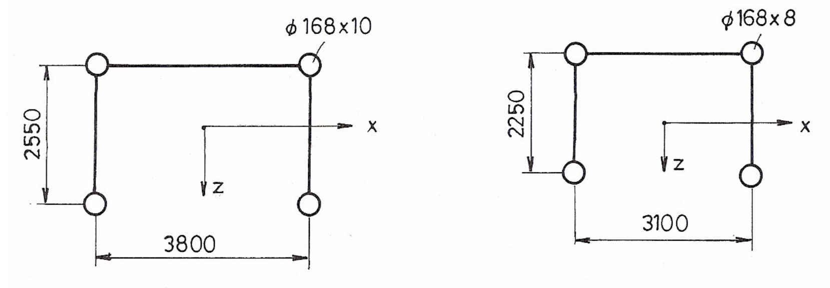



**Fig. 1.** MU200 mast schema




**a)** **b)**

**Fig. 2.** MU200 cross sections

## The Influence of the Natural Frequencies

It is well known [2] that as loss of the dynamic stability, i.e. the quick augmentation of the amplitude of the oscillations, is in progress with the values of the parameter $\frac{\omega}{2p_i} = \frac{1}{i}$. In this way, an infinity of areas of instability is obtained, among which first (the fundamental area) is the most important. Consequently, for the $\omega$ parameter (the frequency of the perturbation force) the following expression is adopted $\omega = 2p$ .

The algorithm used to obtain the instability chart, in this case, for the MU200 mast is presented in [1]. Using the above mentioned algorithm, it was possible to obtain the stability chart of the MU200 mast, stability chart which is presented in figure 3. The algorithm applies only onto the first three natural frequencies because it is observed from the analysis in figure 3 that the areas of instability get narrower as the natural frequencies grow becoming insignificant when reaching values of natural frequencies superior to the natural frequency 3.

Inside the zones delimited by the curves of stability, the MU200 mast operates unstably. Consequently, these zones must be avoided.

## The Influence of the α Parameter

For this case, the zones of stability are obtained for the first natural frequency. Also, for the parameter $\omega$ (the frequency of the perturbation force) the following expression is adopted $\omega = 2p$ .

The algorithm used to obtain the influence of the α parameter to the instability chart for the MU200 mast is presented in [1]. Using the above mentioned algorithm, it was possible to obtain the influence of the α parameter to the stability chart of the MU200 mast as shown in figure 4.

From the analysis of the figure 4, it is noticed that the static component of the perturbation force decreases the sprocket of the zones of instability, without modifying the form of the instability zones.

# Conclusions

Analysing the pictures 3 and 4, the following conclusions can be drawn:

- as the natural frequencies augment, the zones of instability are narrowed, the most dangerous frequencies being the first two, because these frequencies present the most stretched zones of instability behaviour;

- inside the zones delimited by the curves of stability, the MU200 mast operates unstably. The zones of stable operation are found outside these zones;
- the static component of the perturbation force decreases the sprocket of the zones of instability, without modifying the form of the instability zones;
- for a static component of the perturbation force increasing from 10% of the critical buckling force to 50%, the zones of instability are displaced with approximate 5% down.

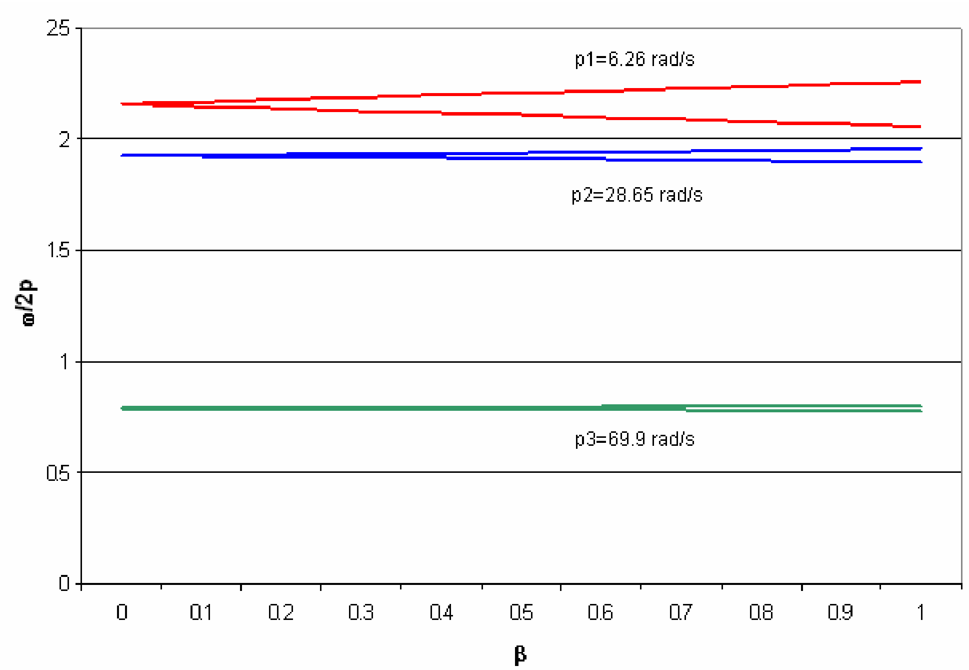



**Fig. 3.** Stability chart for the MU200 mast

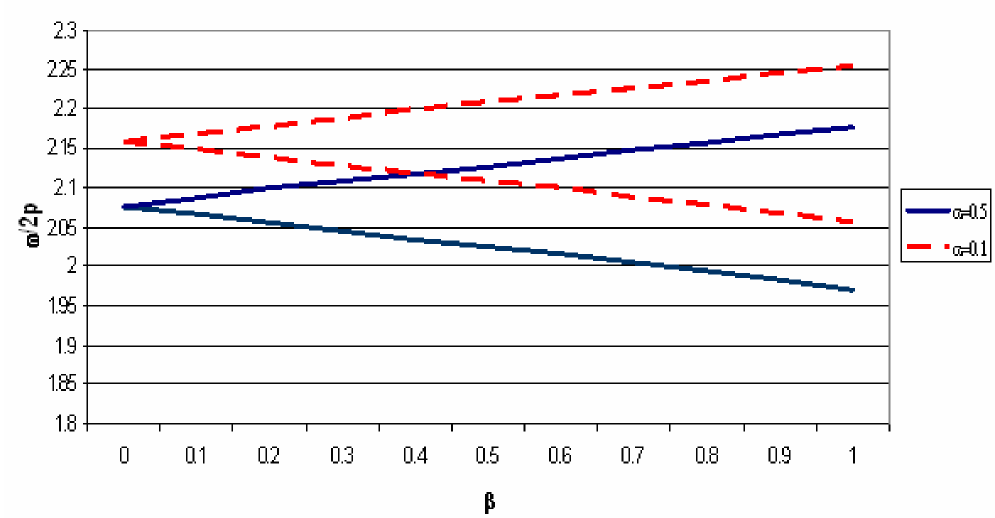



**Fig. 4.** Influence of the α parameter on the stability chart for the MU200 mast

This article presents an efficient method in order to study the behaviour of the way a structure operates, using the curves of dynamic stability. The presented algorithm used to obtain the zones of stability for the MU200 mast is easy to be transposed into practice through implementation into computing programs.

This paper has presented a study method of the zones of dynamic stability of the MU200 mast.

These zones of stability are of big practical importance because they specify the zones in which the structure (MU200 mast) can operate in the safe seat conditions.

## References


1. Gherghe, Gh., *Contribuţii privind stabilitatea dinamică a masturilor*, Teza de doctorat, Universitatea Petrol-Gaze Ploieşti, 2006.
2. Bolotin, V.V., *Kinetische Stabilitat Elasticher Systeme*, Berlin, 1961.
3. Marinov, R., *Probleme de stabilitate dinamica in constructii*, Editura Tehnică, Bucureşti, 1985.
4. Crisfield, M.A., *Non-linear finite element analysis of solids and structures*, 1991.
5. Zienkiewicz, O.C., *The Finite Element Method in Engineering Science*, McGraw-Hill, London, 1971.
6. Timoshenko, S., Gere J., *Teoria stabilităţii elastice*, Editura Tehnică, Bucureşti, 1967.


# Studiul curbelor de stabilitate dinamică ale mastului MU200

## Rezumat


*In cadrul acestui articol sunt prezentate cele mai importante aspecte ale studiului curbelor de stabilitate dinamică ale mastului MU200. Pe baza acestor zone de stabilitate dinamică sunt stabilite condiţiile în care mastul MU200 poate să funcţioneze în condiţii de siguranţă. Ţinând cont de importanţa acestor zone de stabilitate, se propune un algoritm care permite obţinerea curbelor de stabilitate dinamică pentru mastul MU200, algoritm uşor de implementat în practică. Acest mod de analiză prezentat poate fi cu uşurinţă extins la toate masturile de foraj din industria petrolieră.*