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A Research Study on Estimating the Performance of an Oil Reservoir with Active Aquifer

Ion Crețu, Eugen Mihail Ionescu, Aurelian Dobrescu

Universitatea Petrol-Gaze din Ploieşti, Bd. Bucureşti, 39, Ploieşti
e-mail: i_cretu@upg-ploiesti.ro

Abstract

Hydrocarbon reservoirs associated with aquifers having significant elastic expansion capacities can be managed so that high recovery factor values are obtained. To accomplish this task, it is necessary to recognize the action of the aquifer early in the reservoir's producing life, and also to properly estimate the water influx. This paper presents a procedure for calculating the performance of an oil reservoir with active aquifer and initial gas cap. The algorithm is illustrated by a case study.

Key words: *oil reservoir, active aquifer, cumulative water influx, oil recovery factor*

Basic Aspects

An oil reservoir with active aquifer or with elastic water drive is characterized by a hydraulic connection with a porous rock saturated with water that is called aquifer, in conditions in which the respective aquifer is set, partially or entirely, under the reservoir.

The importance of the aquifer water advance by elastic relaxation as a production mechanism of the adjacent oil reservoir consists in the degree in which this advance is put in evidence and used prior to the stage of the mentioned reservoir depletion by solution gas drive. Consequently, it is necessary to take into account not only the ability of recognizing the relatively early presence in the reservoir's life of the natural water influx, but also the possibility of evaluating this influx [1].

While the reservoir pressure decreases as a result of the oil production, water, as a compressible liquid, expands, generating a water-sweep process on the reservoir–aquifer boundary [2]. The reservoir energy is also supplemented by the aquifer rock compressibility. When the reservoir has large dimensions and contains a quantity of energy which is big enough, the whole reservoir can be swept by water through properly managing fluid production rates.

In some reservoirs with active aquifers it is possible to obtain values of the oil recovery efficiency ranging between 70% and 80%. The reservoir geology, the heterogeneity and the structural position constitute important variables which affect the oil recovery efficiency in each specific case.

Reservoirs with strong water push were found all over the world. Thus, we may mention the oil field East Texas, the reservoirs Arbuckle from Kansas, Tensleep from Wyoming and Titu from Romania.

To estimate the performance of an oil reservoir with active aquifer and initial gas cap, the macroscopic material balance equation can be written under the form [3]

$$N = \frac{N_p\left[b_o + \left(R_p - R_s\right)b_g\right] - W_e + W_p b_w}{b_o - b_{oi} + \left(R_{si} - R_s\right)b_g + \dfrac{r\,b_{oi}}{b_{gi}}\left(b_g - b_{gi}\right)}\,, \tag{1}$$

where

$$r = \frac{G\,b_{gi}}{N\,b_{oi}}\,, \tag{2}$$

N is the oil resource, N_p – the cumulative oil production, G – the gas resource from the gas cap, W_e – the cumulative water influx in the reservoir at a given exploitation moment, b_o, b_g, b_{oi}, b_{gi} – the volume factors of oil and gas respectively, at a given time t or in the initial moment i, R_s, R_{si} – solution gas–oil ratios at time t and at $t = 0$ respectively, R_p – produced gas–oil ratio.

The use of equation (1) to estimate the reservoir production evolution involves the availability of production and pressure data, PVT analyses, knowing oil and gas (from gas cap) resources N and G respectively, as well as water influx W_e and cumulative water production versus time.

To determine the cumulative water influx W_e it is necessary to solve the fundamental equations of aquifer water flow, represented by the filtration equation, the microscopic continuity equation and the equation of state written as [4, 5]

$$\vec{v} = -\frac{k}{\mu}\nabla p\,, \tag{3}$$

$$\nabla\left(\rho\vec{v}\right) + m\frac{\partial\rho}{\partial t} = 0\,, \tag{4}$$

$$\rho = \rho_0\, e^{\beta(p - p_0)}\,, \tag{5}$$

where $\vec{v}$ is the filtration velocity expressed as a vector, k – the permeability, μ – the dynamic viscosity of water, ρ_0 – the water density in isothermal reservoir conditions, β – the compressibility coefficient of water, m – the porosity and ∇ – the nabla operator having the expression

$$\nabla = \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} + \vec{k}\,\frac{\partial}{\partial z}\,. \tag{6}$$

If the porous medium is homogeneous and isotropic, and the fluid properties μ and β present negligible variations, the system of equations (3), (4) and (5) is reduced to the form

$$\Delta p = \frac{1}{a}\frac{\partial p}{\partial t}\,, \tag{7}$$

which constitutes a Fourier type equation, where Laplace's operator Δ has the expression

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{8}$$

and

$$a = \frac{k}{m\mu\beta} \tag{9}$$

is the hydraulic piezo-conductibility coefficient.

If the aquifer and the oil reservoir have comparable dimensions, the cumulative water influx at a given time can be expressed, according to the definition equation of the compressibility coefficient, as

$$W_e = \beta_t W_i (p_i - p), \tag{10}$$

where W_i is the initial volume of water in the aquifer, p_i – the reservoir initial pressure, p – the current pressure at the water–oil front, and the total compressibility coefficient β_t has the expression

$$\beta_t = \beta_a + \beta_r , \tag{11}$$

in which β_a is the water compressibility coefficient, and β_r corresponds to the rock pores.

In the case of aquifers of large dimensions compared with those of the oil region, in order to solve equation (7) the macro-well model introduced by van Everdingen and Hurst [6] can be used, by referring to Laplace transformation, obtaining the following formula for W_e, by admitting a constant pressure drop Δp at aquifer's boundary:

$$W_e = 2\pi m \beta_t h r_d^2 \Delta p \overline{W}(\bar{t}), \tag{12}$$

where

$$\overline{W}(\bar{t}) = \frac{4}{\pi^2} \int_0^\infty \frac{\left(1 - \mathrm{e}^{-u^2 \bar{t}}\right) \mathrm{d}u}{u^3 \left[J_0^2(u) + Y_0^2(u)\right]}, \tag{13}$$

$$\bar{t} = \frac{k\,t}{m \mu \beta r_d^2}, \tag{14}$$

$$r_d = \sqrt{A/\pi}, \tag{15}$$

J_0 – Bessel function of first species zero order, Y_0 – Bessel function of second species zero order, and A – oil region area.

If the aquifer has large but finite dimensions, a specific $\overline{W}(\bar{r}_e, \bar{t})$ relationship is established, whose application is facilitated by the diagrams presented in [3], pp. 275…276 for various values of the dimensionless radius

$$\bar{r}_e = r_e / r_d , \tag{16}$$

including for $\bar{r}_e = \infty$, where r_e is the external radius of the aquifer boundary approximated as being of circular shape. As an illustration, in Figure 1 one on the mentioned diagrams is shown.

In these conditions, equation (12) can be written as:

$$W_e = U \Delta p \overline{W} , \tag{17}$$

where

$$U = 2\pi f m \beta h r_d^2 , \tag{18}$$

$$f = \frac{\alpha}{360°}, \tag{19}$$

and α is the angle (in degrees) corresponding to the reservoir edge by means of which the water gets into the reservoir.

Eq. (17) can also be applied for a finite linear aquifer–reservoir system, but in this case we have:

$$U = b\,l\,h\,m , \tag{20}$$

$$\bar{t} = \frac{k\,t}{m \beta \mu l^2}, \tag{21}$$

and $\overline{W}$ can be read in Figure 5.9 [3], on the curve for finite one-dimensional aquifer.

If the average reservoir pressure varies continuously, the curve of this pressure will be approximated using a step variation and by applying the superposition principle, the following formula is obtained:

$$W_e = U \sum_{j=0}^{n=1} \Delta p_j \overline{W}\left(\bar{r}_e, \bar{t} - \bar{t}_j\right), \tag{22}$$

where $\Delta p_j = p_j - p_{j-1}$, with $\Delta p_0 = p_i - p_0$ and $t_0 = 0$.

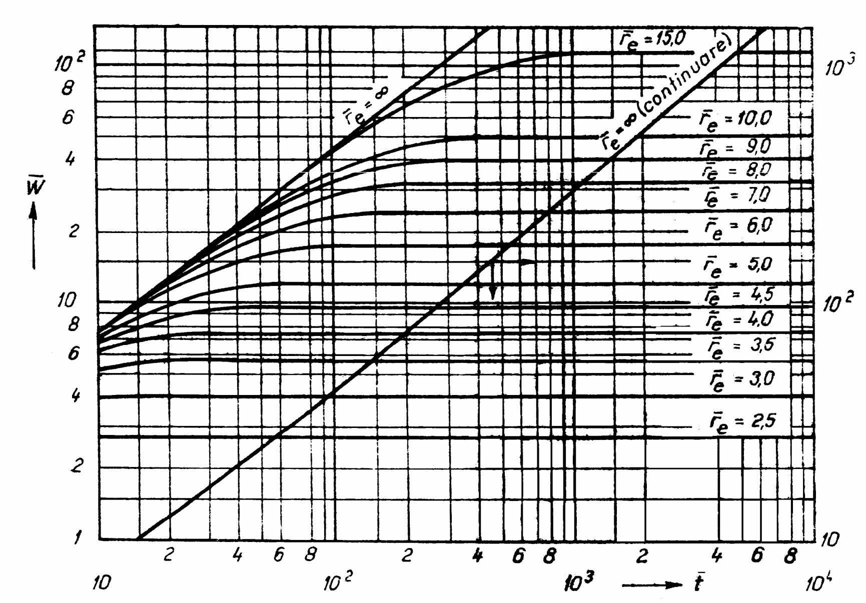

Fig. 1. Plot of $\overline{W}\left(\bar{r}_e, \bar{t}\right)$ for $10 \le \bar{t} \le 10^4$ and $\bar{r}$ ranging from 1.5 to ∞.

Case Study

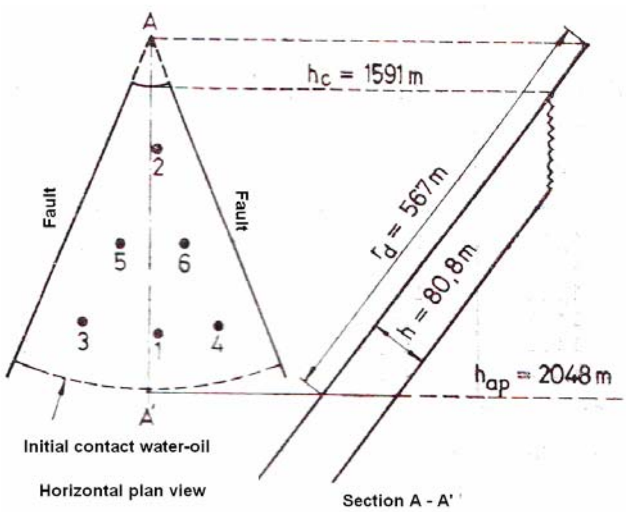

Fig. 2. Reservoir configuration for the case study

Let us consider an oil reservoir having the configuration in Figure 2 and the initial pressure p_i = 17.273 MPa [7]. The reservoir raw volume increases with depth as shown in Figure 3 and the oil PVT data are listed in Table 1. The aquifer estimated coefficient is U = 0.00519 m^3/Pa.

The six wells in Figure 2 produced initially at a flow rate Q = 119.25 m^3/(day·well), with a constant average dynamic pressure p_s = 14.8238 MPa.

The average oil saturation in the water-flooded area has been estimated as having the value s_{o0} = 0.40. The oil resource N = 2.6855·10^6 m^3, permeability k = 100 mD, interstitial water saturation s_{wi} = 0.22, oil, rock and water compressibilities $\beta_o = 11.6 \cdot 10^{-10}$ Pa^{-1}, $\beta_r = 5.8 \cdot 10^{-10}$ Pa^{-1}, $\beta_w = 4.35 \cdot 10^{-10}$ Pa^{-1}, water viscosity μ = 0.3 mPa·s and porosity m = 0.25 are also known.

The relative permeabilities can be approximated by means of Wyllie's relation corresponding to unconsolidated aquifer sands.

Our aim is to estimate the cumulative oil and gas output after 500 days, admitting that the water has not been produced by wells and the initial saturation of free gas has been zero. We also take into consideration the fact that the aquifer has an infinite action, the initial transition zone being negligible, the displacement is piston-like, the water cones and digitations miss and from the practical point of view, we cannot take into consideration a time step longer than 50 days.

Table 1. PVT and viscosity data for the hydrocarbon system

p, Pa	b_o	R_s, m_N^3/m^3	b_g	μ_o/μ_g	μ_o, mPa·s
17.237.000	1.325	115.76	0.004469	56.60	0.38
15.858.040	1.311	110.06	0.004733	61.46	0.39
14.479.080	1.296	104.36	0.005093	67.35	0.40
13.100.120	1.281	98.49	0.005621	74.33	0.41
11.721.160	1.266	92.61	0.006379	81.96	0.42
10.342.200	1.250	86.55	0.007496	91.56	0.43

Given the above mentioned conditions, in order to predict the oil reservoir performance, we need to do the following:

a) To estimate the oil reservoir average pressure in the un-invaded area of the reservoir at the end of the next forecast period. For this purpose, the average reservoir pressure plot is extrapolated until the end of the respective evaluation period.

b) To calculate the water cumulative influx W_{en} using relation (22) written as

$$W_{en} = U\left[\sum_{j=1}^{n-1} \Delta p_j \, \overline{W}\left(\bar{r}_e, \bar{t} - \bar{t}_j\right) + \right. \left. + \frac{p_{n-1} - p_{n+1}}{2} \overline{W}\left(\bar{r}_e, \bar{t} - \bar{t}_n\right)\right], \tag{23}$$

where, admitting that the pressure varies linearly, we considered that

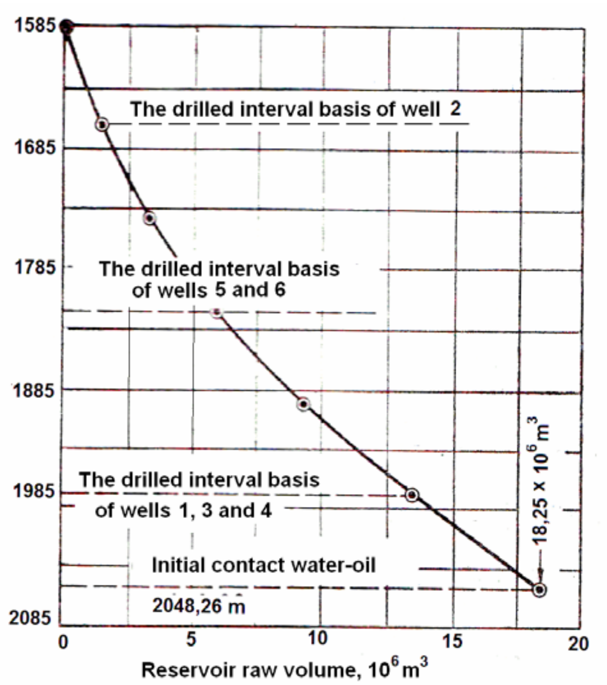

Fig. 3. Reservoir raw volume versus thickness

$$p_n - p_{n+1} = \frac{p_{n-1} + p_n}{2} - \frac{p_n + p_{n+1}}{2} = \frac{1}{2}\left(p_{n-1} - p_{n+1}\right). \tag{24}$$

c) To establish the producing wells during the next forecast period.

d) To estimate the cumulative oil production N_{pn} by extrapolating the $N_p(t)$ curve obtained from the realized or forecasted performance.

e) To evaluate the oil saturation s_{one} using the relations

$$s_{one} = V_{on}/V_{ozn}, \tag{25}$$

$$V_{on} = b_o\left(N - N_p\right) - s_{o0}\left(W_e - W_p\right)/\left(1 - s_{oe} - s_{wi}\right), \tag{26}$$

$$V_{ozn} = \left[N b_{oi}/\left(1 - s_{wi}\right)\right] - \left(W_e - W_P\right)/\left(1 - s_{oe} - s_{wi}\right), \tag{27}$$

where V_{on} is the oil volume in the un-flooded area, W_p – the volume of water produced through wells, and V_{ozn} – the volume of the pores in the un-flooded area.

f) To estimate the average productivity index I_p, during the forecast period for the productive wells using the relation

$$\frac{I_{p1}}{I_{p2}} = \left(\frac{k_o}{\mu_o b_o}\right)_1 \Big/ \left(\frac{k_o}{\mu_o b_o}\right)_2. \tag{28}$$

g) To calculate the cumulative oil production N_{pn} as a function of productivity index calculated above, average value of differential pressure ($p_m - p_s$) and number of producing wells.

If the calculated N_{pn} does not correspond to N_{pn} estimated in step d), the calculation will be repeated between steps d) and g) until reaching an acceptable concordance.

h) To calculate the gas cumulative production G_{pn} as a function of s_{one} saturation, using the equation

$$G_{pn} = G_{p(n-1)} + \left[\left(R_n + R_{n-1}\right)/2\right]\left(N_{pn} - N_{p(n-1)}\right), , \tag{29}$$

with

$$R_n = R_{sn} + \left(\frac{k_g\,\mu_o\,b_o}{k_o\,\mu_g\,b_g}\right)_n \ . \tag{30}$$

i) To calculate the cumulative water influx W_e using the material balance equation (1) completed with the term corresponding to the elastic expansion of interstitial water and reservoir rock, written as

$$\begin{aligned} W_e = N_p\,b_0 + b_g\left(G_p - N_p\,R_s\right) + G\left(b_g - b_{gi}\right) - N\left[b_o - b_{oi} + \left(R_{si} - R_s\right)b_g + \right.\\ \left. + \left(s_{wi}\,\beta_w + \beta_r\right)\Delta p\,b_{oi}/\left(1 - s_{wi}\right)\right]. \end{aligned} \tag{31}$$

j_1) To pass to the next forecast period if $W_{en} = W_e$.

j_2) To repeat the steps a)…i), taking a higher pressure for the first step a), if $W_{en} > W_e$.

j_3) To repeat the steps a)…i), taking a lower pressure for the first step a), if $W_{en} < W_e$.

When applying this procedure, the following suppositions and results are obtained:

a) It is supposed that, for the production time t = 500 days, the pressure at the initial water–oil contact will be p_2 = 15.858 MPa, corresponding to position 2 in Table 1.

b) According to relation (14), for time t = 500 days we get the dimensionless time $\bar{t}$ = 176.49 which corresponds to $\overline{W}$ = 68.4 for $\bar{r}_e = \infty$ in figure 1. Thus, from equation (23) we obtain

$$W_{e2} = 0.00519 \cdot \frac{1}{2}\left(17.237 - 15.858\right)\cdot 10^6 \cdot 68.4 = 244{,}769.7\ \mathrm{m}^3\ .$$

c) The reservoir volume V_{zi} in the flooded area is given by the relation

$$V_{zi} = \frac{W_e - W_p}{m\left(1 - s_{o0} - s_{wi}\right)}\ , \tag{32}$$

which gets the value

$$V_{zi} = \frac{244{,}769.7}{0.25\left(1 - 0.40 - 0.22\right)} = 2{,}576{,}523\ \mathrm{m}^3\ .$$

The un-flooded area volume V_{zn} is expressed as

$$V_{zn} = V_z - V_{zi} \tag{33}$$

where, according to Figure 3, the reservoir volume is $V_z = 18.25\cdot 10^6$ m^3.

Figure 3 also shows that the un-flooded region's volume is higher than the volume of the region situated above the first well line. Therefore, we can say that, during the first 500 production days, all six wells are productive and, from relation (33), V_{zn} = 15,673,477 m^3 is obtained.

d) In absence of the dependence $N_p(t)$, we suppose that, after 500 days, the average flow rate per well decreases from the initial value Q_1 = 119.25 m^3/day to Q_2 = 119.25/3 m^3/day. Therefore, the cumulative oil production after 500 exploitation days has the estimative value of

$$N_{p2} = \left[\left(119.25 + 119.25/3\right)/2\right]\cdot 6 \cdot 500 = 238{,}500\ \mathrm{m}^3\ .$$

e) Out of the relations (25)…(27) it yields

$$V_{pn} = 1.311(2.6855 - 0.2385)\cdot 10^6 - 0.4\cdot 244{,}769.7/(1-1.4-0.22) = 2{,}950{,}365 \text{ m}^3 ,$$

$$V_{pzn} = 2.6855\cdot 10^6 \cdot 1.325/(1-0.22) - 244{,}769.7/(1-0.4-0.22) = 3{,}917{,}776 \text{ m}^3 ,$$

$$s_{one} = 2{,}950{,}365/3{,}917{,}776 = 0.7531 .$$

f) Using Wyllie's equation for unconsolidated well sorted sands according to their granulation, written as

$$k_{ro} = s^{*3} , \quad k_{rg} = (1-s^*)^3 , \tag{34}$$

with

$$s^* = s_{one}/(1-s_{wi}) , \tag{35}$$

we get $s^* = 0.9655$ and $k_{ro} = 0.90$. Then, from equation (28) for

$$I_{P1} = Q/(p_i - p_s) , \tag{36}$$

$p_i = 17.273$ MPa and $p_s = 14.8238$ MPa, the value $I_{P2} = 7.3796\cdot 10^{-5}$ m^3/(Pa·day) is obtained.

g) The cumulative oil production after 500 exploitation days is given by the relation

$$N_p = \lfloor Q_1 + I_{p2}(p_2 - p_s) \rfloor n t/2 , \tag{37}$$

which leads to $N_p = 247{,}193$ m^3 for $n = 6$ wells. Because $N_{p\ calc} = 247{,}193 \text{ m}^3 > 238{,}500 \text{ m}^3$, we consider $N_{p2} = 247{,}193$ m^3 and steps e, f and g are repeated. The results are summarized here: e) $V_{pn} = 2{,}938{,}968$ m^3, $s_{one} = 2{,}938{,}968/3/917{,}776 = 0.7502$; f) $s^* = 0.96179$, $k_{ro} = 0.88969$ and $I_{p2} = 4.3295\cdot 10^{-5}$ m^3/(Pa·day); g) $N_p = 246{,}041 \text{ m}^3 < 247{,}193 \text{ m}^3$ is obtained. Therefore, we have to repeat steps e…g as follows: e) $V_{pn} = 2{,}940{,}478$ m^3, $s_{one} = 0.7505$; f) $s^* = 0.9622$, $k_{ro} = 0.8909$, $I_{p2} = 4.3354\cdot 10^{-5}$ m^3/(Pa·day); g) $N_p = 246{,}133 \text{ m}^3 \approx 246{,}041 \text{ m}^3$.

h) By approximating the basis of the layer with an isosceles triangle, admitting that $\rho_o = 800$ kg/m^3 and $\rho_w = 1{,}000$ kg/m^3, and taking into consideration that the thickness of the layer is $h = 80.8$ m, the average reservoir pressure at the time $t = 500$ days is estimated with the relation

$$p_m = p_2 - g\left[\rho_w \Delta h + \frac{1}{3}\rho_o(h_{wo} - h_c - \Delta h) + \frac{1}{2}\rho_o h \cos\alpha\right] , \tag{38}$$

where the lifting distance of the water–oil contact $\Delta h \approx 60$ m was read from Figure 3 and $h_c = 1{,}591$ m was taken from Figure 2, for $\alpha = 53.8°$. Therefore, from equation (38) we obtain $p_m = 14.573563$ MPa. The second expression (34) leads to $k_{rg} = 5.4\cdot 10^{-5}$, then equations (30) and (29) give the values

$$R_2 = 104.75 + \frac{5.4\cdot 10^{-5}}{0.8909}\cdot\frac{0.399\cdot 1.2973}{5.92\cdot 10^{-3}\cdot 0.005068} = 105.795 \text{ m}_\text{N}^3/\text{m}^3 ,$$

$$G_{p2} = 0 + [(105.795 + 115.76)/2](246{,}133 - 0) = 27{,}265{,}988 \text{ m}^3 .$$

i) From equation (31) we have

$$W_e = 246{,}133\cdot 1{,}297 + 0.005068\cdot(27{,}265{,}998 - 246{,}133\cdot 104.75) - 2.6855\cdot 10^6 \cdot$$
$$\cdot[1.297 - 1.325 + (115.76 - 104.75)\cdot 0.005068 + (0.22\cdot 4.35 + 5.8)\cdot 10^{-10}\cdot$$
$$\cdot(17{,}237{,}000 - 15{,}858{,}040)\cdot 1.325//(1-0.22)] = 244.947 \text{ m}^3 .$$

j1) As $W_{e2} - W_e = -247$ m^3, we consider $W_{e2} \cong W_e$ and the solution of this case study is $W_e = 244{,}900$ m^3, $N_p = 246{,}000$ m^3, $p_m = 14.57$ MPa and $G_p = 27{,}266{,}000$ m_N^3.

The case study achieved in this way can be improved by using process data gathered at the beginning of the forecast period and applying the imposed corrections.

Conclusions

Generally speaking, hydrocarbon reservoirs can be found in the vicinity of aquifers with dimensions big enough so that the elastic volume growth energy of the water–rock system plays an important part within the respective reservoir exploitation.

It is possible that values of between 70% and 80% of the oil recovery factor are obtained in some hydrocarbon reservoirs associated with active aquifer.

For estimating the performance of a hydrocarbon reservoir under the substantial action of an aquifer's elastic expansion, one can use the macroscopic material balance equation, when calculating the water cumulative influx evolution means solving the system of fundamental equations of water unsteady influx, system made up of Darcy's equation, continuity macroscopic equation and state equation for water as compressible liquid. This system of fundamental equations can be reduced to an equation containing partial derivatives of parabolic type under the conditions of two-dimensional influx.

The solution of the pressure's parabolic equation by using Laplace transformation, allowed the determination of pressure and water influx variation laws, in radial plane and one-dimensional systems.

The case study illustrated in this paper shows the application of a procedure which establishes the performance of a hydrocarbon reservoir associated with an active aquifer, considering that the average reservoir pressure can decline under the saturation pressure value.

The results of the procedure can be improved by using process data gathered before the beginning of the forecast period, along with the introduction of the imposed corrections.

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Cercetări privind estimarea performanței unui zăcământ de țiței cu acvifer activ

Rezumat

Zăcămintele de hidrocarburi asociate cu acvifere care dispun de capacități însemnate de destindere elastică pot fi exploatate astfel încât să se obțină valori mari ale factorului final de recuperare. Pentru aceasta este necesar, pe de o parte, să se identifice acțiunea acviferului în faza inițială a exploatării zăcământului, iar pe de altă parte să se estimeze corect volumul cumulativ al apei de influx. Lucrarea prezintă o metodă de estimare a performanței unui zăcământ de țiței cu cap de gaze şi acvifer activ.